Structural Design and Analysis

for Aerospace Engineers

Course Book

Last revised November 2024

Developed and taught by

Thomas P. Sarafin
Instar Engineering and Consulting, Inc.
tom.sarafin@instarengineering.com

Offered jointly by

Instar Engineering and Consulting, Inc.
Highlands Ranch, Colorado
(303) 903-9581
Email: tom.sarafin@instarengineering.com
www.instarengineering.com

Applied Technology Institute
349 Berkshire Drive, Riva, Maryland 21140
Toll Free: (888) 501-2100 • Local: (410) 956-8805
Email: ati@aticourses.com
www.aticourses.com

See next page for copyright notice and restrictions, acknowledgements, and disclaimer

Tom Sarafin, President and Chief Engineer
Instar Engineering and Consulting, Inc.
tom.sarafin@instarengineering.com

If you want the full experience—with oral explanations, class exercises, relevant stories from the teacher's experience, and ability to ask questions—you will need to take the course.

Acknowledgements

This course book draws heavily from the book *Spacecraft Structures and Mechanisms, from Concept to Launch*, 1995 [Sarafin, Thomas P., editor]. Credit for material used herein especially goes to the following contributors to that book: Leonard J. Demchak, Peter (Poti) G. Doukas, Harold R. Feldman, Walter F. Habicht, Todd A. Harmel, Alfred Herzl, Robert J. Heymans, Kelly D. Lewis, John L. MacCoun, James R. McCandless, Donald L. Palmer, Robert Von Sabin, and Robert G. Wendt.

I wish to acknowledge and thank Scott Malaznik for his detailed review and comments, particularly on Sec. 4. Section 5 includes contributions from Mike Browning, Poti Doukas, and John MacCoun. Section 6 herein draws heavily from an unpublished manuscript called "Right for Flight: The Structural and Architectural Design of Machines that Fly," written in the late 1980s by O. P. Harwood.

Disclaimer

Structural Design and Analysis

for Aerospace Engineers

3-day course

Objectives:
- Strengthen your understanding of …
  - statics, dynamics, and mechanics of materials, and how to apply them
  - strength analysis, including buckling
  - fatigue and how to assess life
  - structural design with a systems engineering approach and the use of analysis
- Make you a better engineer!

Audience: Structural and mechanical design engineers, stress analysts, and others interested in the topic

Course Developer and Teacher: Tom Sarafin, Instar Engineering

- Engineering consultant, structures and mechanical systems
- President and chief engineer of Instar; founded Instar in 1993
- Has consulted for NASA, DARPA, DOD Space Test Program, Lockheed Martin, DigitalGlobe (Maxar), U.S. Air Force Academy, Sierra Nevada Corp (Sierra Space), Planetary Systems Corp (Rocket Lab USA), and many other organizations
- Key member of the team that developed NASA-STD-5020 for design, analysis, quality assurance, and verification of threaded fastening systems
- At Martin Marietta, Denver, (now Lockheed Martin) from 1979 to 1993 (stress analysis, preliminary structural design, structural test engineering, verification planning; technical lead; and project manager)
- Editor and principal author of the book, *Spacecraft Structures and Mechanisms: From Concept to Launch* [1995]. Contributing author to *Space Mission Analysis and Design* (1st, 2nd, and 3rd editions) [1991, 1992, & 1999] and *Human Spaceflight: Mission Analysis and Design* [1999]
- Has taught over 300 short courses (2 to 4 days) to more than 6000 engineers and managers
  - SCS—Spacecraft Structures, from Concept to Launch (formerly SMS—Space Mission Structures)
  - SDA—Structural Design and Analysis for Aerospace Engineers
  - DABJ—Design and Analysis of Bolted Joints for Aerospace Engineers
  - STDI—Structural Test Design and Interpretation
  - VTSS—Vibration Testing of Small Satellites
  - TenP—Ten Principles for Successful Space Programs

Contents

Detailed Course Outline

1. Structural Requirements and Design Criteria
   - Structural requirements: what they are and what they are not
   - Typical structural functions and constraints
   - How flight loading environments affect the structure
   - Standards and criteria
   - Top-level criteria for strength analysis
   - Other commonly used structural design criteria
2. Statics and Dynamics
   - Static equilibrium and free-body diagrams
   - Benefits of a statically determinate (kinematic) interface
   - Examples and class problems
   - Dynamic equilibrium
   - Modes of vibration
   - The equation of motion
   - Transmissibility
   - Mode shapes
   - Applying what we learned
   - Appendix: introduction to random vibration

Detailed Course Outline

3. Mechanics of Materials
   – Stress and strain
   – Combined state of stress
   – Principal stresses and Mohr's circle
   – Beams and bending stress
   – Unsymmetrical bending
   – Torsion and the effects of warping constraint
   – Thermal effects

Detailed Course Outline (continued)

4. Strength Analysis, Including Buckling
   – An important thing to understand
   – Accounting for variation in material strength: allowable stresses
   – Revisiting the margin of safety
   – Failure theories for materials
   – Failure in practice and the benefits of ductility
   – Understanding stress analysis from the engineer's perspective
   – Common pitfalls
   – An effective process for strength analysis
   – Failure modes for fastened joints
   – Class exercise: identifying potential failure modes
   – Forms of buckling
   – Elastic buckling of columns
   – Modified Johnson Parabola, inelastic buckling, and eccentric loading
   – Buckling of plates and shells

Detailed Course Outline (continued)

5. Fatigue Assessment
   - What is fatigue?
   - Brief history of fatigue failures and ensuing research
   - Stress concentration factor
   - Terms defining a loading cycle
   - Fatigue analysis process
   - Quantifying fatigue life by test
   - Presentation of fatigue data
   - High-cycle vs. low-cycle fatigue
   - Miner's rule
   - Life (scatter) factor and fatigue analysis factor
   - Fatigue notch factor and notch sensitivity factor
   - The Goodman method and equivalent alternating stress
   - Linear-elastic fracture mechanics and fracture control
   - Generating a loading spectrum

Detailed Course Outline (continued)

6. Structural Design
   - Opening thoughts on structural design
   - Material selection
   - Types of structures and important things to understand when designing them
     - Beams
     - Trusses and frames
     - Forms of lightweight panels and shells
     - Monocoque and semi-monocoque cylinders
     - Skin-stringer and panel-frame structures
   - Methods of attachment
   - Design of bolted joints
   - Reducing cost by reducing the number of parts
   - Designing an adaptable structure
   - Summary: Structural design guidelines from a master

Summary

References

1. Sarafin, Thomas P., ed. 1995. *Spacecraft Structures and Mechanisms: From Concept to Launch*. Torrance, CA: Microcosm, Inc. (referred to as "SSAM" herein) (available at astrobooks.com)
2. NASA-STD-5020B. August 6, 2021. "Requirements for Threaded Fastening Systems in Spaceflight Hardware."
3. Boresi, Arthur P., and Richard J. Schmidt. 2002. *Advanced Mechanics of Materials*, 6th ed. Wiley.
4. Young, Warren C., and Richard G. Budynas. 2002. *Roark's Formulas for Stress and Strain*, 7th ed. McGraw-Hill.
5. a. MMPDS-08. April 2013. *Metallic Materials Properties Development and Standardization* (MMPDS). National Technical Information Service.

   b. MIL-HDBK-5J. January 2003. *Metallic Materials and Elements for Aerospace Vehicle Structures*. Department of Defense.

In 2003, MMPDS (an FAA document) replaced MIL-HDBK-5 (a USAF document), starting with the same content and format. MMPDS has been revised periodically since then.

MMPDS is copyright protected (and costs money), whereas MIL-HDBK-5 is not (and is in the public domain at no charge).

Thus, this course presents plots copied from MIL-HDBK-5 rather than from MMPDS.

continued

References, continued

6. Ryan, Robert S. “Lessons Learned in Engineering.” NASA Marshall Spaceflight Center.
7. “Columbia Accident Investigation Board, Report Volume I”, August 2003. NASA and the Government Printing Office.
8. Bruhn, E. F. 1973. *Analysis and Design of Flight Vehicle Structures*. Indianapolis, IN: S. R. Jacobs & Associates, Inc. (available at www.jacobspublishing.net)
9. NASA Technical Memorandum NASA TM X-73305. August 1975. “Astronautic Structures Manual.” (Out of print but available in pdf format through search at ntrs.nasa.gov/search.jsp)
10. McGinty, Bob. continuummechanics.org/eccentriccolumnbuckling.html.
11. Gerard, George, and Herbert Becker. NACA TN 3781. 1957. “Handbook of Structural Stability, Part I, Buckling of Flat Plates.” National Advisory Committee for Aeronautics.
12. Gordon, J. E. *Structures, or Why Things Don’t Fall Down*. 1978. Da Capo Press.
13. Pilkey, Walter D., and Deborah F. Pilkey. 2008. *Peterson’s Stress Concentration Factors*, 3rd ed. John Wiley & Sons, Inc.
14. JSC 65828, Rev. B. July 15, 2014. “Structural Design Requirements and Factors of Safety for Spaceflight Hardware, for Human Spaceflight.” NASA Johnson Space Center.
15. SSP 30559, Rev. C. September 29, 2000. “Structural Design and Verification Requirements, International Space Station.” NASA Johnson Space Center, European Space Agency, National Space Development Agency of Japan, and Italian Space Agency.
16. Shigley, Joseph E. and Charles R. Mischke. 1989. *Mechanical Engineering Design*, 5th ed. McGraw-Hill.

continued

References, continued

17. Boyer, Howard E., ed. *Atlas of Fatigue Curves*. ASM International.
18. Shahzad, M., et al. March 2010. "Influence of Anodizing Process on Fatigue Life of Machined Aluminum Alloy." Elsevier Ltd.
19. Rateick, R. G., Jr, et al. August 1996. "Effect of Hard Anodize Thickness on the Fatigue of AA6061 and C355 Aluminum." Journal of Materials Science Letters.
20. Peery, David J. 1950. *Aircraft Structures*. Original publisher: McGraw-Hill. Reprinted in 2011 by Dover Puplications, Inc.
21. Rolfe, Stanley T. and John M. Barsom. 1977. *Fracture and Fatigue Control in Structures*. Prentice-Hall, Inc.
22. MIL-STD-1530C. November 1, 2005. "Aircraft Structural Integrity Program (ASIP)," Department of Defense Standard Practice.
23. NASA-STD-5019A. February 26, 2016. "Fracture Control Requirements for Spaceflight Hardware."
24. NASA-STD-5009B. May 8, 2019. "Nondestructive Evaluation Requirements for Fracture-Critical Metallic Components."
25. Clough, R.W. and J. Penzien. 1975. *Dynamics of Structures*. McGraw-Hill.
26. Wertz, James R. and Wiley J. Larson. 1999. *Space Mission Analysis and Design*, 3rd ed. Microcosm.
27. AIAA S-110-2005. "Standard: Space Systems—Structures, Structural Components, and Structural Assemblies." 12 July 2005. American Institute of Aeronautics and Astronautics.

continued

References, continued

28. SMC-S-016. Space and Missile Systems Center Standard. "Test Requirements for Launch, Upper-Stage, and Space Vehicles." 5 September 2014. Air Force Space Command.
29. DOD-HDBK-343. Military Handbook. "Design, Construction, and Testing Requirements for One of a Kind Space Equipment." 01 February 1986. Department of the Air Force.
30. NASA-STD-5001B. "Structural Design and Test Factors of Safety for Spaceflight Hardware." 08-06-2014.
31. Letter from USAF Col. Norman Lee to Glynn Lunney, JSC Manager of Shuttle Payload Integration. February 29, 1980.
32. Lang, George Fox and Philip Van Baren. "How Well Does 3 Approximate ∞? Understanding +/-3σ Clipping in Random shake Tests." Sound & Vibration Magazine, March 2009. (http://www.sandv.com/downloads/0903lang.pdf
33. Sarafin, Thomas P. "DABJ—Design and Analysis of Bolted Joints for Aerospace Engineers" course book. March 2024. (Available for free download at https://instarengineering.com/resources.html)
34. NASA CR-124075 Rev. A. February 1973. "Isogrid Design Handbook."
35. Gerard, George, and Herbert Becker. NACA TN 3781. "Handbook of Structural Stability, Part I—Buckling of Flat Plates." July 1957. National Advisory Committee for Aeronautics.
36. Seide, P., V.I. Weingarten, and E. J. Morgan, Space Technology Lab. "The Development of Design Criteria for Elastic Stability of Thin Shell Structures." 1960.

