Vibration Testing of Small Satellites

This series of papers provides a tutorial along with guidelines and recommendations for vibration testing of small satellites. Our aim with these papers is to help you (a) ensure the test meets its objectives in demonstrating flight worthiness and (b) avoid test failures, whether associated with a design deficiency or with excessive loading during test. Addressed are sine-burst testing, random vibration testing, and low-level diagnostic sine sweeps. Although much of the guidance provided in this series applies to CubeSats, the series is primarily aimed at satellites in the 50 – 500 lb (23 – 230 kg) range. Most of the guidance applies to larger satellites as well if they will be tested on a shaker.

The plan is for this series to include seven parts, each of which will be released when completed:

1. Introduction to Vibration Testing (released April 11, 2014; last revised July 19, 2017)
2. Test Configuration, Fixtures, and Instrumentation (released April 11, 2014)
3. Low-level Sine-Sweep Testing (released May 13, 2015)
4. Sine-Burst Testing (released April 28, 2017; last revised July 19, 2017)
5. Random Vibration Testing (released April 7, 2016; last revised July 19, 2017)
6. Notching and Force Limiting (released May 13, 2015)
7. Designing a Small Satellite to Pass the Vibration Test (yet to be released)

The most recent versions of these papers are available for free download at

http://instarengineering.com/vibration_testing_of_small_satellites.html.

Part 4: Sine-Burst Testing

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4.1 Introduction

In a ***sine-burst test***, the shaker introduces sinusoidal acceleration at the base of the test article, with gradually increasing, then maintained, and finally decreasing amplitudes, all at constant frequency. The sinusoidal frequency is selected to be significantly lower than the fundamental vibration frequency of the test article (when in test configuration) in order to minimize dynamic response or amplification of the acceleration. In this way, we subject the test article to near-uniform acceleration.

The objective of a sine-burst test is to verify that the satellite's ***primary structure***—the body structure and the structure carrying loads from the launch vehicle (LV) interface—has enough strength to withstand the maximum expected loads during launch, which we refer to as ***limit loads***. The test may be run to some factor above limit loads, depending on the test philosophy or criteria.

Limit loads for a small satellite are typically defined either as quasi-static loads[1] or as ***load factors***, which are multiples of weight on Earth and which represent quasi-static loads. Quasi-static loads are often defined in g units; however, inertia loads act in a direction opposite that of the acceleration and thus are actually of opposite sign from the acceleration. A load factor, which has no units, also represents inertia load. The distinction between signs of acceleration and inertia load is not important in a sine-burst test because acceleration is reversible as the shaker moves back and forth.

[1] The difference between a static load and a quasi-static load is that a ***static load*** is a steady-state external load (e.g., force), typically associated with static equilibrium, in which there is no acceleration, whereas a ***quasi-static load*** is a steady-state (or near-steady-state) internal (inertia) load resisting uniform acceleration. In comparison, a ***dynamic load*** varies with time and either causes or is associated with vibration.

A sine-burst test is conducted over short duration in order to avoid unnecessary fatigue damage to the structural materials; a sine-burst test is not intended as verification of fatigue life. GEVS (Ref. 1) requires at least five cycles at full acceleration. Figure 4-1 shows an example, with full acceleration at 12 *g*.

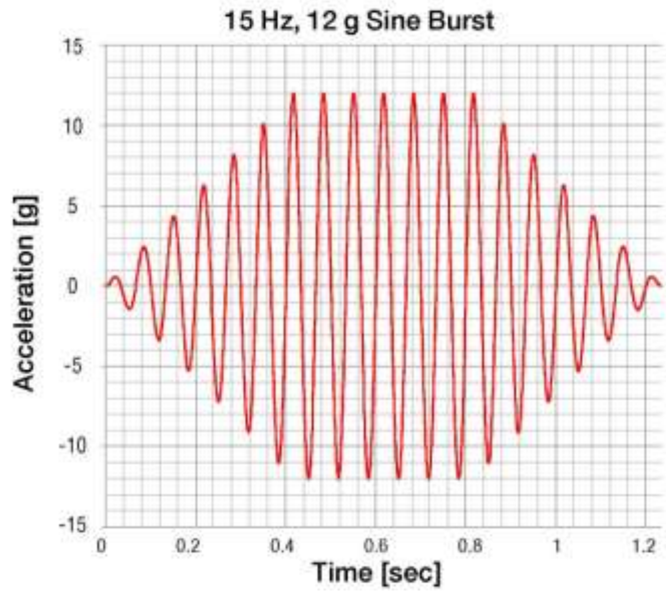


Fig. 4-1. Example Input Acceleration for a Sine-Burst Test.

A sine burst may be the preferred method of strength testing if …

– the structure will be on a shaker for other vibration tests (e.g., random vibration), which makes it convenient to do a sine-burst test as well;

– the primary structure may not be adequately stressed by the other vibration tests;

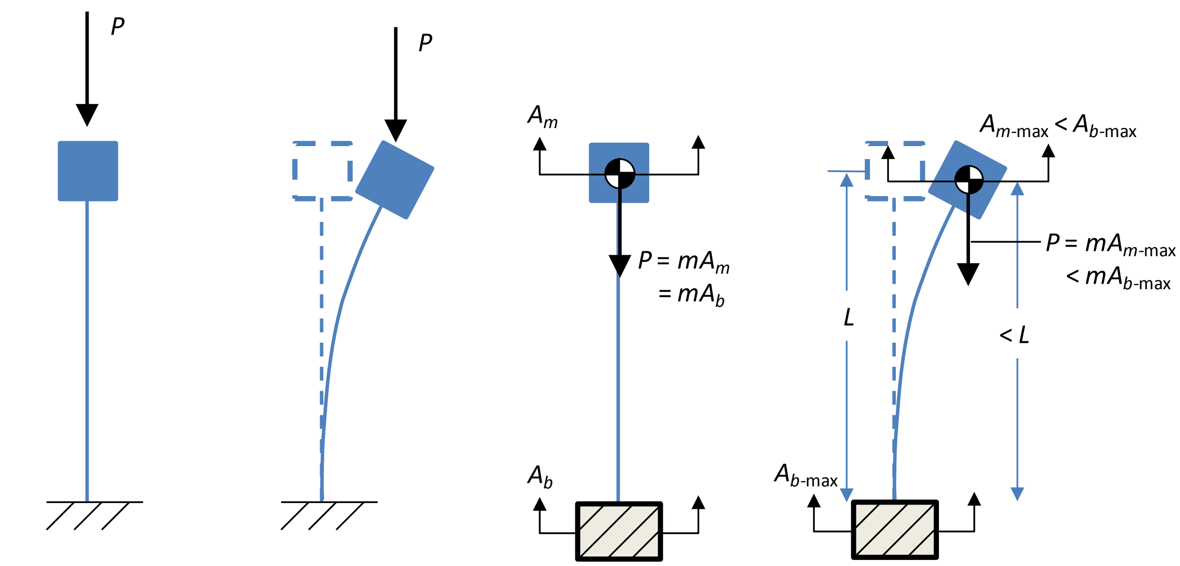
– and the potential modes of failure in the primary structure can be adequately screened out by cyclic loading from its base, making static or quasi-static loading unnecessary. (See upcoming discussion of limitations of sine-burst testing in Sec. 4.2.)

If you choose to verify strength with a sine-burst test, we recommend doing the test prior to vehicle integration, with mass simulators in place of flight components and harness. Not only is this approach smart from a risk-reduction standpoint, as noted in Part 1 of this paper series, but it also makes sense when considering test effectiveness. In a sine-burst test of an integrated spacecraft, some of the mass associated with wires, cables, and thermal blankets ("floppy" masses) won't move with the shaker as intended. The frequency at which we do the test may be low relative to the structure's fundamental frequency, but probably not low enough to fully accelerate the "floppy" masses. Adding the mass of harness and blankets to component mass simulators (making the simulators slightly heavier than the simulated components) helps ensure all the mass accelerates as intended. Still, even with that approach, not all of the mass may be adequately loaded for the reasons discussed below.