Other Sources of Information

- Malaznik, Scott. Multiple educational slide packages (especially for buckling and crippling analysis) for free download: https://collieraerospace.com/historical-aerospace-analysis-methods/
- Shigley, Joseph E. and Charles R. Mischke. 1986. *Standard Handbook of Machine Design*. McGraw-Hill.
- Maddux, Gene E., et al. August 1969. "Stress Analysis Manual." Air Force Flight Dynamics Laboratory. https://apps.dtic.mil/sti/pdfs/AD0759199.pdf
- "Structures Analysis Manual." April 1988. General Dynamics Convair Division and Space Systems Division. (not marked as proprietary) https://archive.org/stream/STRUCTURESANALYSISMANUALVOLI/STRUCTURES_ANALYSIS_MANUAL_VOL_I_djvu.txt and https://archive.org/stream/STRUCTURESANALYSISMANUALVOLII/STRUCTURES_ANALYSIS_MANUAL_VOL_II_djvu.txt
- "Detail Aircraft Structural Analysis." 1945. The Glenn L. Martin Company. (not marked as proprietary)
- "Structural Design Manual." Various dates for different sections. Chance Vought Aircraft, Inc. (not marked as proprietary)
- "Structures Manual." 1974. Rockwell International Space Division. (not marked as proprietary)
- "Engineering Practices Manual, Section M10.0, Stress Analysis Handbook." Circa 1973. Martin Marietta. (not marked as proprietary)
- "Structural Analysis Standards, Dynamics and Stress." August 1993. Martin Marietta Astro Space. (not marked as proprietary)
- Sarafin, Thomas P. PDF files for free download at https://instarengineering.com/resources.html for the following courses:
  - Spacecraft Structures, from Concept to Launch (SMS)
  - Structural Test Design and Interpretation for Aerospace Programs (STDI)
  - Vibration Testing of Small Satellites (VTSS)
  - Design and Analysis of Bolted Joints (DABJ) (listed as Ref. 33 on previous page)

Conversion of Units

- One inch (in or ”) = 2.54 centimeters (cm) = 0.0254 meters (m)
- One mil = 0.001 in
- One micron (μm) = 10^{-6} m = 3.937E-5 in
- One foot (ft) = 12 in
- One nautical mile (nmi) = 1.151 mile (mi) = 1.852 kilometers (km)
- One pound (lb) = 4.4482 Newton (N)
- One pound of weight corresponds to 0.45359 kilogram (kg) of mass
  – Something having a mass of one kg will weigh about 2.2 lb
- One lb-s^2/in of mass = 12 slug (sg) $\cong$ 175 kg
- Mass moment of inertia: one lb-s^2-in = 0.11298 kg-m^2
- One kip = 1000 lb
- One pound per square inch (psi) = 6894.8 Pa = 6894.8 N/m^2
- One kip per square inch (ksi) = 6.8948 MPa
- One msi = 1000 ksi
- g = gravitational acceleration $\cong$ 386.1 in/s^2 $\cong$ 9.81 m/s^2
- To convert between Fahrenheit (˚F), Celsius (˚C), and Kelvin (K):

$$°C = (5/9)(°F - 32)$$
$$K = °C + 273.15$$

1. Structural Requirements and Design Criteria

- Structural Requirements: What They Are and What They Are Not
- Typical Structural Functions and Constraints
- How Flight Loading Environments Affect the Structure
- Standards and Criteria
- Top-Level Criteria for Strength Analysis
- Other Commonly Used Structural Design Criteria

Structural Requirements

To a stress analyst, positive margin of safety may be considered the top-level structural requirement.

Others may perceive the requirement for, say, a random vibration test as the driving structural requirement.

But analyses and tests are <u>not</u> the requirements for the structure.

We do these activities to <u>verify compliance</u> with requirements.*

Let's put on our systems engineering hats to identify the true structural requirements …

*We also do analyses and tests to understand a problem and steer a design.

Requirements Stem From Functions and Constraints

Functions

– What the product must do or be able to do

Constraints

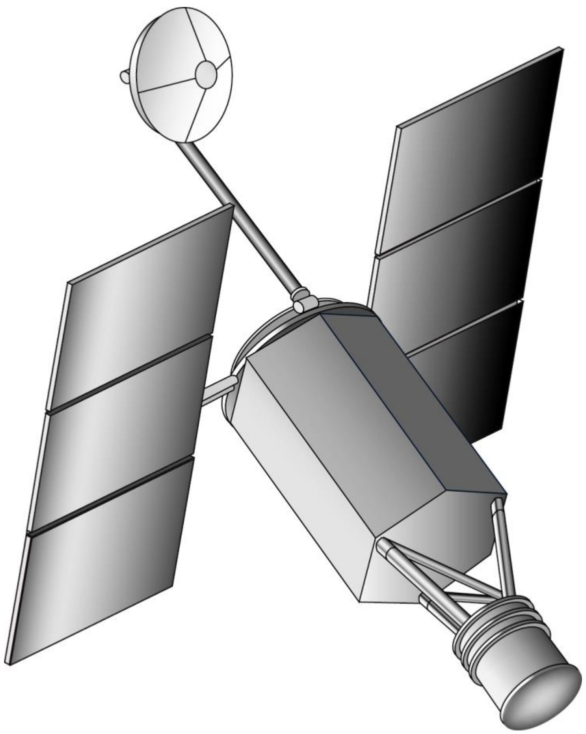
– Limitations, driven mostly by …

- interfaces
- allocations from higher-level requirements

So what are some examples of structural functions and constraints?

From my course "Spacecraft Structures, from Concept to Launch" (SCS)

Common Structural Functions



- Physically support something
- Attach two or more items
- Protect sensitive components or people from environments, such as vibration or radiation
- Maintain alignment and dimensional stability of sensors, antennas, and moving parts, as needed for system performance, considering …
  – permanent displacement (e.g., from launch),
  – temporary displacement (such as from temperature changes in orbit),
  – and ***jitter***: on-orbit vibration from operating equipment

The structure may serve nonstructural functions as well, such as conducting electricity and transferring or isolating from heat.

From my course "Spacecraft Structures, from Concept to Launch" (SCS)

Typical Constraints

- ***Dynamic envelope***—the physical space the item must stay within while deflecting under launch loads
  - ***Static envelope***—a smaller envelope derived from the dynamic envelope and an allocation for deformation under loads
- Mass properties (mass, center of mass (or CG), moments of inertia)
- Mechanical interface
  - Bolt pattern
  - Flatness
- Constraints driven by the need for access (e.g., for installation and removal)
- Fundamental frequency (lowest natural frequency for a structure)
- Environments
- Programmatic constraints such as cost and schedule

Flight Vehicles Must Function Despite Exposure to Life-cycle Environments

- Corrosive environments on the ground
- Gravitational loading
- Steady-state acceleration
- Transient loads, e.g., impulses
- Pressure, vacuum
- Aerodynamic turbulence
- Aerodynamic heating
- Vibration
- Shock
- Thermal radiation
- Charged-particle radiation
- Electromagnetic interference
- Magnetic fields
- Atomic oxygen

F-15 (courtesy U. S. Air Force)

Atlas V (courtesy U.S. Air Force)

Environments are constraints under which the product must function.

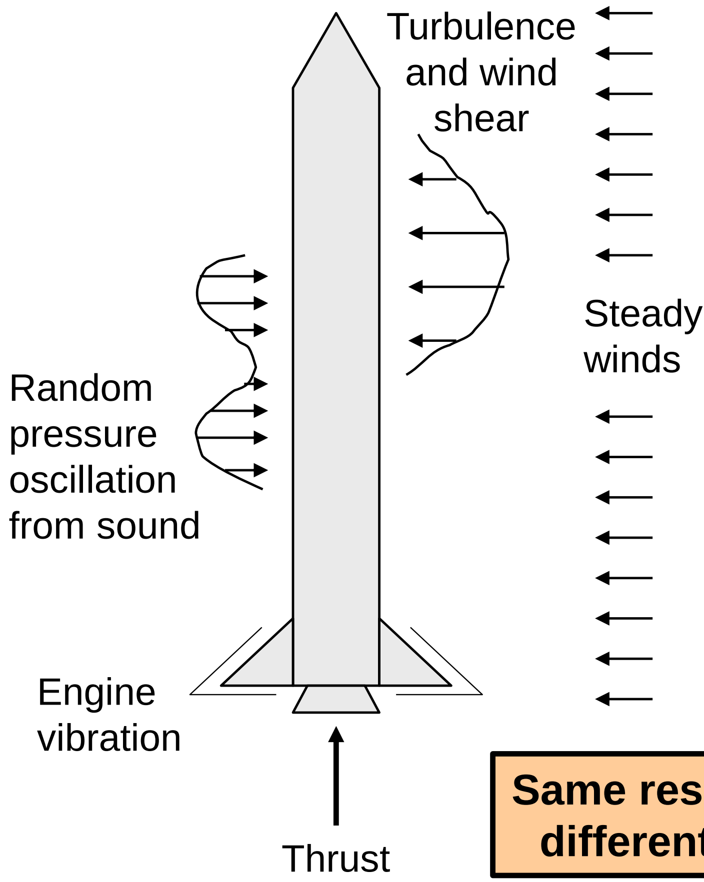
An important structural function can be to protect something or someone from environments.

The Product's Life Cycle Is a Source of Requirements

- Fabrication and Assembly
- Component Handling
- Component Testing
- Transportation
- Storage
- Spacecraft Integration & Test
- Spacecraft Handling
- Ground Transportation
- Air Transportation
- Integration with Launch Vehicle
- Prelaunch Preparations
- Launch and Spacecraft Separation
- Mission Operations
- Landing (sometimes)
- Post-Landing

As soon as possible, identify the life-cycle events and environments for your hardware, and continue to document them as you acquire more information.

How Flight Loading Environments Affect the Structure



Two types of loads act on a flight vehicle: steady-state (constant) and time-varying.

- Steady-state loads cause uniform acceleration along with resisting ***quasi-static*** inertia loads.
  – Inertia loads cause stress in the materials.
- Time-varying loads cause structures to vibrate.
  – Acceleration and corresponding inertia loads vary according to the shapes of the excited modes of vibration.
  – Inertia loads cause stress in the materials.

Same result at the material level—but materials can fail differently when stress is cyclic, e.g., from vibration.

Effects of Stress on Materials

- A material can take only so much stress before failure occurs.
- Under one-time application of stress, failure can be …
  - Rupture
  - Collapse (a structural failure rather than a material failure)

    (Rupture and Collapse are referred to as ***ultimate failure***)
  - Yielding, resulting in permanent deformation that is detrimental to form, fit, or function — ***yield failure***

Yielding is considered failure only if it is detrimental.

- Under cyclic stress, for metals …
  - a crack may form and then grow to rupture (***fatigue***)

Two structural characteristics required of all hardware:

Strength (ultimate or yield): the highest load a structure can withstand (or highest stress a material can withstand) without failure

Life: the number of cycles (or duration) of load or stress before failure

Accounting for Random Variables and Uncertainty

- Flight environments and loads are largely random.
- Strength and fatigue life of materials are also random.
- And there other random variables and sources of uncertainty as well:
  - Manufacturing variables: dimensional tolerances, processing, bolt preload
  - Analysis uncertainty: loads, temperatures, strength, life, human error
- We want to characterize and account for these random variables and uncertainties in design to make sure our products work.
  - Flight measurements, environmental tests, and engine firings
  - Strength and fatigue tests of multiple specimens
- But we still have to keeps things light enough to fly.