4.2 Limitations of Sine-Burst Testing

Ironically, even though the objective is to verify strength, a sine-burst test is limited in its ability to detect strength deficiencies. Some people refer to a sine-burst test as a "static loads test," but it's not a static loads test. A centrifuge test, in which the peak acceleration is maintained to cause quasi-static loading, is much more like a static loads test. A structure that would fail a static or quasi-static loads test may withstand a sine-burst test that has been designed to introduce the same load. This is because the loading in a sine-burst test is introduced by time-varying base acceleration rather than by sustained discrete or inertia loads.

Example 4-1: Consider a mass supported by a long, slender column, as shown in Fig. 4-2. The shaker moves vertically with sinusoidal acceleration. When the compressive inertia load on the mass nears its peak intended level, the column buckles. Such buckling is not necessarily catastrophic. Once the column starts to buckle, it no longer provides a stiff load path between the shaker and the mass. Because of relative vertical displacement between the mass and the shaker, the peak acceleration of the mass is less than the peak acceleration of the shaker. Accordingly, the column may not fully collapse and thus may withstand the test. In comparison, a static load applied to the mass remains constant once buckling begins and thus normally causes the column to collapse. Figure 4-3 shows a hypothetical example of what a plot of measured response acceleration in a sine-burst test might look like when the structure is buckling without collapsing.



a. Static load, P, applied to mass, m.

b. Column buckles. The static load then typically causes the column to collapse.

c. Column under base acceleration, A_b, as a sinusoidal function of time. If the column is rigid, $A_m = A_b$.

d. Column buckles. Because of relative displacement between mass and base, peak mass acceleration $A_{m\text{-max}}$ is less than peak base acceleration $A_{b\text{-max}}$.

Fig. 4-2. Buckling of a Column in a Sine-burst Test. A column that would buckle and collapse under a static load may not collapse under a sinusoidal base acceleration that is intended to cause the same peak load.

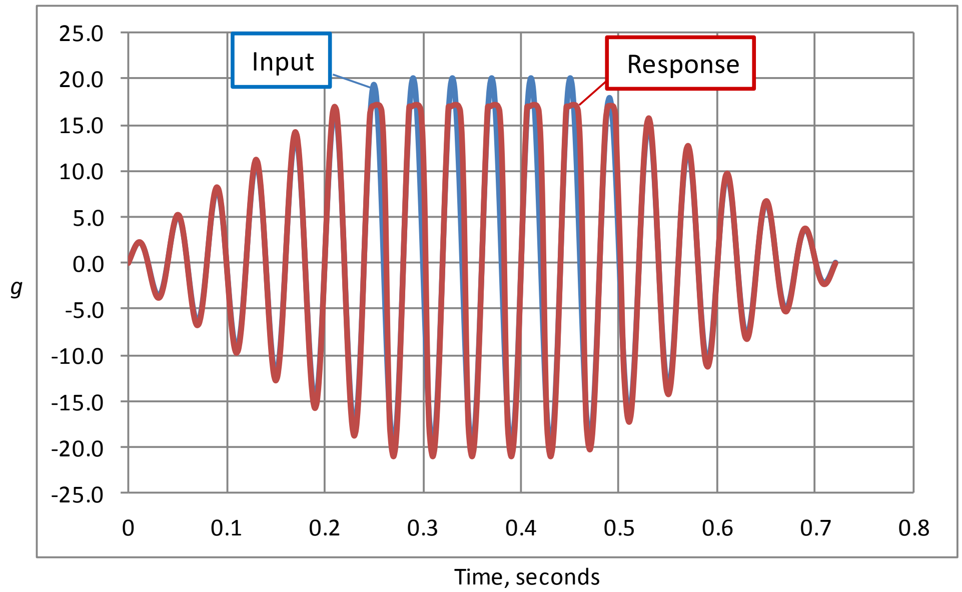


Fig. 4-3. What the Response Might Look Like if the Structure Starts to Buckle. If any mass within the test article does not see acceleration that is at least as high as the input acceleration, part of the structure may be buckling or yielding. Here, the structure may be buckling at about 17 *g*.

Similarly, ultimate failure of a material (rupture) that may occur in a static loads test may not occur in a sine-burst test. In a ductile material, yielding precedes rupture, essentially introducing compliance between the shaker and the mass. Such yielding may prevent the mass from seeing the full input acceleration, with the difference depending on how much plastic deformation occurs.

Example 4-2: The column shown in Fig. 4-2 is shortened so that buckling is not a concern; let's now call this structure a "post" rather than a "column" because the latter term implies buckling concerns. The post is made of a ductile aluminum alloy, such as 6061-T6, with a constant cross section. The static tensile loads that cause the material to begin to yield over the full length of the post and to rupture are 3000 lb and 3900 lb, respectively. The mass at the top of the post weighs 400 lb, the post itself has no mass in this hypothetical case, and the shaker imparts a 10 *g* sine burst at a frequency that is low relative to the post's fundamental axial frequency. (We'll ignore gravitational loading in this example.)

Our first thought is that the post will rupture in test because it will see a load of 10 x 400 = 4000 lb, which exceeds the post's ultimate strength. However, in such a case, yielding over the full length of the post would begin once the input acceleration hits 7.5 *g* (7.5 x 400 = 3000) and would continue until the shaker hits its peak acceleration of 10 *g*. The material may not rupture because the post becomes much less stiff as the material yields, which means the mass does not see the full 10 *g* acceleration, much like with buckling in Example 4-1. When the acceleration reverses in sign and nears its peak level, the material yields in compression. The next loading cycle causes the same sequence of tensile then compressive yielding. Such cyclic yielding is very damaging to the material in a fatigue sense, but the column may pass the test with no sign of rupture—without the mass seeing the full 10 g required acceleration.

Example 4-3: The post in Example 4-2 is strengthened, either by use of a stronger material or by increasing its cross-sectional area, which increases its onset-of-yield strength to 5000 lb. However, the bolts that attach the post at its base have combined onset-of-yield strength and ultimate strength of 3000 lb and 3900 lb, respectively. In this case, it's far less likely that the post assembly would pass a sine-burst test to 10 *g* than would the post in Example 4-2. In other words, for this example, the sine-burst test becomes more like a static loads test in terms of its ability to cause a deficient structure to fail. This is because, even though the bolt material yields before rupturing, the bolts are short relative to the length of the column. Yielding occurs over a very small length, and the failure exhibits little ***apparent ductility***, which we define here as plastic displacement prior to rupture as a percentage of the post's length. Thus, the mass should see nearly the full target acceleration, up to the point at which the bolts break.

We referred in Example 4-3 to a "deficient structure." But are the structures in Examples 4-1 and 4-2 actually deficient for a design ultimate load of ±10 *g*? Clearly they are deficient if the load is static or quasi-static, but launch is a dynamic event. Some of the load during launch is quasi-static, such as from steady-state engine thrust, and some is dynamic, such as from random vibration. As a result, neither test—static loads nor sine burst—is truly flight-like. Because of the reasons explained in the above examples, a static loads test tends to be overly conservative—i.e., more severe, strength-wise, than the flight loading—whereas a sine-burst test tends to be unconservative.

This is the main reason that relatively large spacecraft should be tested with static or quasi-static loads. As a launch-vehicle payload gets more massive, a higher percentage of loading for its primary structure is quasi-static during launch. (Steady-state loads are the same for all items during launch, regardless of mass, whereas dynamic decrease with mass; it takes more energy to accelerate a larger mass.) On the other hand, the majority of launch loading in the primary structure of most small satellites is dynamic[2], which makes sine-burst testing more realistic.