To balance the need for low weight against the risk of failure, the aerospace industry has adopted standards and criteria for how the above random variables are addressed.

Some of these standards and criteria may be levied by your customer or some governing body. Others your organization may need to self impose.

Standards and Criteria

Aerospace programs use many types of standards and criteria :

- Design standards and criteria
  - Material selection
  - Statistical levels for loads and material properties
  - Factors of safety
  - Fit (can parts be assembled?)
- Manufacturing criteria
  - Process control and inspection
- Test criteria
  - Test levels and duration
  - Flight hardware or test-dedicated hardware
  - Pass/fail

Keep requirements—which derive from functions and constraints—separate and distinct from criteria—which specify how to do the engineering or otherwise ensure quality (responsibility of the product developer).

Doing so allows the engineering team to understand and focus on what the product must do.

Combining criteria with requirements chokes the program with too many requirements. Engineers become bookkeepers with no sense of ownership or responsibility.

Top-Level Structural Design Criteria Read Like Requirements

(for Space and Launch Vehicles, adapted from aircraft industry)

The structure shall be able to …	as shown by (usually) …
• withstand ***limit loads*** (highest expected applied loads) – without detrimental elastic deformation ➢ such as the structure making contact with something else during launch ➢ or a tension joint gapping and significantly losing stiffness, making linear loads analysis less dependable	analysis and test
– without fatigue failure when accounting for all loading cycles expected over the product's life cycle ➢ Some programs require use of a fatigue analysis factor (a multiplier on stress) for this assessment (discussed in Sec. 5).	analysis alone*
• withstand a single application of ***design yield loads*** (limit loads multiplied by a specified or adopted yield factor of safety) without detrimental yielding or detrimental <u>permanent</u> deformation – Such as yielding or joint slipping that impairs function or performance	analysis and test
• withstand a single application of ***design ultimate loads*** (limit loads multiplied by a specified or adopted ultimate factor of safety) without catastrophic rupture or collapse	analysis alone*

Many aerospace structural engineers focus on ultimate strength, but any of the above criteria can drive the design.

*unless testing a dedicated (sacrificial) structure

A Closer Look at Criteria for Strength Analysis

Make sure the ***design load*** (limit load multiplied by a designated factor of safety) does not exceed the allowable applied load.

- ***Limit load***—The maximum expected applied load (acceleration, pressure, force, or moment), typically defined at a target statistical probability, usually 99% or mean plus 3 standard deviations (99.87% for a normal distribution). Applied load does not include preload (e.g., for a bolt) or residual stresses from fabrication.
- ***Allowable applied load***— The highest permissible applied load, which usually is derived either from the hardware specification (e.g., for bolts) or from the material's allowable stress (minimum strength) defined at a statistical basis specified by the program's design criteria (usually 99% probability). Derivation of the allowable applied load must account for the effects of preloads or residual stresses.
- ***Factor of safety***, *FS*—A factor applied to the limit load for use in analysis to decrease the chance of failure. The ultimate factor of safety is higher than the yield factor of safety because of the more severe consequences of ultimate failure.

The ***margin of safety*** is a measure of how much the design load can increase before the applicable criteria are not satisfied:

$$MS = \frac{\text{Allowable applied load}}{\text{Limit load} \times \text{Factor of safety}} - 1 \qquad \textbf{(Eq. 1-1)}$$

$$\boxed{\text{Criterion: } MS \geq 0}$$

See Sec. 4 for further discussion on the margin of safety

Factors of Safety for Structural Design

A factor of safety is historically an ignorance factor.

When structures have failed unexpectedly, the natural tendency has been to increase the factor of safety, settling in on the level that avoids such failures in the future.

In response to failures in connecting rods in locomotives as recently as 1910, a factor of safety of 18 was used for design!

(Ref. 12)

Engineers didn't know much about fatigue back then.

Factors of safety came down significantly as we learned more about how materials and structures fail, particularly during the first half of the 20th century.

Let's look at the factors of safety used for flight hardware …

Factors of Safety for Flight Hardware

Factors of safety used in aerospace programs depend on type of vehicle, type of structure, consequence of failure, responsible government organization, and whether the structure will be tested.

Typical ultimate factors of safety:

- Airframes—1.5
- Missiles—1.5 for events in which human safety is at risk, such as ground transportation and captive carry by an aircraft, and 1.25 for events away from humans and other vehicles
- Launch vehicles and spacecraft (if a structural test will be performed):
  - NASA programs: 1.4
  - U.S. DOD programs: 1.4 when human safety is at risk; 1.25 otherwise
- If there will be no test: 2.0 or higher

Yield factors of safety for flight structures can be as low as 1.0 but for most space programs are somewhat higher (e.g., 1.1 to 1.25, going as high as 2.0 for untested structures).

Test Options and Factors of Safety for Spaceflight Structures, Military Programs

Ref. 27 (AIAA S-110-2005) in combination with Ref. 28 (SMC-S-016), derived from the inactive Ref. 29 (DOD-HDBK-343)

Option (Design and Test Strategy)	Design Factors of Safety			
	Human-Rated Vehicles		Non-Human-Rated Vehicles	
	Yield	Ultimate	Yield	Ultimate
1) Qualification test: Test a dedicated article to the design ultimate loads	1.1	1.4	1.1	1.25
2) Protoqualification test: Test one flight unit of a fleet to 1.25 times limit	1.25	1.4	1.25	1.4
3) Proof test each flight structure to 1.1 times limit, in combination with option 1 or 2, above	1.1	1.4	1.1	1.25
4) No structural test	1.6	2.25	1.6	2.0

Use with caution. Originally intended to apply only to fail-safe designs.
(Ref. 31, February 29,1980 letter from USAF Col. Norman Lee to Glynn Lunney, JSC Manager of Shuttle Payload Integration)

Original Applicability of the "No Test" Option

1.1 New Structures

Option 4 may be applied to the development of new structures when the following conditions are met:

a. The program has a single flight article.

b. **The structures are of fail-safe design** with sufficient residual strength after failure of a single load-carrying element to permit mission completion without jeopardizing crew safety.

c. The structures have **well-defined failure modes** and can be adequately characterized by comprehensive load and stress analyses.

d. The designs are based on data and experience **obtained with similar design/manufacturing technology**.

e. Structural weight is not a critical design constraint.

f. **Development tests are performed**, as required, to provide necessary design data.

Quoted (emphasis added) from the attachment to Ref. 31 (Lee-Lunney letter)

Factors of Safety for NASA Programs (Ref. 30, NASA-STD-5001B)

		Critical for Personnel Safety				Not Critical for Personnel Safety			
		Factors of Safety		Test Factors		Factors of Safety		Test Factors	
		Ultimate	Yield	Qual	Proof	Ultimate	Yield	Qual	Proof
Prototype	Metallic structures	1.4	1.0	1.4	N/A or 1.05*	1.4	1.0	1.4	N/A or 1.05*
	Composite/bonded structures:								
	Discontinuity areas	2.0	N/A	1.4	1.05	1.4	N/A	1.4	1.05
	Uniform material	1.4	N/A	1.4	1.05	1.4	N/A	1.4	1.05
Protoflight	Metallic structures	1.4	1.25	1.2	N/A or 1.05*	1.4	1.25	1.2	N/A or 1.05*
	Composite/bonded structures:								
	Discontinuity areas	2.0	N/A	1.2	1.2	1.4	N/A	1.2	1.2
	Uniform material	1.5	N/A	1.2	1.2	1.4	N/A	1.2	1.2

*Proof testing of metallic structures applies only to propellant tanks and solid rocket motor cases.

Per NASA-STD-5001B: "In order to use the 'no-test' approach, project-specific criteria and rationale shall be developed for review and approval by the responsible Technical Authority."

Other Commonly Used Structural Design Criteria

- Discussed in the following slides in this section:
  - Fitting factor
  - Stability factor
  - How to address dimensional tolerances
  - Preload for bolted joints and mechanisms
  - Friction
- Discussed in Sec. 4:
  - Statistical basis for allowable stresses
- Discussed in Sec. 5:
  - Life factor to account for variation in fatigue life
  - Fatigue analysis factor
- Discussed in Sec. 6:
  - A closer look at preload for bolted joints

Commonly Used Structural Design Criteria:
Fitting Factor

A ***fitting factor*** is an extra factor of safety (most commonly 1.15) applied to the calculated load acting on a fastener (not preload) and the associated stresses in the bolt and the attached members (fittings).

- Why use a fitting factor? Because failure occurs most often in joints.
  - Uncertain load distribution between fasteners
  - Sudden changes in cross section
  - Inelastic behavior of ductile materials—historically, empirical methods of analysis are more reliable than linear-elastic finite element analysis, and we don't have empirical methods for all types of joints.
- Apply the fitting factor to strength analysis for any potential failure modes that are (a) associated with predicted load distribution between fasteners or (b) not fully characterized with test data or empirical methods.
- Normally used for ultimate-strength analysis, but it may be wise to use it for yield as well, especially when the design is alignment critical.
- A fitting factor higher than 1.15 may be warranted for joints in which rupture can occur with little apparent ductility—e.g., the highest loaded bolt breaks before the other bolts can carry their share of load.
  - In Sec. 6 we'll look at design guidelines intended to ensure failure would be ductile enough to spread load between fasteners.

Commonly Used Structural Design Criteria:
Some Programs Use a Stability Factor

A ***stability factor*** is an extra factor of safety (often 1.1 or 1.15) on applied load, used for buckling analysis. Rationale:

- Buckling is difficult to predict, and linear-elastic analysis tends to be unconservative.
- Compressive strength is greatly affected by dimensional tolerances.
- Compressive failures are usually self-propagating and catastrophic.

A stability factor may not be warranted if you …

- use appropriate nonlinear methods when allowing ultimate stresses to exceed the material's proportional limit;
- adequately account for variation, misalignments, and dimensional tolerances;
- and use appropriate empirical methods.
- Addressed in Sec. 4.

But don't rely too heavily on a stability factor. It can entice engineers to depend solely on linear-elastic buckling solutions in FEA, not accounting for the effects of dimensional tolerances and inelastic behavior.

A 1.1 or 1.15 stability factor is not nearly enough to account for these effects, so account for them directly.

Commonly Used Structural Design Criteria:
Accounting for Dimensional Tolerances

- Tolerance stack-up for ensuring fit:
  - Varies between companies or organizations
  - Some do a worst-case combination of max/min tolerances.
  - Others less conservatively assess a root sum square (RSS) of tolerances.
  - Your company should standardize a method to ensure consistency.
- Part thickness for strength or life analysis:
  - Some analyze minimum thickness, others analyze nominal only.
  - One philosophy I like is to limit the effect of tolerances to 10%. Examples: Use the lower of nominal thickness or …
    - 1.1 times minimum thickness for tensile or shear stress calculations
    - 1.05 times minimum for bending of a solid cross section
    - 1.03 times minimum for buckling of a plate or a shell
- Part thickness for finite element models used for loads analysis:
  - Use nominal thickness—predicted modes of vibration and load distribution should be "best estimate" rather than based on minimum thickness.

Section 4 addresses the effects of misalignment for compressive loading.

Commonly Used Structural Design Criteria:
Preload Criteria

Preload bolts and mechanisms highly enough to …

- prevent gapping at limit loads in order to (a) ensure stiffness is relatively linear (load vs. displacement), so that linear loads analysis is valid, and (b) avoid fatigue failure of parts such as bolts.
- prevent gapping at design ultimate loads if the consequence of gapping is potentially catastrophic (e.g., releasing propellant at a pressurized joint with gaskets).
- reduce likelihood of joints slipping through clearance holes at limit loads, which can lead to failure.

Bolt preload and its benefits are explored in Sec. 6 herein, drawing from my course "Design and Analysis of Bolted Joints" (DABJ), Ref. 33.

Commonly Used Structural Design Criteria:
Criteria for Use of Friction in Bolted-Joint Analysis

- Common practice in the space industry is not to count on any beneficial effects of friction for ultimate strength analysis.
  - This is a requirement in NASA-STD-5020B for threaded fastening systems.
  - Rationale: Tests show that ultimate shear strength is the same for a preloaded joint as it is for a non-preloaded joint.
- But friction is a desirable load path, as it can prevent joint slip and make fatigue failure in the joint highly unlikely. (See Sec. 6 herein.)
  - So design engineers should try to maximize friction load-carrying capacity.
  - NASA-STD-5020B permits the use of friction for all analyses other than ultimate strength.
- The design team should (a) strive for high friction capability in shear joints; (b) decide on when, if ever, friction can be counted on; and (c) establish criteria for its use.
  - How much preload can you count on, given the uncertainty in the torque-preload relationship? See NASA-STD-5020B.
  - What coefficient of friction will you assume when assessing joint slip? Appendix A in NASA-STD-5020B provides guidance.

Regardless, be sure to account for friction if it's detrimental.

Responsibility of the Product Developer

As the organization designing a structure, don't wait for your customer to tell you how to do the engineering.

Take responsibility for your products by adopting standards and criteria intended to ensure your products work.

Then sell those standards and criteria to your customer and other stakeholders.

- Government and industry standards should be considered the customer expectation.
- If the contractor wants to do something different, by tailoring or replacing a government or industry standard, the contractor should be expected to convince the customer that it's the right thing to do.
- When paying for the development of a system, the customer has the right of approval for any criteria related to risk.

Personal Responsibility Is Just as Important

Image courtesy NASA

Manned Maneuvering Unit (MMU)

"You'd better be willing to swing from that structure over a den of alligators."

Paul Knox, stress-analysis supervisor for the Manned Maneuvering Unit program at Martin Marietta Astronautics in early 1980, to a junior stress analyst, Tom Sarafin, who was about to sign an engineering drawing for the first time

Key Points from Section 1

- Structural and environmental tests are **NOT** the true requirements.
  - We do them to verify compliance with requirements.
- Positive margin of safety is **NOT** a true requirement.
  - It's a criterion for verification, which means it's related to risk.
- True requirements relate to functions and constraints.
- Take time to understand the requirements.
  - We can't use the same design and analysis practices for a structure that must maintain optical alignment as for the primary structure of a launch vehicle.
- Take responsibility for your products by adopting standards and criteria intended to ensure quality and high likelihood of success.

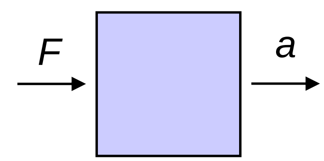
2. Statics and Dynamics

- Static Equilibrium and Free-Body Diagrams
- Benefits of a Statically Determinate (Kinematic) Interface
- Examples and Class Problems
- Dynamic Equilibrium
- Modes of Vibration
- The Equation of Motion
- Transmissibility
- Mode Shapes
- Applying What We Learned
- Appendix: Introduction to Random Vibration

Static Equilibrium and Newton's Second Law of Motion

Recall Newton's second law of motion: $F = ma$ **(Eq. 2-1)**

– If there is a resultant force acting on an object, the object will accelerate in the direction of the force.



Therefore, if the object has no acceleration, it must have no resultant force acting upon it.

> ***F = ma*** **is the main principle behind structural analysis.**

An object under no acceleration is in ***static equilibrium***, which means the external forces and moments sum to zero in all directions.

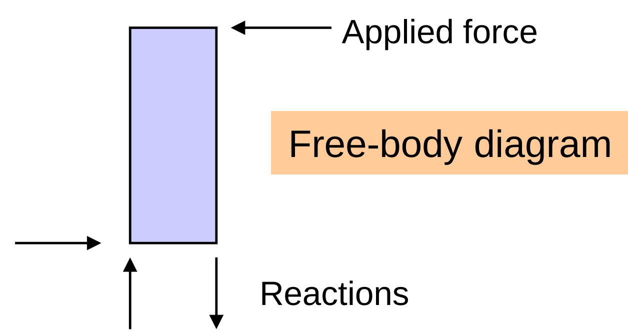
– There are six equations of static equilibrium for a rigid body, one for each degree of freedom (DOF):

Forces:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

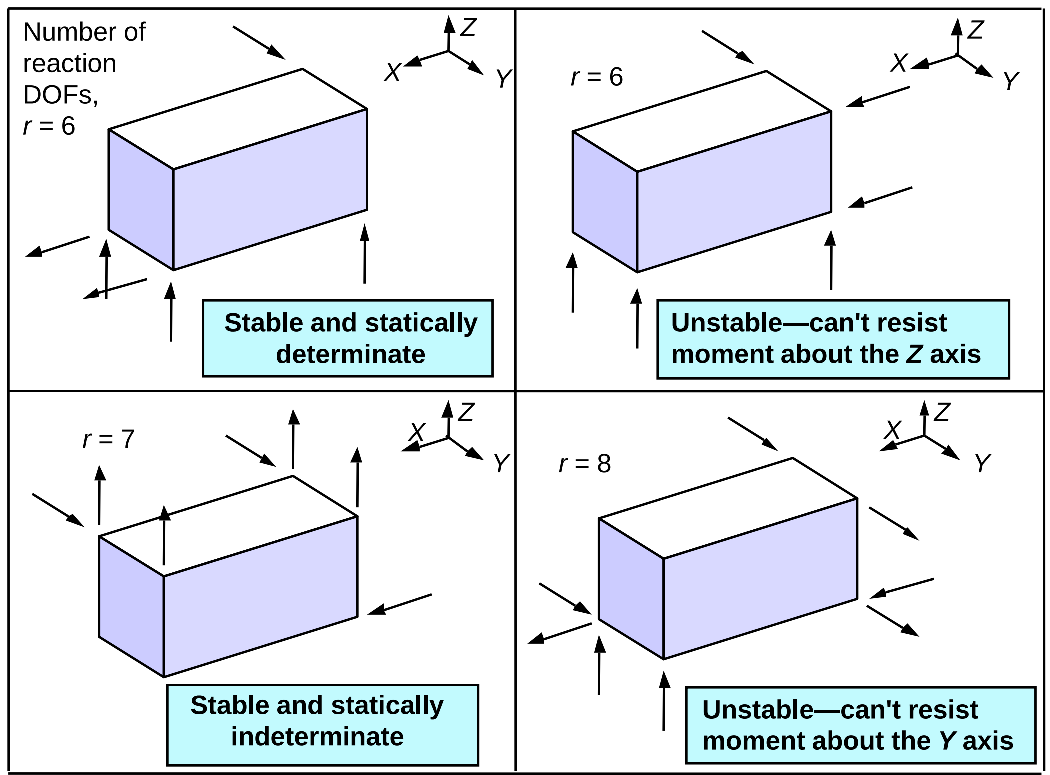
Moments:

$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$



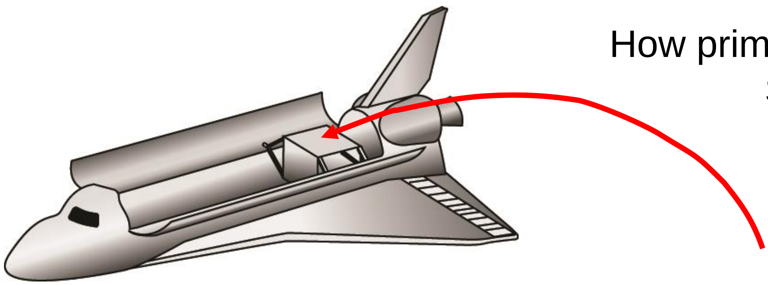
Statically Determinate and Indeterminate Interfaces

- For a structure at rest, any applied forces and moments are balanced by ***reactions*** (forces and moments at the structure's supported interface) in such a way as to satisfy the six equations of static equilibrium.
- If a structure has a ***statically determinate*** (a.k.a ***kinematic***) interface, only one set of reactions will satisfy the six equations for any loading condition. This requires exactly six reaction DOFs.
- A ***statically indeterminate*** interface has more reaction DOFs than are necessary for equilibrium.
- An interface is ***unstable*** if the reaction DOFs are insufficient for equilibrium, which means no load can be applied without acceleration.

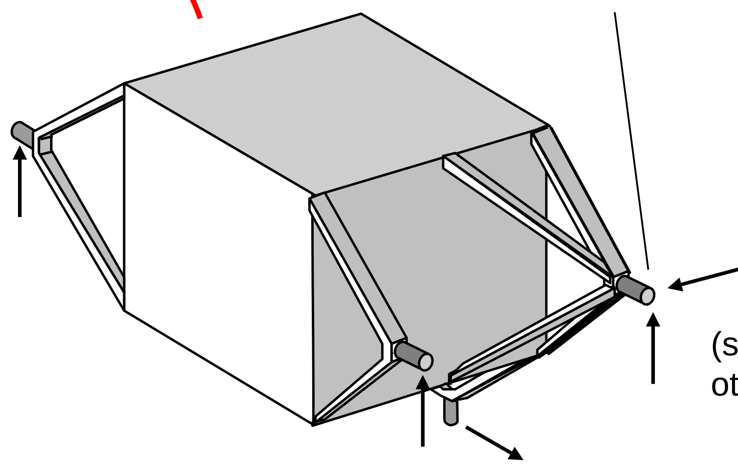


SSAM Fig. 4.4

The Best Interface Often Is a Statically Determinate Interface



How primary payloads were mounted in the Shuttle Orbiter cargo bay



With a statically determinate interface, the payload was not loaded by distortion of the Orbiter.

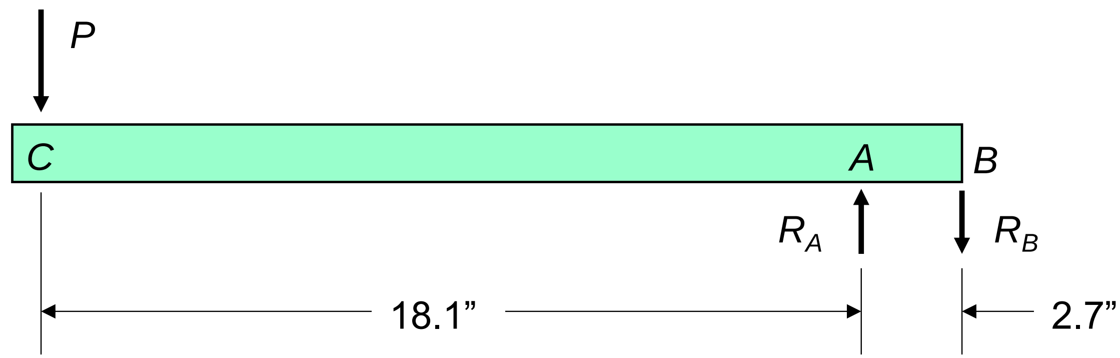
Unfortunately, the keel interface could not carry much load in the thrust direction, so each relatively large payload required a second plane of support structure, and thus a statically indeterminate interface.

With a statically indeterminate interface, distortion of the Orbiter drove loads into the payload.

Design of the fuselage was released for manufacturing before the payload interface was defined. If timing had been different, the fuselage could have been strengthened for keel loads.