Unfortunately, designing and performing a proper static loads test or centrifuge test can be expensive, especially if there are no suitable test facilities nearby. Such tests may require unique fixtures and instrumentation. In contrast, a small satellite will be tested on a shaker for random vibration; it's quite easy to add a sine burst while in test configuration. Cost and schedule considerations normally drive small-satellite programs to do all structural testing on shakers.

4.3 Pass/Fail Criteria

To compensate for the possibility that a sine-burst test doesn't adequately encompass the effects of launch loading, for the reasons noted above, we need pass/fail criteria for ultimate strength that are more stringent than simply ensuring the test article withstands the test without rupture or collapse. For us to confidently state that withstanding the sine-burst test means the structure will not rupture or collapse during the mission, we should make sure that all of the items of relatively high mass in the test article actually see the target (required) acceleration, as opposed to the hypothetical example shown in Fig. 4-3, or that the total interface load or moment, as measured by force gages, achieve the target loads. If the test does not include force gages at the mounting interface, we should put an accelerometer on each relatively high-mass component, as well as at various locations on the primary structure, and add the pass/fail

[2] An exception can be a small satellite that has a highly damped structure or that is mounted on a vibration-isolation system.

criterion that the measured acceleration at each accelerometer location is at least as high as the required acceleration.

Failure to meet this criterion on the first attempt at a sine-burst test does not necessarily mean the test article failed the test. We should investigate the reason that a particular location in the structure failed to see the target acceleration. It may be that the accelerometer malfunctioned or had the wrong calibration factor. If the data is deemed valid and ultimate failure does not seem imminent, we could try repeating the test with higher input acceleration to see if the response at the location being investigated hits the target acceleration. If so, even though yielding or the onset of buckling may have occurred within the structure, we may conclude that the structure passed the criteria for ultimate strength. However, if this is a flight structure rather than one dedicated for test, we'll need to be sure that any yielding that may have occurred at the full test load is not detrimental.

4.4 Designing the Sine-Burst Test Environment

The launch vehicle contractor (LVC) or a customer organization typically specifies limit loads and maximum predicted launch environments for small satellites, along with test factors and margins applicable to these loads and environments, but the satellite developer is responsible for ensuring the satellite is adequately tested. The specified random vibration environment, with the appropriate margin (e.g., 3 dB) included, is the same as the test environment, but the same is not always true for the specified quasi-static loads. Such loads are often specified as acting in multiple axes simultaneously, whereas a sine-burst test is done one axis at a time. In such a case, the satellite developer must design a sine-burst test that will meet the objective of verifying strength for the combined multi-axis loads.

The sine-burst test environment is defined by the following:

- Axis
- Acceleration
- Frequency

Table 4-1 defines a process for designing these aspects of a sine-burst test.

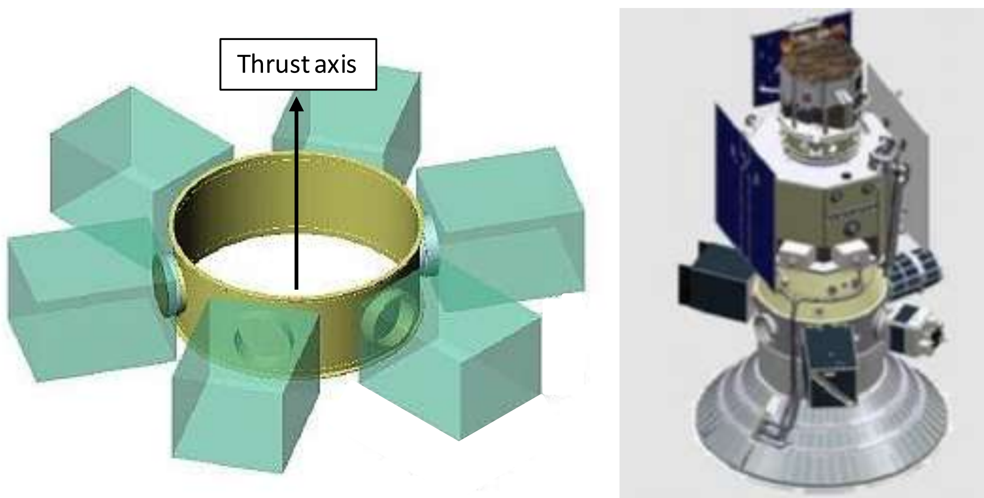
Table 4-1. Process for Designing a Sine-Burst Test Environment.

Step	Where discussed
1. Establish the test axes relative to the test article's coordinate system.	Sec. 4.4.1
2. Derive the target acceleration for the test article.	Sec. 4.4.1
3. Select the sine-burst frequency.	Sec. 4.4.2
4. Adjust the input acceleration to account for dynamic amplification.	Sec. 4.4.3

4.4.1 Establishing the Test Axes and Deriving the Target Acceleration for the Test Article

Let's discuss axis and acceleration together, using the 2007 STP-1[3] mission as an example.

The STP-1 mission was the first flight of the ESPA[4], with five auxiliary payloads (formerly referred to as "secondary payloads"), each of which was a satellite weighing less than 400 lb, and one empty ESPA port. Orbital Express was the primary payload, mounted on top of the ESPA. (Figure 4-4 shows a generic configuration in which all six ports are populated with auxiliary payloads along with the STP-1 configuration.) For this mission, limit loads for the auxiliary payloads were specified as follows: +/- 8.5 *g* in the launch vehicle's thrust axis acting simultaneously with 8.5 *g* acting in any LV lateral axis. Each of these five spacecraft was required to be strength tested to 1.25 times limit loads.



A. ESPA with Six Auxiliary Payloads B. STP-1 Configuration

Fig. 4-4. ESPA Configurations. (Image credit: AFRL.)

With loads specified in this manner, clearly it would not be adequate to do a sine-burst test at 1.25 x 8.5 = 10.6 g, one axis at a time. Doing so would not stress the primary structure as much as the specified combined loads. To simulate the specified loads in test, we would have to clock the spacecraft at 45° relative to the shaker's axis and then test to 1.25 x 12 = 15 *g*, where 12 *g* is the magnitude of the resultant load vector of 8.5 *g* acting in two axes simultaneously ($\sqrt{(8.5)^2+(8.5)^2}=12$). This may be easy to do for a lateral test on the slip table, but not for a vertical test, which would require a relatively expensive, wedge-like fixture and a difficult procedure for setting up the test configuration (mounting the spacecraft to the 45° wedge).

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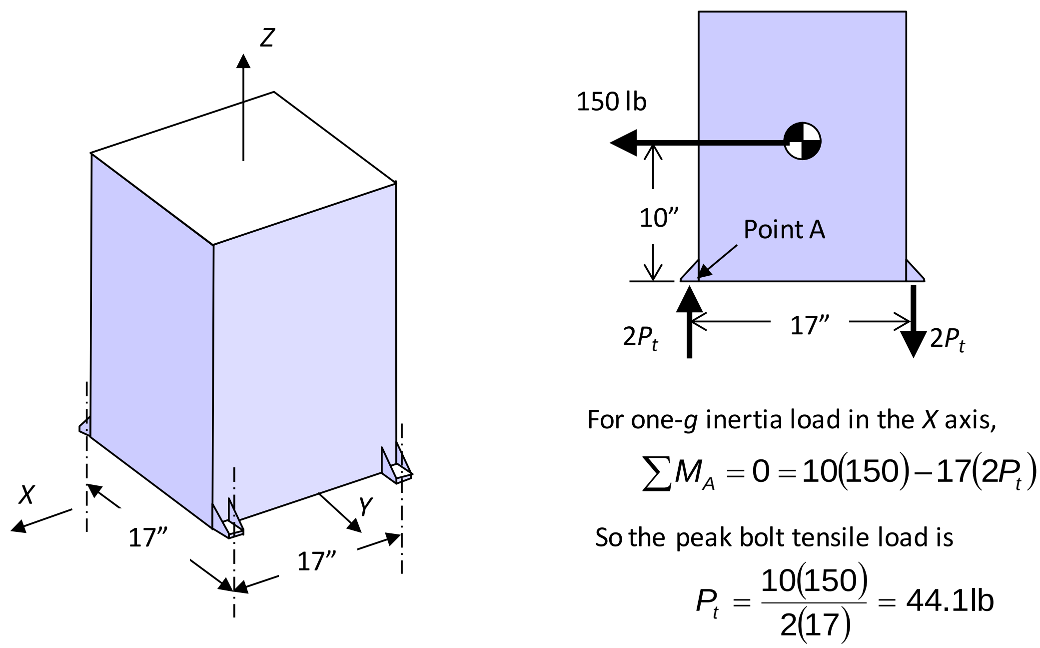
[3] STP stands for Space Test Program, which is part of the Department of Defense.