Example Problem 2-a: Statics

Calculate reactions R_A and R_B in terms of applied load P



Pick a convenient point about which to sum moments:

At point *A*: $\sum M_A = 0 = 18.1P - 2.7R_B$ so $R_B = \left(\frac{18.1}{2.7}\right)P = 6.70P$

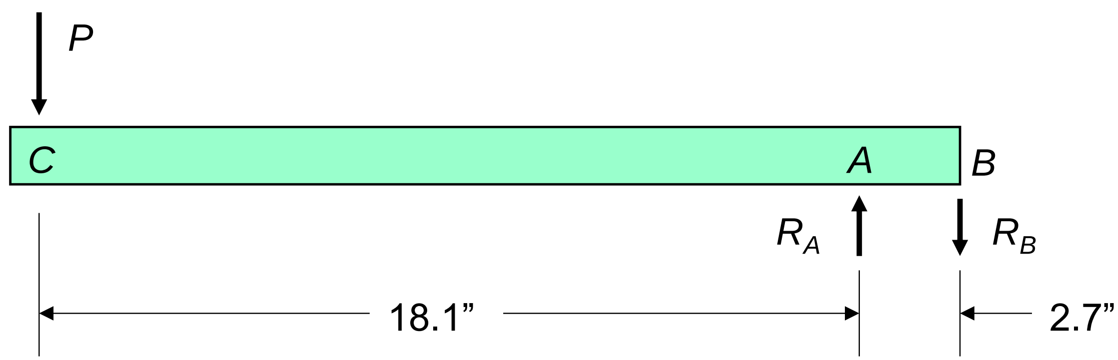
$\sum F = 0 = -P - 6.7P + R_A$ so $R_A = 7.70P$

This may look simple, but many errors are made by not satisfying equilibrium.

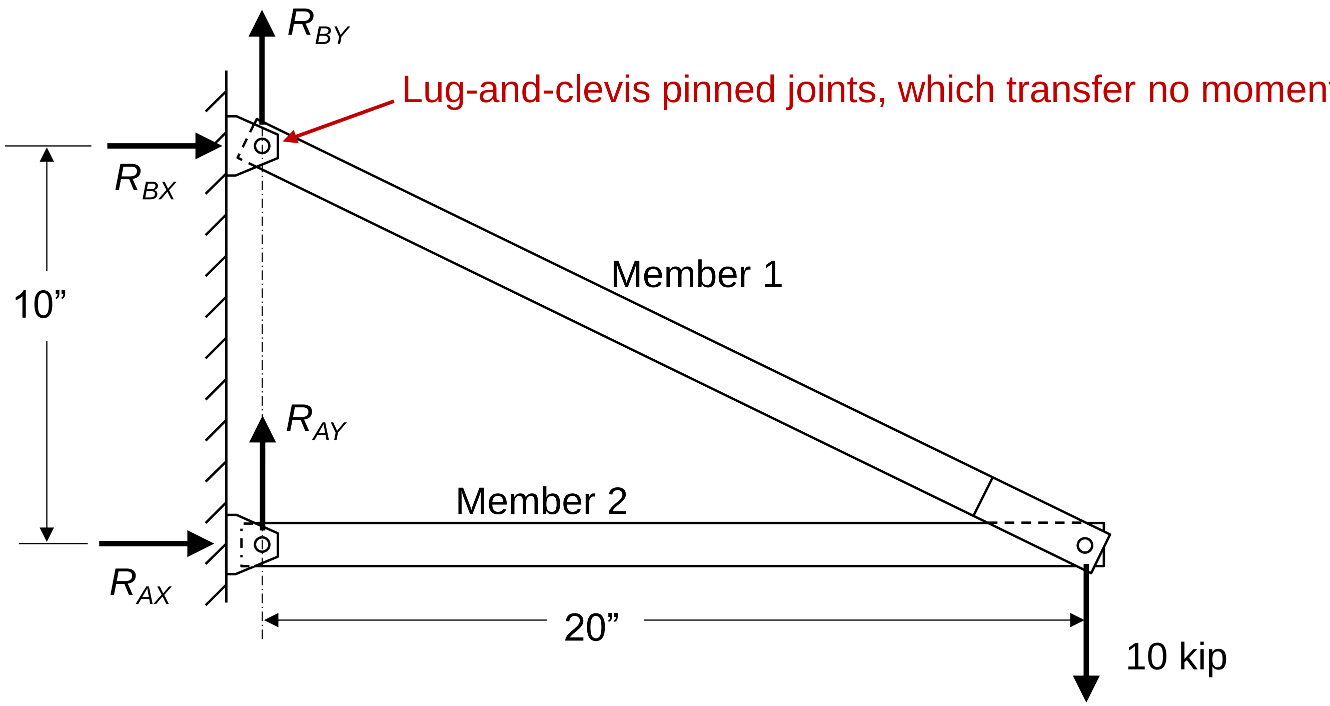
Get in the habit of drawing free-body diagrams!

Class Problem 2-1: Same As Example Problem 2-a but Worked by Summing Moments at a Different Location

Calculate reactions R_A and R_B by summing moments at point C.



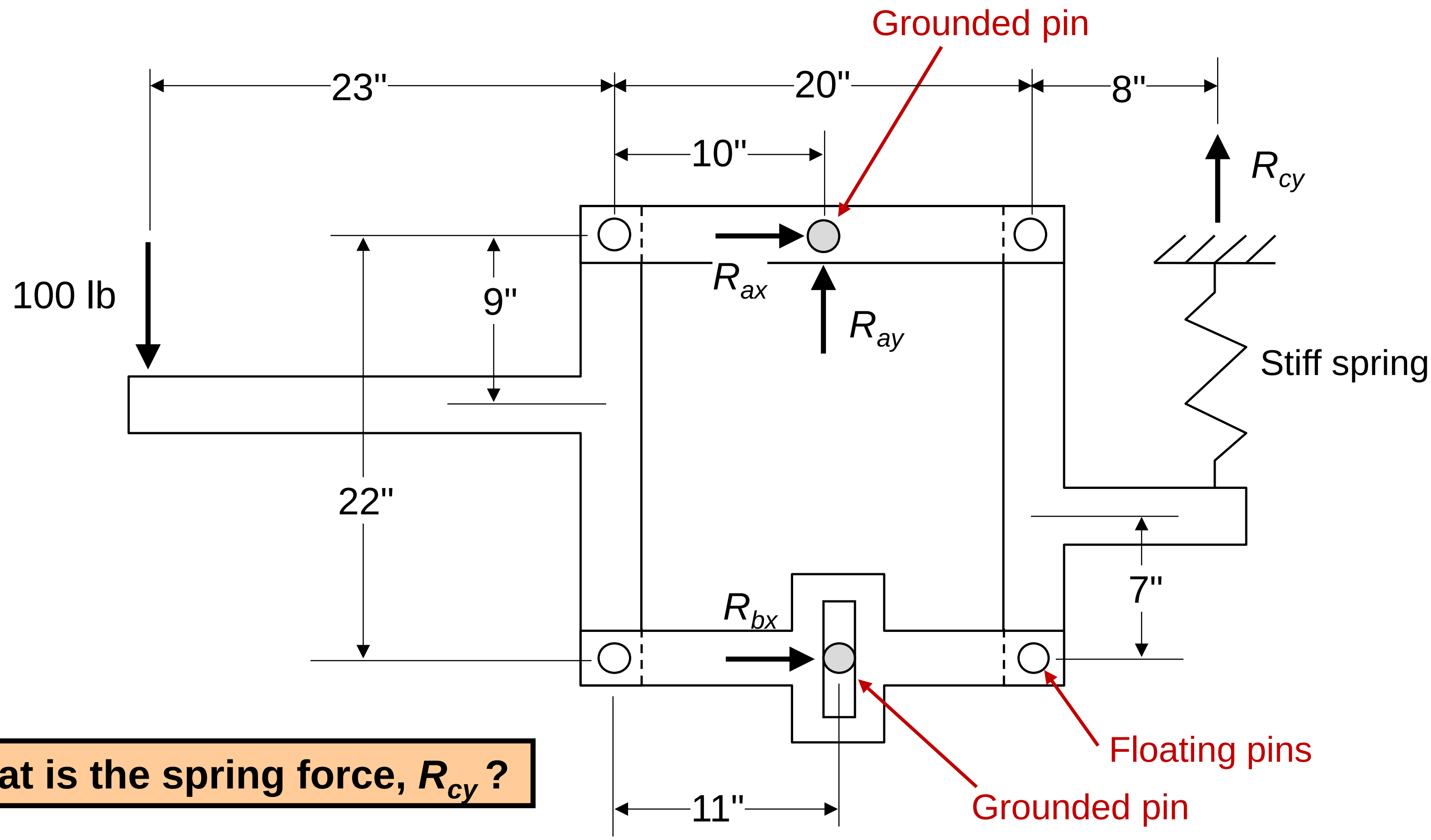
Class Problem 2-2: A Simple Truss



Calculate all reactions and member loads.

1 kip = 1000 lb

Class Problem 2-3: A More Challenging Statics Problem



What is the spring force, R_{cy} ?

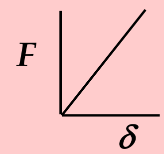
Load and Displacement

A key assumption in most structural analysis is that displacement is proportional to load.

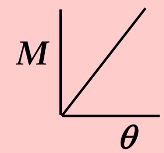
This is what we mean by the term ***linear structure*** or ***linear system***.

- Nonlinear structural analysis is possible but can be time consuming and is often impractical.
- A nonlinear structure is a less predictable structure.
- And a less predictable structure is less dependable or less safe (usually).

Two types of loads: force and moment



Two types of displacements: translations and rotations



Note: In stress analysis, P is the traditional symbol for force, f (or σ) is calculated stress, and F is allowable stress.

Unfortunately, any structure with bolted joints is nonlinear to some extent!

One of the goals in design is to minimize nonlinearity by…

- **using materials within the linear-elastic region, stress vs. strain;**
- **minimizing the difference in load paths when load reverses, e.g., from tension to compression in joints;**
- **preloading joints and avoiding physical gaps and clearances.**

Quasi-Static Loads and Load Factors

- A ***static load*** is a steady-state external load (force or moment) acting on an object and is a term typically associated with static equilibrium, with the object having no acceleration.
- An ***inertia load*** is the internal load that resists acceleration.
- A ***quasi-static load*** is a term used for (a) uniform steady-state acceleration and (b) the inertia load associated with uniform acceleration.
- A ***dynamic load*** varies over time and either causes or is associated with vibration.

Limit load is the maximum expected load during the mission or some other event.

- A ***load factor*** is a multiple of *g* (Earth's gravitational acceleration), representing a quasi-static inertia load; thus, it's also a multiple of weight on Earth.
- Quasi-static loads are often defined in *g* units; however, an inertia load acts in a direction opposite that of the acceleration and thus is of opposite sign from the acceleration:

$$\boxed{a = -ng} \quad \textbf{(Eq. 2-2)}$$

where a = acceleration (in/s^2 or m/s^2)
n = load factor
g = gravitational acceleration (in/s^2 or m/s^2)

Before we review the mechanics of vibration,

Remember: Weight Is Not the Same As Mass!

The weight of an object is less on the moon than on Earth, but the mass is the same.

$$F = ma \quad \text{(Ref. Eq. 2-1)}$$

Force Mass Acceleration

$$w = mg \quad \textbf{(Eq. 2-3)}$$

Weight (force) Gravitational acceleration

	US customary units	SI units
Weight	lb	N
Acceleration	in/s^2	m/s^2
Mass	lb-s^2/in (="slinch")	N-s^2/m (= kg)

Weight is in units of force.

Kilogram is a unit of mass, not force.

Pound is a unit of force, not mass.

In this course, I do not recognize the "pound mass" unit. It's unnecessary, and I believe it's caused a great deal of confusion.

The Convenience of Using Acceleration in "*g*" Units

When using *g* as the unit for acceleration, we use weight (lb or *N* units) in place of mass and load factor (or number of *g*'s) in place of acceleration.

$$F = ma$$

$$F = \left(\frac{w}{g}\right)(-ng)$$

Recall: $m = w/g$ from Eq. 2-3, and $a = -ng$ from Eq. 2-2

External force ⇨ $F = -nw$ **(Eq. 2-4a)**

Internal force ⇨ $F_i = nw$ **(Eq. 2-4b)**

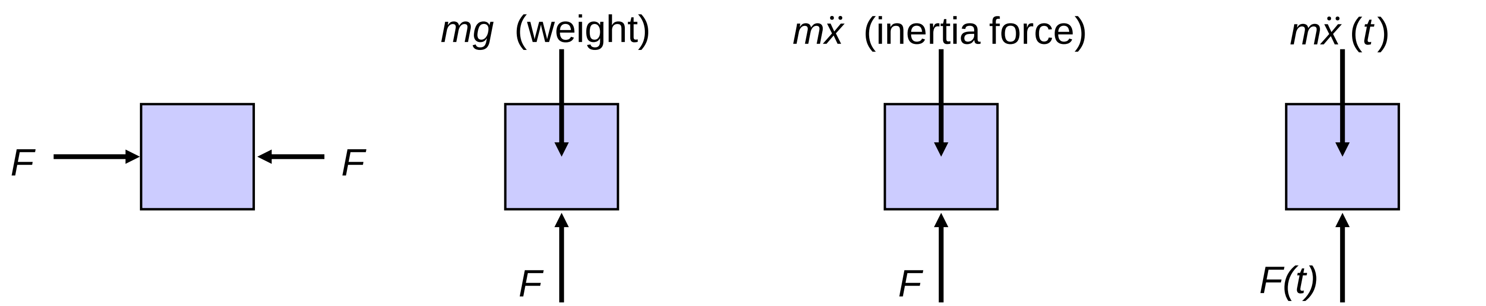
where
- m = mass
- w = weight
- a = acceleration
- g = gravitational acceleration
- n = load factor

Example:	An object weighs 100 lb (445 N).	w = 100 lb
	The object is under 5g acceleration.	$a = 5g$
	The load factor for that object is -5.	n = -5
	The inertia force is -500 lb (-2224 N).	$F_i = nw$ = -500 lb
	The external force that causes 5g acceleration is 500 lb.	$F = -nw$ = 500 lb

Force Balance for Accelerating Objects

Static equilibrium applies when there is no acceleration: $\Sigma F = 0$, where F is external force.

For an accelerating object, external force is balanced by an equal and opposite internal force:



Body at rest; external forces balance (no acceleration and no gravity)

Body at rest under gravitational force (e.g., a rock sitting on the ground)

Body under steady-state acceleration

Applied force and acceleration vary with time

F = force

m = mass

g = gravitational acceleration

$\ddot{x}$ = acceleration

t = time

Everything is in force equilibrium—always. External forces, gravitational forces, and inertia forces always sum to zero.

From Instar's course "Spacecraft Structures, From Concept to Launch" (SCS)

Modes of Vibration

Every structure has a near-infinite number of DOFs, with a ***mode of vibration*** for each DOF. Each mode is characterized by …

- ***Natural frequency***: frequency at which the structure vibrates once excited and after the excitation is gone
- ***Mode shape***: deformed shape of the vibrating structure
- ***Damping***: loss of energy in a vibrating structure

The ***fundamental frequency*** is the structure's lowest natural frequency, corresponding to the fundamental (first) mode of vibration, which usually …

- has the most mass participation
- and is the easiest mode to excite

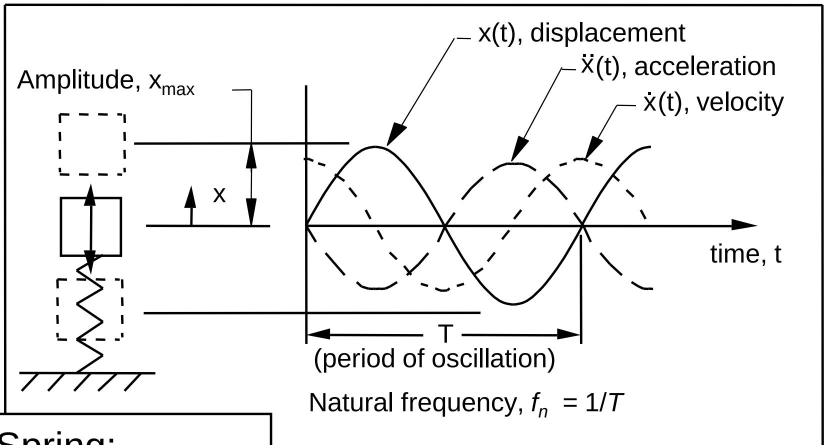
Note: If the first mode for a spacecraft is bending of a small antenna, that mode is considered the fundamental mode of the antenna but not of the spacecraft.

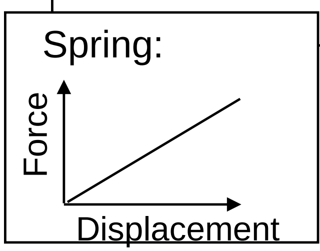
To simplify structural analysis, we try to idealize or model a structure with as few DOFs as needed for the desired accuracy.

Often we can use just one with what we call a *single-degree-of-freedom system (SDFS)* (mass on a spring).

Harmonic Motion of a Mass on a Spring
(Single-Degree-of-Freedom system, SDFS)

- Once disturbed by a time-varying force, a structure will vibrate.
- ***Free vibration*** is vibration that continues after external forces are removed.
- In absence of damping, each point in a freely vibrating, linear structural system experiences ***harmonic motion***, in which displacement is a sinusoidal function of time.
- Velocity and acceleration are the first and second derivatives of displacement with respect to time, and are thus also sinusoidal functions of time.





Displacement, $x(t) = x_{max} \sin(\omega_n t)$ **(Eq. 2-5)**

Velocity, $\dot{x}(t) = \omega_n x_{max} \cos(\omega_n t)$ **(Eq. 2-6)**

Acceleration, $\ddot{x}(t) = -\omega_n^2 x_{max} \sin(\omega_n t)$ **(Eq. 2-7)**

Note: Acceleration is proportional to displacement.

ω_n = natural frequency in rad/s

$= \sqrt{k/m}$ **(Eq. 2-8)**

k = stiffness
m = mass

Natural frequency in Hz:

$$f_n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{k/m} = \frac{\omega_n}{2\pi} \text{ (Hz)}$$ **(Eq. 2-9)**

Damping

- If there were no energy-dissipating forces, a structure, once disturbed, would vibrate forever.
- Damping, which is the loss of energy in a vibrating structure (e.g., from friction), allows vibrations to die out by applying forces that resist motion.
- ***Viscous damping***, the most commonly assumed damping model, is proportional to velocity:

$$\text{Damping force} = -c\,\dot{x}(t) \qquad \textbf{(Eq. 2-10)}$$

$c =$ damping factor
$\dot{x}(t) =$ velocity

- The ***critical damping factor*** is the value that would allow the mass, once disturbed, to return to its initial position without vibration:

$$\text{Critical damping factor, } c_c = 2m\omega_n = 2\sqrt{km} \qquad \textbf{(Eq. 2-11)}$$

m = mass, k = stiffness
ω_n = natural frequency in rad/s

- The ***damping ratio***, ζ, is a measure of a structure's damping relative to the critical damping:

$$\zeta = \frac{c}{c_c} \qquad \textbf{(Eq. 2-12)}$$

The damping ratio is difficult to predict for most structures. Until we test the structure, we must rely on experience with similar structures.

The Equation of Motion for a Linear SDFS

All forces acting on an object (mass) must always sum to zero:

Applied force + inertia force + damping force + spring force = 0

$$F(t)+\left[-m\ddot{x}(t)\right]+\left[-c\dot{x}(t)\right]+\left[-kx(t)\right]=0 \qquad \textbf{(Eq. 2-13)}$$

m = mass
c = damping factor
k = stiffness
t = time

$x =$ displacement
$\dot{x} =$ velocity (dx/dt)
$\ddot{x} =$ acceleration (d^2x/dt^2)

Rearranging terms results in the ***equation of motion*** (an ***equation of equilibrium***) for a linear SDFS:

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=F(t) \qquad \textbf{(Eq. 2-14)}$$

or

$$m\ddot{x}(t)+2\zeta\omega_n m\dot{x}(t)+\omega_n^2 mx(t)=F(t)$$

ω_n = natural frequency (rad/s)
ζ = damping ratio

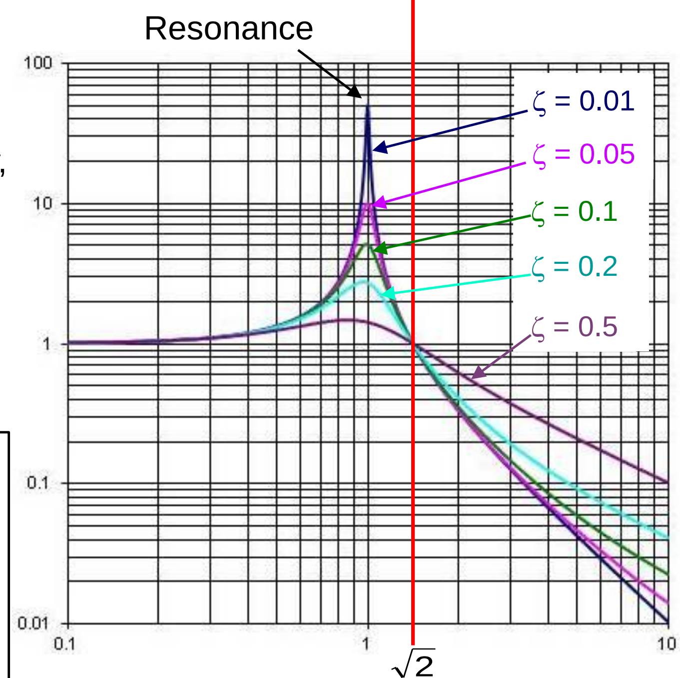
The above equation of motion applies to each degree of freedom in a linear system and is the basis for dynamic analysis.

Response of an SDFS to Base-driven Sinusoidal Vibration: Transmissibility

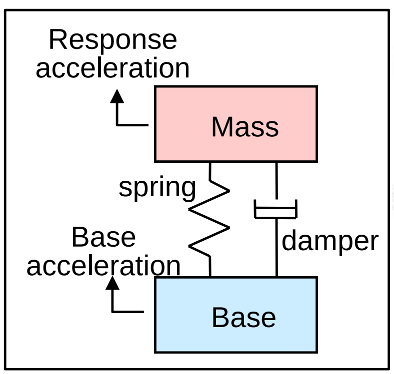
Transmissibility is the ***dynamic gain*** in a sinusoidally base-driven linear SDFS: peak response acceleration divided by peak base (input) acceleration, as a function of frequency.

Amplification ⟷ Attenuation (isolation)

Transmissibility, *TR* (ratio of peak response acceleration to peak base acceleration)



Ratio of forcing frequency to natural frequency, f/f_n



$$TR = \sqrt{\frac{1+\left(\frac{2\zeta f}{f_n}\right)^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left(\frac{2\zeta f}{f_n}\right)^2}} \quad \textbf{(Eq. 2-15)}$$

(SSAM Eq. 5.30)

where

f = Forcing frequency in Hz

f_n = System's natural frequency in Hz

ζ = Damping ratio

Quality factor, $Q \cong \dfrac{1}{2\zeta}$

= transmissibility at resonance for a mass on a spring **(Eq. 2-16)**

Class Problem 2-4:

Predicting Dynamic Loads for Alternate Designs

The instrument you are designing will be subjected to a sinusoidal vibration test, with peak input acceleration of 2.5 *g* and frequency gradually increasing from 10 Hz to 100 Hz.