[4] ESPA stands for EELV Secondary Payload Adapter, where EELV is Evolved Expendable Launch Vehicle. The ESPA has six equally spaced ports, each with a 15"-diameter bolt circle to serve as the interface for an auxiliary payload. The primary payload for a mission typically mounts on top of the ESPA, with the separation mechanism (or a structural adapter to the mechanism) bolting to the upper 62"-diameter bolt circle. Multiple ESPAs can be stacked, in which case there may not be a primary payload, which in turn means none of the payloads is "auxiliary."

For the STP-1 program, several of the auxiliary-payload developers initially planned to do sine-burst tests to 10.6 *g*, one axis at a time. Recognizing that such a test would be inadequate, the program added a requirement that the test be done to 15 *g*, one axis at a time. There was no requirement to clock the spacecraft at different 45° angles relative to the test axis, so the specified test level of 15 *g* did not necessarily envelop the effects of the specified limit loads with a 1.25 factor, as shown in Example 4-4. Regardless, the program decided the test would adequately envelop the actual flight loads.

Example 4-4: Consider the box shown in Fig. 4-5, which is mounted to its support structure by four bolts that make up a square bolt pattern. Here, the inertia load from 1 *g* acceleration in the *X* axis causes 44.1 lb of tensile load in each of two of the mounting bolts. One of those two bolts also sees a 44.1 lb tensile load from 1 *g* acceleration in *Y*. If this box has limit loads defined as described above, with 8.5 *g* in each axis simultaneously, the limit (maximum) tensile load in the most highly loaded bolt is 8.5(44.1 + 44.1) = 750 lb. If we do the sine-burst test to 12 *g* in the *X* and *Y* axes, one axis at a time, the bolt would see a tensile load of 15(44.1) = 529 lb in each test. To adequately test the square bolt pattern with sine bursts in *X* and *Y* separately, we'd have to increase the acceleration to 8.5 + 8.5 = 17 *g* (17(44.1) = 750 lb). With a 1.25 test factor, the test acceleration becomes 1.25(17) = 21.3 *g*, as compared with 15 *g* used for the STP-1 program.



a. Geometry and coordinate system. b. Bolt tensile load for one *g* in *X*.

Fig. 4-5. Calculation of Bolt Tensile Load for One-*g* Inertia Loading for Example 4-4.

For the STP-1 mission, the program authority specified the test loads, but spacecraft developers should not assume this will be the case for every mission. As stated previously herein, it's the responsibility of the spacecraft developer to design a test to meet its objectives, starting with the as-specified limit loads. Let's look at an effective process for deriving the target acceleration.

When the limit quasi-static loads are specified in a manner similar to how they were for the STP-1 mission, discussed above, with loads acting simultaneously in multiple axes, but the test will be in one of those axes at a time, we recommend the process defined in Table 4-2 for deriving the appropriate test acceleration for each axis.

Table 4-2. Process for Designing the Target Acceleration for the Test Article.

Step	Comments
a. Identify all the potentially critical modes of failure in the structure that will be tested, along with the ***indicator parameters*** that relate to them.	In Example 4-4, we used bolt tensile load (discounting preload) as an indicator parameter for certain failure modes that could occur in the joint. To ensure you fully test the structure, identify indicator parameters at multiple locations within the structure, corresponding to different potential failure modes.
b. Calculate values for the indicator parameters for the specified limit loads. These values become the ***target loads*** for designing the test.	In Example 4-4, the target bolt load was 750 lb for the test, prior to inclusion of the 1.25 test factor.
c. Identify the test axis that would most efficiently achieve the target load for each of the indicator parameters.	In Example 4-4, we used the lateral axes, *X* and *Y*. A simple check shows that loading in these axes is more severe for the bolt than loading in the *Z* axis: 1 *g* load in *Z* causes a bolt load of 150/4 = 37.5 lb, which is less than the 44.1 lb caused by 1 *g* in *X* or *Y*.
d. Derive the acceleration required in each axis to achieve all the target loads efficiently, and then multiply that acceleration by the test factor (1.25 in Example 4-4). Remember to account for the 1-*g* bias from gravity, if significant.	This derived acceleration in the *X* and *Y* axes, including the test factor, was 21.3 *g* in Example 4-4, when ignoring the 1 *g* bias in *Z*, which causes 37.5 lb of compressive load at each bolt location. To negate the 1 *g* effects, with a target bolt tensile load of 1.25(750) = 937 lb, the required acceleration in *X* or *Y* is (937 + 37.5)/44.1 = 22.1 *g*.

The acceleration derived in each test axis, with the process described in Table 4-2, is the target uniform acceleration of the test article. However, this is not necessarily the acceleration we want the shaker to impart. Depending on the frequency we select for the sine-burst test (Sec. 4.4.2), we may want to reduce the input acceleration in order to achieve the target response acceleration without excessive loading (Sec. 4.4.3).

4.4.2 Selecting the Sine-Burst Frequency

Two main considerations drive the decision regarding sine-burst frequency: the desire to minimize dynamic amplification and the shaker's ***stroke***, which is its range of motion. (A third consideration, which relates to harmonic frequencies, is addressed in Section 4.5.)

To understand the first consideration, refer back to the transmissibility function, as defined in Eq. 1.1. This equation is repeated here.

$$T_R(f_{\text{ratio}}) = \sqrt{\frac{1+(2\zeta f_{\text{ratio}})^2}{\left[1-f_{\text{ratio}}^2\right]^2+(2\zeta f_{\text{ratio}})^2}} \tag{1.1}$$

where $T_R(f_{\text{ratio}})$ is the ratio of peak response acceleration to peak sinusoidal input (base) acceleration as a function of f_{ratio}—which is the ratio of forcing (input) frequency, f, to natural frequency, f_n, of a mass-spring system—and ζ is the damping ratio. Figure 4-6, which originally appeared as Fig. 1-2 in Part 1 of this paper series, shows plots of transmissibility for various damping ratios.