Assuming the instrument can be idealized adequately as a mass on a spring (SDFS), and given the following two sets of properties for the instrument, estimate the highest response acceleration you expect it to see during the test. (Assume notching* is not justifiable and thus not permitted.)

a. Fundamental frequency = 79 Hz and damping = 2% of critical

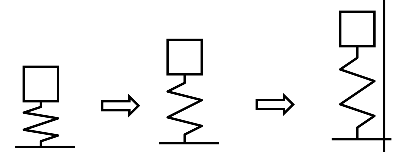
b. Fundamental frequency = 112 Hz and damping = 1% of critical

Notching means reducing the input acceleration at and near the test article's fundamental frequency to avoid unrealistically high response. (Addressed in great detail in my courses "Structural Test Design and Interpretation" (STDI) and "Vibration Testing of Small Satellites" (VTSS))

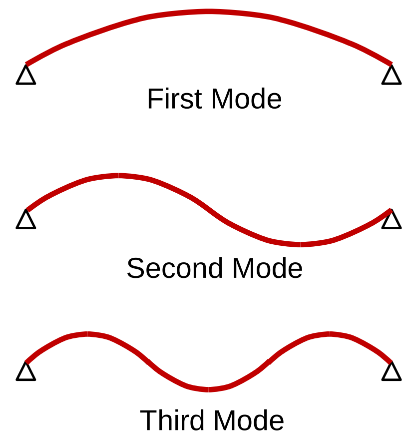
Mode Shapes

- Each mode of vibration has a different shape.
- Some modes are analogous to that of a mass on a spring.
  - Concentrated mass, concentrated spring
- Others are more like the bending modes of a beam
  - Distributed mass, distributed spring.

Mass on a spring:

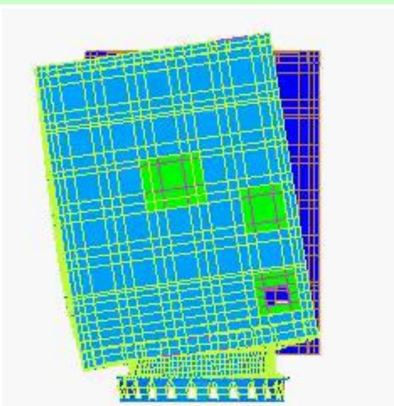


Beam:

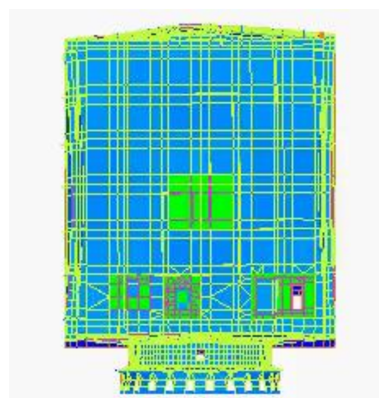


For a structure vibrating in a single mode, when ignoring damping, each point moves as a sinusoidal function of time.

Example: FalconSat-3 engineering model, grounded base



Fundamental lateral (rocking) mode



Fundamental axial mode

Normal Modes and Complex Modes

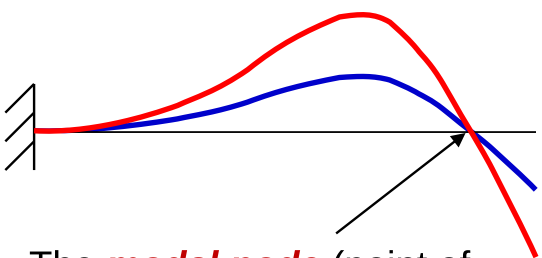
For low to moderate damping (less than about 10% of critical), we can assume the structure exhibits ***normal modes***, in which the mode shape is constant as amplitude varies.

- Each point in a structure vibrating in a normal mode moves in a sinusoidal function of time, as if it were an SDFS.

Because of damping, modes of vibration actually are ***complex modes***, having shapes that vary with amplitude.

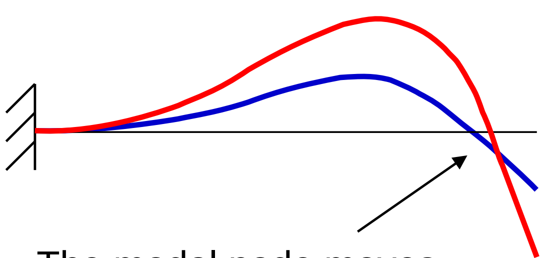
- Complex modal behavior becomes significant only at high damping.

Normal mode



The ***modal node*** (point of zero displacement) does not change with amplitude

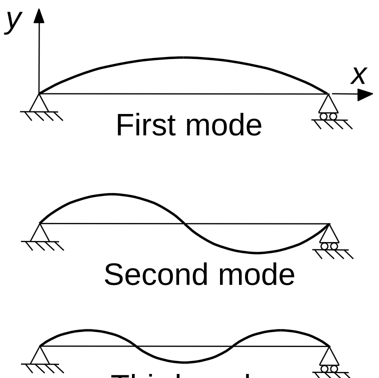
Complex mode



The modal node moves with amplitude

Simplifying with the Assumption of Normal Modes

First three modes of a simply supported, uniform beam:



With a normal mode, we can predict dynamic response of any point in the structure by treating the mode as a single-DOF system:

$$y_i(x,t) = \Phi_i(x)q_i(t) \quad \textbf{(Eq. 2-17)}$$

t = time

i = mode number

Displacement in the *y* direction of location x at time *t*

Mode shape

Generalized (modal) coordinate (analogous to how a mass on a spring would respond)

The total response equals the sum of the responses of the vibration modes.

For a structure freely vibrating in a single mode with low damping, any point other than a modal node moves sinusoidally like a mass on a spring.

Example Problem 2-b:

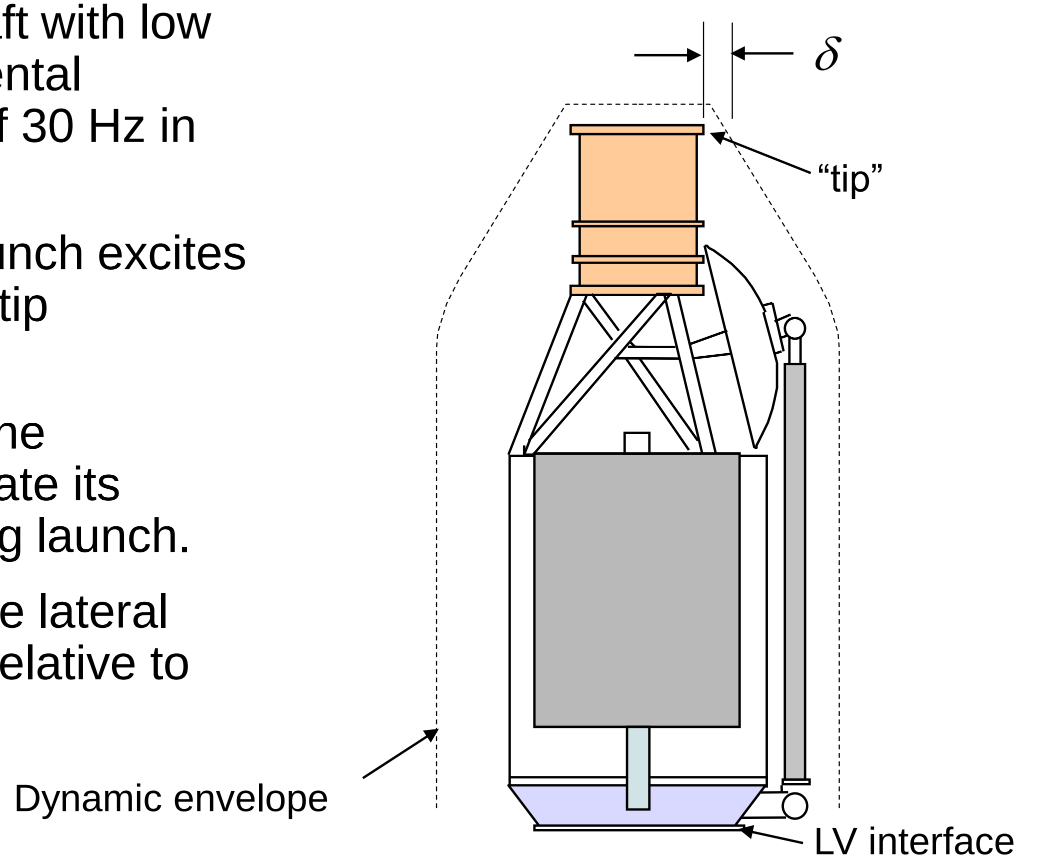
Simple Assessment of Dynamic Envelope

A cantilevered spacecraft with low damping has a fundamental bending frequency, f_n, of 30 Hz in launch configuration.

A disturbance during launch excites this mode such that the tip acceleration is 8*g*.

We want to make sure the spacecraft does not violate its dynamic envelope during launch.

How can we estimate the lateral displacement of the tip relative to the dynamic envelope?



continued

Example Problem 2-b: Solution

Assume the tip of the spacecraft acts like a linear, undamped SDFS (mass on a spring) that is vibrating sinusoidally at 30Hz.

Recall Eqs. 2-5, -6, and -7:

Displacement, $x(t) = x_{\max} \sin(\omega_n t)$

Velocity, $\dot{x}(t) = \omega_n x_{\max} \cos(\omega_n t)$

Acceleration, $\ddot{x}(t) = -\omega_n^2 x_{\max} \sin(\omega_n t)$

Let $\delta = x_{\max}$ (max displacement) and let $a = \ddot{x}_{\max}$ (max acceleration)

$\omega_n = \text{natural frequency} = 2\pi f_n = 2\pi(30) = 188.5\ \text{rad/s}$

$a = \omega_n^2 \delta$ **(Eq. 2-18)**

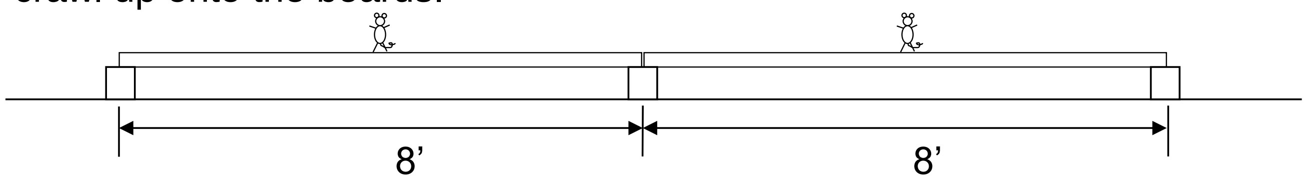
$$\delta = \frac{a}{\omega_n^2} = \frac{8\left(\overset{g}{386.1}\ \text{in}/s^2\right)}{\left(188.5\ \ \text{rad}/s\right)^2} = 0.087\ \text{in} = 2.2\ \text{mm}$$

If acceleration is in g's and displacement is in inches, $\delta \approx \dfrac{10a}{f_n^2}$ **(Eq. 2-19)**

$a \approx \dfrac{\delta f_n^2}{10}$ **(Eq. 2-20)**

Of Mice and Boards

In his garage, Bert lays two boards of the same size out on bricks that are 8 feet (2.44 m) apart (long garage). He then climbs a ladder to get something from the attic, and two mice crawl up onto the boards.



Bert opens the attic door and then slams it behind him. The boards vibrate (first bending mode).

What caused the vibration?

Answer:

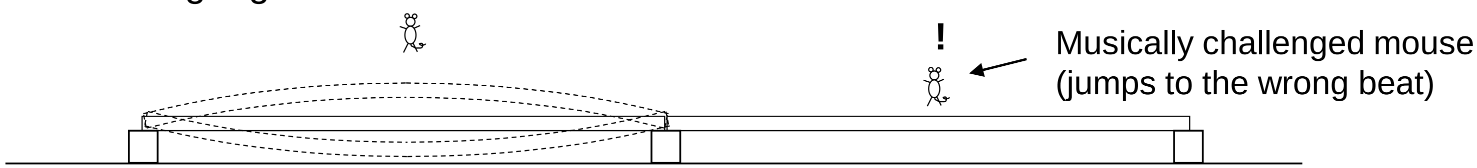
An air-pressure pulse caused the boards to vibrate.