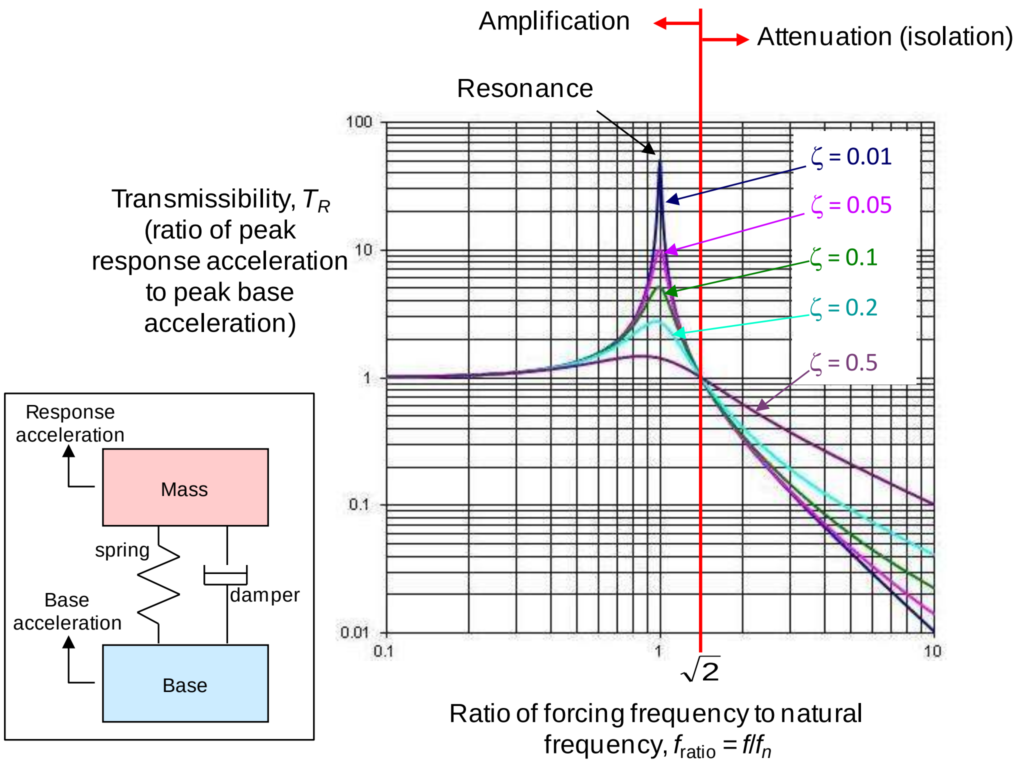


Fig. 4-6. Transmissibility.

A small satellite in test configuration ("test article") is not a mass on a spring, so Eq. 1.1 doesn't apply directly, but the equation is representative of the expected average ***dynamic gain*** (ratio of peak response acceleration to peak input acceleration), a.k.a. ***dynamic amplification***, associated with the test article's fundamental mode of vibration. To achieve near-uniform acceleration and negligible dynamic gain for the test article, we want the ratio of forcing frequency to fundamental frequency to be as low as possible. For example, if the fundamental frequency of the spacecraft is 50 Hz, we'd like the shaker to impart sinusoidal motion at, say, 5 Hz, which gives a ratio of 0.1. As the plot shows, at that ratio, regardless of damping, response acceleration is nearly the same as input acceleration. In other words, the test article should move as a rigid body with the shaker, without dynamic amplification. Unfortunately, our shaker probably doesn't have enough stroke to achieve the desired acceleration at 5 Hz. As frequency drops, the displacement needed to cause a particular sinusoidal acceleration increases.

For an object moving with sinusoidal translation, velocity and acceleration also change as sinusoidal functions of time. Recall that velocity and acceleration are the first and second time derivatives of displacement, respectively. Thus,

$$\text{Displacement, } \delta = x(t) = \delta_{max} \sin(\omega t) \tag{4.1}$$

$$\text{Velocity, } V = \dot{x}(t) = \omega \delta_{max} \cos(\omega t) \tag{4.2}$$

$$\text{Acceleration, } A = \ddot{x}(t) = -\omega^2 \delta_{max} \sin(\omega t) \tag{4.3}$$

where δ_{max} is the peak displacement, t is time, and ω is the frequency in radians per second. Figure 4-7 shows plotted values for one-inch maximum sinusoidal displacement at 0.25 Hz.

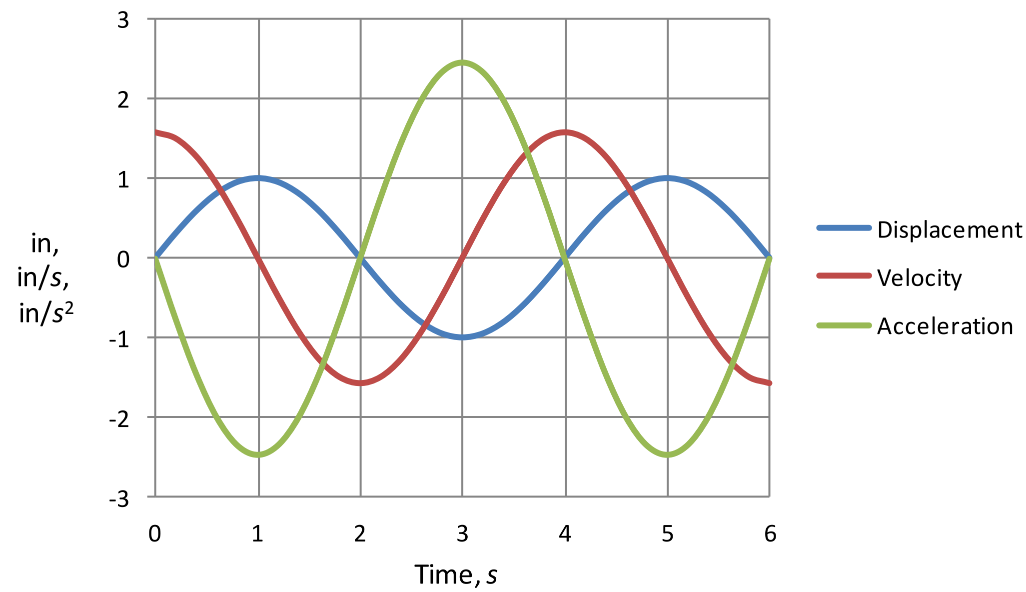


Fig. 4-7. Displacement, Velocity, and Acceleration for Sinusoidal Motion with One-Inch Peak Displacement. The natural frequency is 0.25 Hz for the system used in this example.

The above equations apply for sinusoidal motion of a shaker. From Eq. 4.3, the maximum acceleration is

$$A_{max} = \omega^2 \delta_{max} \tag{4.4}$$

Note that the negative sign in Eq. 4.3 falls out of Eq. 4.4 because $\delta_{max} = -\delta_{min}$.

Converting frequency from rad/s to Hz,

$$A_{max} = (2\pi f)^2 \delta_{max} \tag{4.5}$$

where f is the frequency in Hz of the sine-burst test.

Thus, the shaker's stroke dictates how low in frequency we can go with a sine-burst test in order to achieve a desired input acceleration. We should provide some margin to ensure we don't run out of stroke in test, so let's count on only 90% of the advertised stroke (unless the test lab recommends something less). Setting the target input acceleration to A_{max} and the zero-to-peak stroke to δ_{max}, we can rearrange Eq. 4.5 and insert a 0.9 factor to solve for the minimum test frequency, f, in Hz:

$$f = \frac{1}{2\pi}\sqrt{\frac{A_{max}}{0.9\delta_{max}}} \tag{4.6}$$

We would then round up to the next higher integer value for the test frequency.

Example 4-5: The shaker's stroke is two inches, peak to peak, which is fairly common for shakers used to test small spacecraft, which means δ_{max} is one inch. If we need to achieve a maximum input acceleration of 15 *g* in test, with *g* = 386 in/s^2, the minimum frequency for the test is

$$f = \frac{1}{2\pi}\sqrt{\frac{15(386)}{0.9(1.0)}} = 12.8\,\text{Hz}$$

Rounding up to an integer, the selected test frequency is 13 Hz for this example.

Figure 4-8 shows a plot of Eq. 4.6 for a shaker with a two-inch stroke, peak to peak, for maximum acceleration from 5 to 30 *g*. Table 4-3 presents the data in a different format, showing the maximum acceleration and a suggested allowable acceleration, with the 0.9 reduction factor, for each integer increment of frequency.

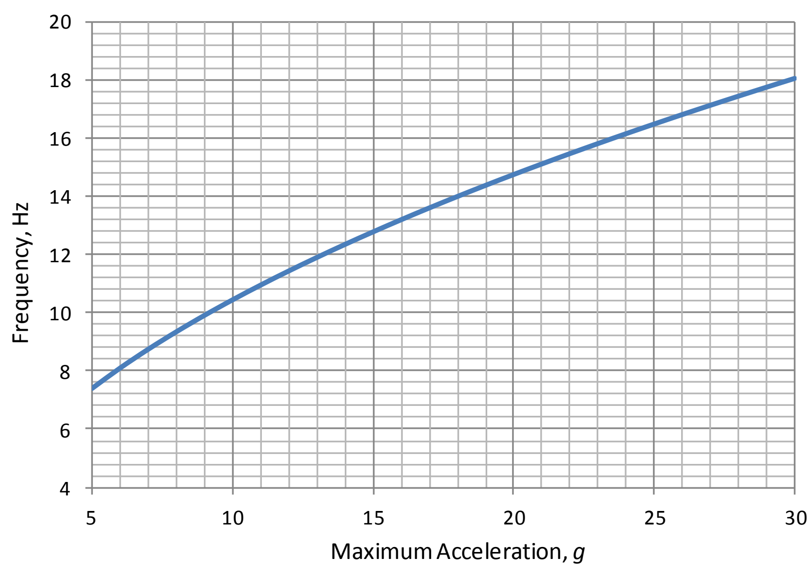


Fig. 4-8. Recommended Minimum Sine-Burst Frequency vs. Desired Maximum Input Acceleration for a Shaker with a Two-Inch Stroke, Peak to Peak. This plot includes the 0.9 reduction factor, per Eq. 4.6.

Table 4-3. Maximum Acceleration vs. Sinusoidal Frequency for a Shaker with a Two-Inch Stroke, Peak to Peak. As indicated by Eq. 4.4, the maximum acceleration at a given frequency is proportional to the shaker's stroke. We recommend using the last column, which is 90% of the theoretical maximum shown in the second column, as an upper limit unless you are told otherwise by the test lab personnel. Note that the peak acceleration a shaker can achieve at any frequency is limited by the shaker's force rating, as discussed in Part 1 of this series, so you might not be able to get the higher acceleration values shown in this table.

f, Hz	A_{max}, g	A_{allow}, g
5	2.56	2.30
6	3.68	3.31
7	5.01	4.51
8	6.55	5.89
9	8.28	7.46
10	10.2	9.20
11	12.4	11.1
12	14.7	13.3
13	17.3	15.6
14	20.0	18.0
15	23.0	20.7
16	26.2	23.6
17	29.6	26.6
18	33.1	29.8
19	36.9	33.2
20	40.9	36.8

4.4.3 Adjusting the Input Acceleration to Account for Dynamic Amplification

Based on the selected test frequency, we can estimate the dynamic amplification of the test article using the transmissibility equation (Eq. 1.1). Table 4-4 shows transmissibility for various damping ratios and values of f_{ratio}.

Recognize that the transmissibility equation (1.1) applies when the sinusoidal input continues long enough for the system to reach equilibrium, which is when, for each cycle, the energy dissipated by damping is equal to the energy introduced by the input acceleration. In a sine-burst test, such equilibrium is not usually reached because of the limited number of cycles involved in the test. Thus, the actual dynamic gain in a sine-burst of a mass on a spring would be somewhat less than predicted by Eq. 1.1. The difference is typically negligible unless f_{ratio} is near one, which is not the intent with a sine-burst test; at f_{ratio} = 1 for a system with low damping, the difference can be quite significant.

Example 4-6: We plan to do a 15-Hz, 10 *g* sine-burst test for a structure with a fundamental frequency of 30 Hz (f_{ratio} = 0.5) and a damping ratio of 2.5% (Q = 20). We would expect a peak response acceleration, as calculated at the test article's center of gravity, of about 1.33 x 10 = 13.3 *g*.

Table 4-4. Transmissibility, T_R, for Different Damping Ratios and Ratios of Forcing Frequency to Natural Frequency. These values apply to a mass-spring system but can be used as a first-cut estimate of dynamic gain at a spacecraft's fundamental frequency. Notice that, if f_{ratio} is less than about 0.60, T_R is insensitive to damping within the damping range covered in this table.

Damping ratio, ζ:	0.01	0.025	0.05	0.075	0.10
Approximate Q :	50	20	10	6.67	5
$f_{\mathrm{ratio}} = f/f_n$	T_R	T_R	T_R	T_R	T_R
1.00	50.01	20.02	10.05	6.74	5.10
0.80	2.78	2.76	2.72	2.65	2.57
0.60	1.56	1.56	1.56	1.55	1.55
0.50	1.33	1.33	1.33	1.33	1.33
0.40	1.19	1.19	1.19	1.19	1.19
0.35	1.14	1.14	1.14	1.14	1.14
0.30	1.10	1.10	1.10	1.10	1.10
0.25	1.07	1.07	1.07	1.07	1.07
0.20	1.04	1.04	1.04	1.04	1.04
0.15	1.02	1.02	1.02	1.02	1.02
0.10	1.01	1.01	1.01	1.01	1.01

When the dynamic amplification is significant, which is the case in Example 4-6, we should decrease the input acceleration to avoid overtesting the structure. However, doing so, while still ensuring the test is adequate, is not always straightforward.

In Example 4-6, with target acceleration of 10 g and expected response of 13.3 g for 10 g input, we can't simply reduce the input to 10/1.33 = 7.5 g because the response acceleration isn't actually uniform, like it is for T_R, which applies to a mass on a spring. The response acceleration varies throughout the test article according to the shape of the excited mode(s). Often, we can convince ourselves the test will adequately verify strength of the primary structure by ensuring the required base force and base moment (loads at the mounting interface) are achieved in test. To do so, we need to either measure the base loads directly or understand the fundamental mode shape well enough to transform measured acceleration to base loads.

As an example, a rocking mode, which is the first lateral mode for most spacecraft when base mounted (from the separation interface), has more acceleration at the top than at the bottom. With the assumption that the structure rocks as a rigid body about its base, we can easily estimate the CG acceleration by interpolating from measured acceleration at top and bottom. But the total base moment is a function not only of the translational acceleration, but also of the angular acceleration. Part 5 of this paper series includes an example of estimating base moment from response data associated with a rocking mode; that example shows how we can significantly underestimate the base moment if we neglect the contribution of angular acceleration.

Using one of the methods described in Parts 5 and 6 of this paper series to determine the base loads, we can tailor the input for a sine-burst test to account for dynamic amplification. We do this by running the test at lower levels, starting at perhaps -12 dB, calculating or measuring the base loads, adjusting the input acceleration to achieve the target base loads, and ramping up in 3 dB increments.

4.5 Potential Problems with Controlling a Sine-Burst Test

Based on our experience, a sine burst is the most difficult environment for a shaker to control. The intended sinusoidal input often turns out to be something other than sinusoidal, and the test article can see acceleration that is considerably higher than expected. The following case histories are examples of problems encountered during sine-burst tests.

Example 4-7—Case History: Consider the acceleration time history shown in Fig. 4-9. This input acceleration was intended to be sinusoidal at 20 Hz in a lateral test of a small satellite structure whose fundamental rocking mode was approximately 62 Hz. Unfortunately, the base input, as measured by the control channel, included 60-Hz acceleration as well. The test started at -12 dB from full level and then ramped up in 3-dB increments, with data monitored between runs. All looked well for the low-level runs, even the -3 dB run. But, at full level, the 60-Hz spikes shown in Fig. 4-9 appeared in the acceleration measured by the control channel. Figure 4-10 shows the acceleration measured at the top of the test article during this test.

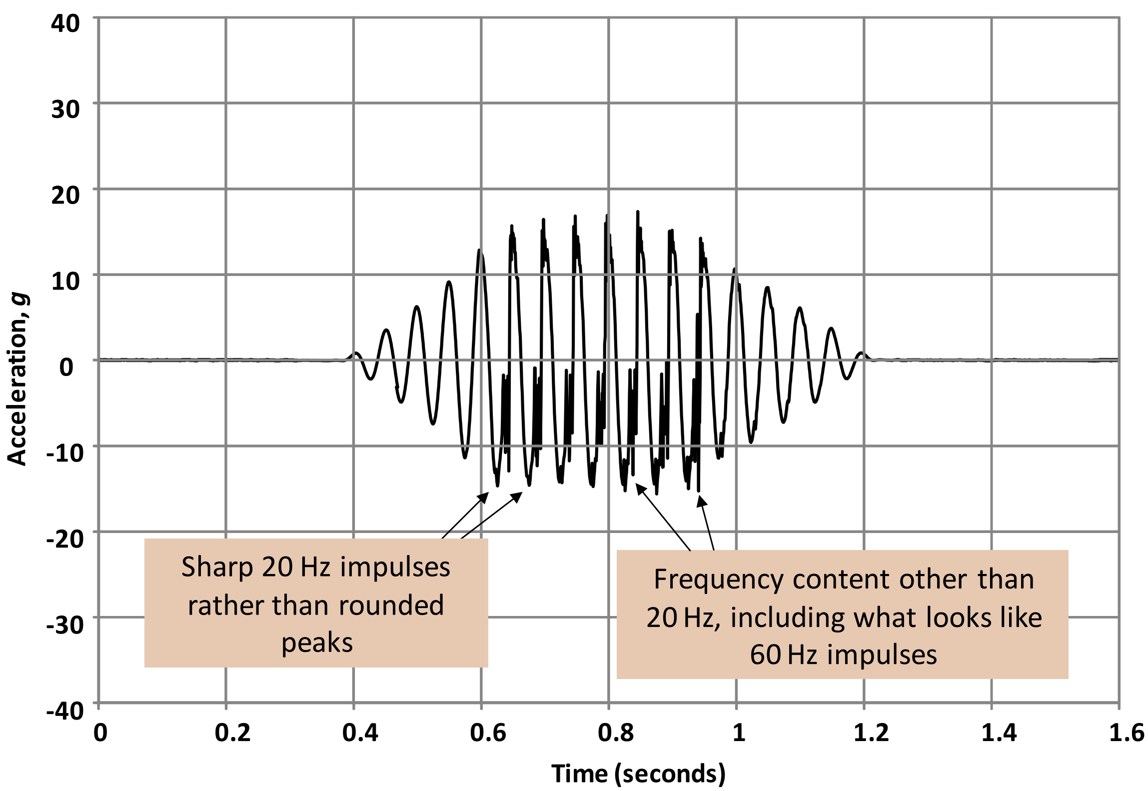


Fig. 4-9. Time History of Acceleration Measured by the Control Channel During a Sine-Burst Test. The input was intended to be sinusoidal, with 14 *g* peak at 20 Hz, aiming at 15 *g* equivalent uniform loading of the test article when accounting for dynamic amplification, but the control channel measured other superimposed frequencies of acceleration as well, including significant levels at what appears to be 60 Hz, based on the plotted spacing of the peaks. The digital data was not available to confirm this observation.

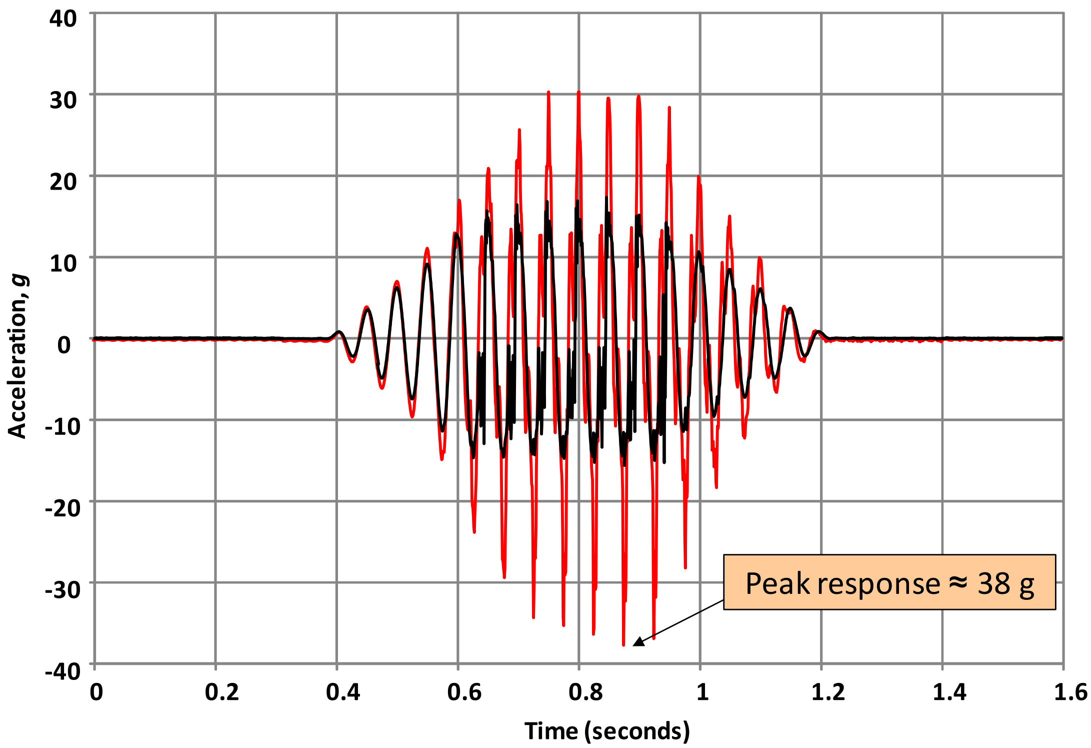


Fig. 4-10. Response Measured at the Top of the Spacecraft Structure, Superimposed With the Input Acceleration from Fig. 4-9. The 60-Hz input excited the 62-Hz response mode, causing 38 *g* peak response as compared to a target of 15 *g* uniform acceleration.

After the test that yielded the data shown in Fig. 4-10, we speculated as to the cause of the problem and concluded there were three likely contributors:

1. As test levels increased, the spacecraft's rocking frequency most likely decreased. Natural frequencies often drop with amplitude of excitation as a result of decreased stiffness and increased damping, both of which are expected in bolted joints to some extent. At full test levels, the rocking frequency may have decreased from 62 Hz to 60 Hz, whereas in the -3 dB run the frequency may have been still high enough for the problem not to occur.
2. 60 Hz is an odd multiple (the third harmonic) of 20 Hz vibration.
3. Another possible contributor to the problem was electrical noise at the test facility. The frequency for the AC power supply was 60 Hz, as is typical in North America (50 Hz in most other locations).

Regarding item 2 from the above list in Example 4-7, an odd-numbered harmonic can be excited easily by intended sinusoidal input if the input degrades from a pure sinusoid. As shown in Fig. 4-11, with even-numbered harmonics, the peaks are of the same sign in one cycle but are of opposite sign for the next cycle, whereas, with odd-numbered harmonics, each peak of the low-frequency input is of the same sign as the peak of the higher-order harmonic. As a result, if anything causes the input acceleration to vary from a pure sinusoid, the test article can see high 60-Hz response.

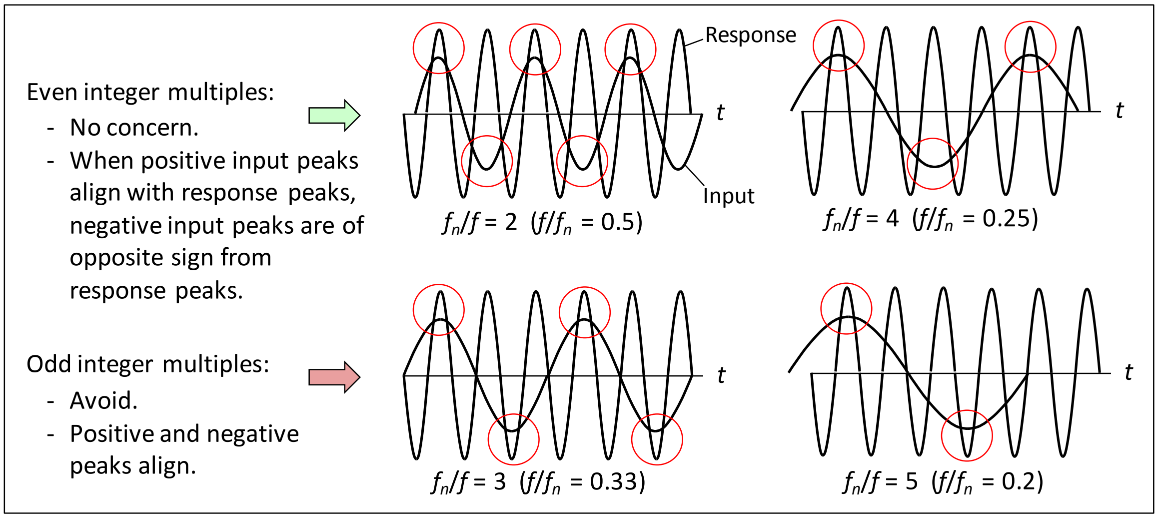


Fig. 4-11. Why Odd-numbered Harmonics Are More Likely to Be Excited than Even-numbered Ones.

Recommendations stemming from the above observations: Avoid a sine-burst frequency …

- for which the frequency of electrical noise is an odd-numbered harmonic; e.g., avoid 20 Hz input when the electrical noise is at 60 Hz.
- for which the test article's fundamental frequency is an odd-numbered harmonic.

Example 4-8—Case History: Figure 4-12 shows a different type of problem encountered during a sine-burst test. Here, the intended acceleration was 35 g, and the test article's fundamental frequency was 182 Hz. The 126 g response was attributed to an impulse or sudden stop of the shaker caused by failure of an amplifier. (We've speculated that a similar response spike may occur if a shaker hits the stops when trying to exceed its stroke.)

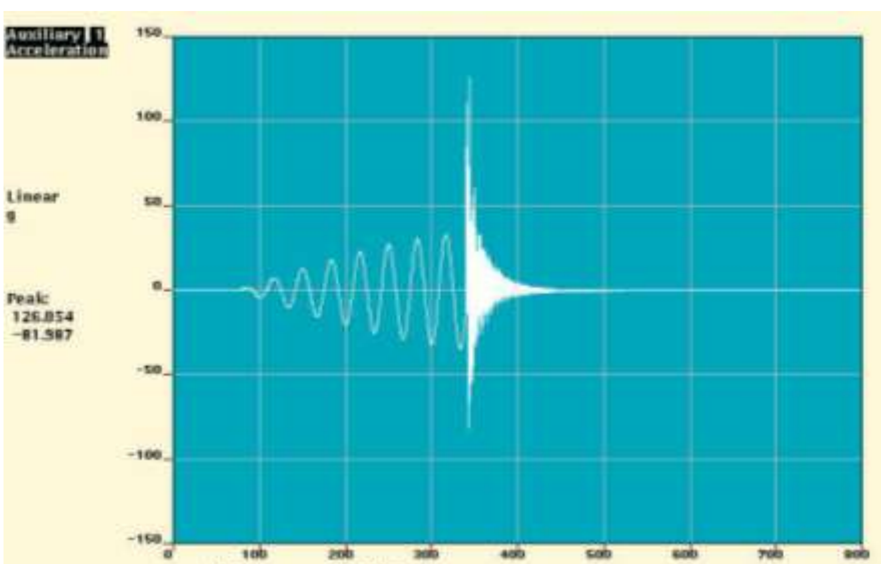


Fig. 4-12. Response Measured at the Top of a Test Article During a Failed Attempt at a Sine-Burst Test. The high sudden response was attributed to failure of the shaker's amplifier. Fortunately this test was conducted on a low-cost engineering model of a spacecraft structure!

Example 4-9—Case History: Overloading of the HESSI Spacecraft in a Sine-Burst Test

In March 2000, NASA's High-Energy Spectroscopic Imager (HESSI) was significantly overloaded during an intended sine-burst test at Jet Propulsion Laboratory, causing over $1M damage to the spacecraft. The test was in the lateral configuration. ***Stiction*** (static friction) in the shaker system prevented free motion and led to an overtest. The shaker was misaligned, partly the result of broken trunnion bearings, and the slip plate rubbed against the granite table. No dry run (validation test) was performed ahead of time, and the pretest self-check of the system was inadequate. Figure 4-13 shows HESSI on the shaker, prior to test. Reference 2 reports on the anomaly investigation; it also provides good descriptions of other things that can go wrong with shaker operation.

Fig. 4-13. High Energy Solar Spectroscopic Imager (HESSI) Spacecraft on Shaker. This photo was taken at JPL in March 2000 shortly before the sine-burst test anomaly that did over one million dollars of damage to the spacecraft. In the initial low-level (-12 dB) run, the slip table did not move freely and then suddenly let loose, with approximately 21 g acceleration, whereas the target acceleration was 1.88 g. (Image courtesy NASA Jet Propulsion Laboratory, from Ref. 3)

A clear conclusion from the above case histories is that sine-burst testing is risky. This is especially true for large test articles, which typically may not have as much structural margin as, say, a CubeSat.

4.6 Summary: Recommendations Regarding Sine-Burst Testing

- Before deciding to do a sine-burst test, consider other tests for verifying strength.
  - Random vibration testing may be sufficient, especially for spacecraft and components weighing less than about 50 lb (<22 kg mass). This may not be the case, however, if you force-limit the test per the guidance provided in Part 6 of this series.
  - A carefully designed sine sweep, aiming for a target response, may be easier for the shaker to control and thus safer for the test article. Ask the opinion of the lab personnel.
  - If dynamic loading on a shaker won't adequately test the structure for certain potentially critical modes of failure, such as buckling, consider doing a static loads test or a centrifuge test instead.
  - For larger and more costly test articles, it is usually better to do a static loads test or a centrifuge test instead of a sine-burst test so that the test is more effective and to avoid risk associated with sine-burst testing.
- If you decide to do a sine-burst test, plan to do it prior to spacecraft integration, with mass simulators mounted on the primary structure in place of flight components. That way, if something goes wrong—shaker being unable to properly control the environment or structure failing during test—you don't incur risk of damaging the flight components.
- Dry run the test before subjecting flight hardware to the environment, preferably with a mass simulator in place of the flight hardware.
- Finally, we repeat a recommendation from above regarding sine-burst frequency: Avoid a sine-burst frequency for which the test article's fundamental frequency is an odd-numbered harmonic; e.g., avoid 15 Hz input when the test article's fundamental frequency is at or near 45 Hz

References

1. GEVS: GSFC-STD-7000A. "General Environmental Verification Standard for GSFC Flight Programs and Projects." NASA Goddard Space Flight Center. April 22, 2013.
2. Worth, Daniel B., NASA Goddard Space Flight Center, and Rodney N. Phillips, NASA Marshall Space Flight Center. "Results of the HESSI Test Mishap Investigation." November 29, 2000.
3. Jet Propulsion Laboratory. "Shake and Break: The HESSI Overtest Mishap." JPL Case Study, July 2014.