A similar pulse occurs when rocket engines ignite on the pad. The pressure pulse reflects off of surrounding structures and hits the launch vehicle, exciting low-frequency modes of vibration. This pressure pulse is referred to as *overpressure*.

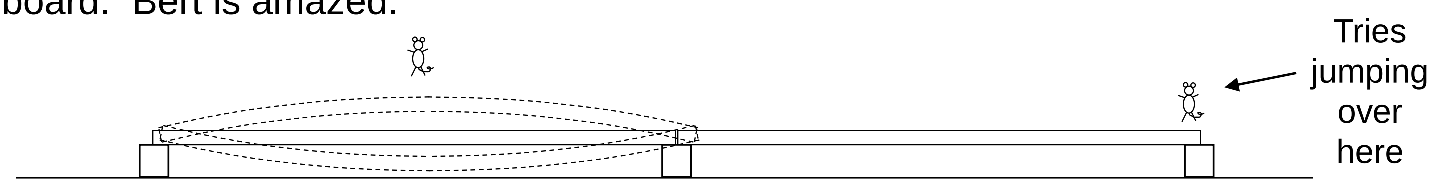
continued

Of Mice and Boards, continued

Of the two mice, one has rhythm, feels the beat, and starts jumping to it. As a result, the vibration of his board begins to amplify, and soon the mouse is flying up and down having a grand time.



Bert comes back down the ladder and sees that one of the boards is bending up and down, with increasing amplitude. He notices the mouse flying up and down with the board. Bert is amazed.



Bert measures and finds the mouse is flying up to a height of 10 inches (25.4 cm) off the floor and is flying up 7 times in a 10-second period. Bert then runs next door to get his neighb~~or, Ern~~ie   Joe.

continued

Of Mice and Boards, continued

When Bert returns with Joe, the one board is vibrating, but the mouse is tiring. Bert measures again and finds the mouse is now reaching a height of 8 inches (20.3 cm) off the floor. He again counts how many times the mouse leaves its feet in a 10-second period.

Assuming the board is a linear system, how many did he count?

Answer: 7

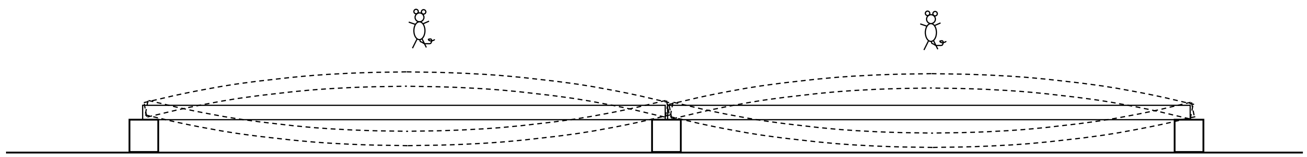
What is the board's fundamental frequency?

Answer: 0.7 Hz

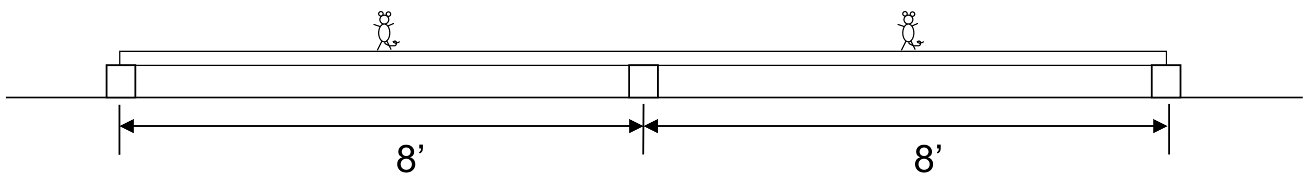
continued

Of Mice and Boards, continued

As Bert and Joe watch, the second mouse finally gets the hang of it and gets its board vibrating also. How many times per 10-second period is he jumping?



Joe runs next door and then comes back with a board of the same wood type and cross section that is 16 feet long, twice the length of the other two boards. Bert and Joe pick up the mice, take the boards off the bricks, place the 16' board on the supports, and then put the mice back in their places.



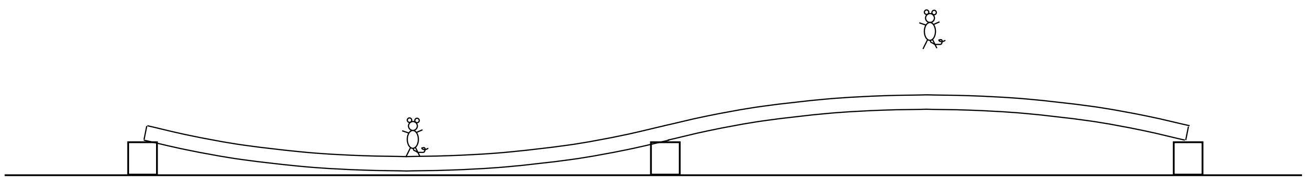
The mouse with rhythm doesn't know what to think, but the musically challenged one immediately starts jumping at a frequency of seven jumps every ten seconds. (Once trained, it's hard to break the habit.)

What happens?

continued

Of Mice and Boards, continued

The board vibrates in response to the mouse (0.7 Hz), and this motion gets the mouse with rhythm jumping at the same frequency but out of phase with the challenged mouse. Soon the mice are flying high again.



Why does the 16’ board have the same fundamental frequency as the 8’ board?

Answer: The mode shape is the same (zero bending moment in the beam at the center support)

continued

Of Mice and Boards, continued

Sensing something wrong, the mice quit jumping, and the board gradually stops vibrating.

What caused the vibration to stop?

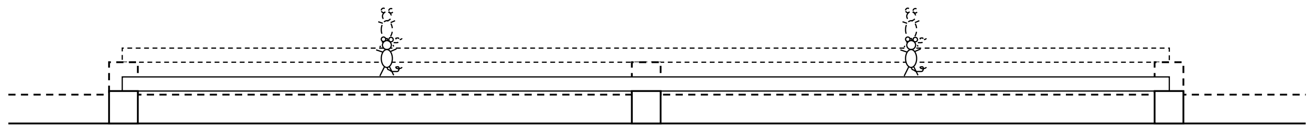
Answer: Damping
(mainly the inherent damping in the wood)

An earthquake begins. The garage floor starts to move up and down in phase (e.g., the entire floor up at the same time) at a frequency of 0.7 Hz.

What happens?

continued

Of Mice and Boards, conclusion



The board moves with the floor nearly as a rigid body, with very little dynamic response. The 0.7-Hz mode is not excited.

Why?

Answer: The 0.7-Hz mode has no modal effective mass (half the mass moves up while the other half is moves down). Without modal effective mass, a mode can't be excited by base acceleration.

Remember this when planning a test on a shaker. If one of the test's objectives requires excitation of a mode that has no modal effective mass in test configuration, you'll have to design a different test.

What would have happened to the two 8' boards?

Answer: The 0.7-Hz modes will be excited, with both boards moving up and then down simultaneously.

Key Points from This Section

- Get in the habit of drawing free-body diagrams to build understanding of how your structures carry loads.
- Statically determinate interfaces ensure distortion in one structure does not cause loads and distortion in the other structure.
- Everything, whether standing still or accelerating, is always in force equilibrium.
  - An object accelerates when the <u>external</u> forces don't balance, in which case the external force is balanced by an internal (inertia) force.
- The equation of motion is simply an equation of equilibrium.
- When damping is low, any point in a linear structure vibrating in a single mode of vibration acts like a mass on a spring.
- The extent to which a dynamically applied force excites a mode of vibration for a structure depends on where and how the force is applied as well as the frequency at which the force is applied.

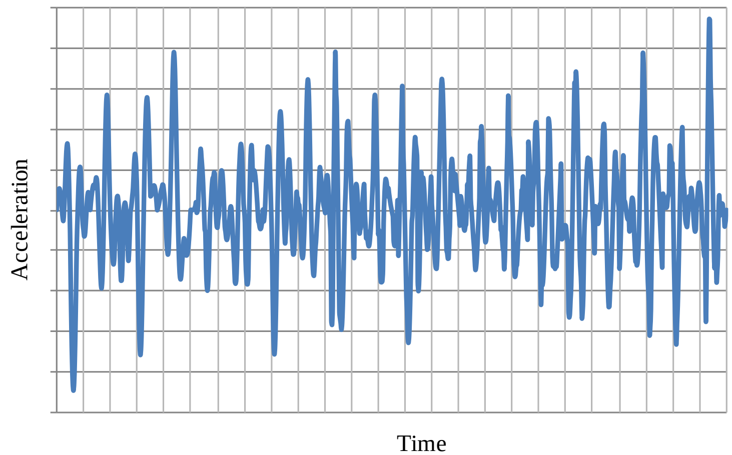
Appendix to Section 2

Introduction to Random Vibration

from my course Spacecraft Structures, from Concept to Launch (SCS)

Introduction to Random Vibration (RV)

Random vibration consists of multiple frequencies of acceleration, each with random amplitude, acting simultaneously.



Example random combination of waves of different frequency and amplitude

In the space industry, we traditionally quantify RV over the frequency range of 20 – 2000 Hz.

- During launch, RV is generated mostly by response of panels and shells to random acoustic pressure, but also by engine vibration and random aerodynamic pressure (turbulence).
  - – The launch-vehicle and spacecraft structures transmit vibration to mounted components.
- Acoustic levels peak at liftoff or during the transonic phase, with total duration of significant levels typically less than 15 seconds.
- Lower frequency RV from turbulence can last longer, perhaps 30 seconds.

Material Rupture as a Result of Random Vibration

Rupture of brittle materials is a result of the peak load (strength failure).

Rupture of ductile materials is usually from the accumulation of loading cycles (fatigue).

When a ductile material fails during random vibration testing, it's usually a fatigue failure.

A main objective of qualification testing of test-dedicated (nonflight) hardware is to verify (or build confidence in) fatigue life.

The duration of exposure to the environment is thus important.

Acceleration Power Spectral Density (PSD)

- By definition, random acceleration is not predictable at any point in time, so we don't use a time history of acceleration to define the environment.
- Instead, we define the environment by its frequency content with PSD.
- Starting with a representative time history, we break the full frequency range into bands and calculate the PSD for each band, including only the frequency content of acceleration within that band.
- For a given frequency band, with f as the center, the PSD is the mean-square acceleration within that frequency band divided by the bandwidth, Δf.

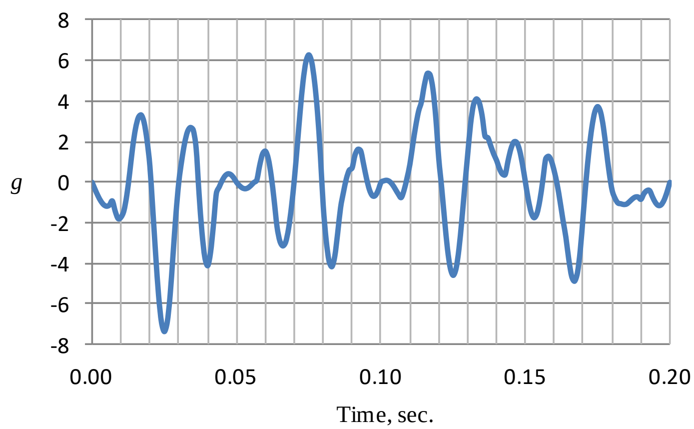
$$\text{PSD, } W(f) = \frac{1}{\Delta f}\overbrace{\left[\frac{1}{T}\int_0^T a^2 dt\right]}^{\textbf{mean square}}$$

where a is the acceleration within the frequency band, and T is the duration of the sample time history

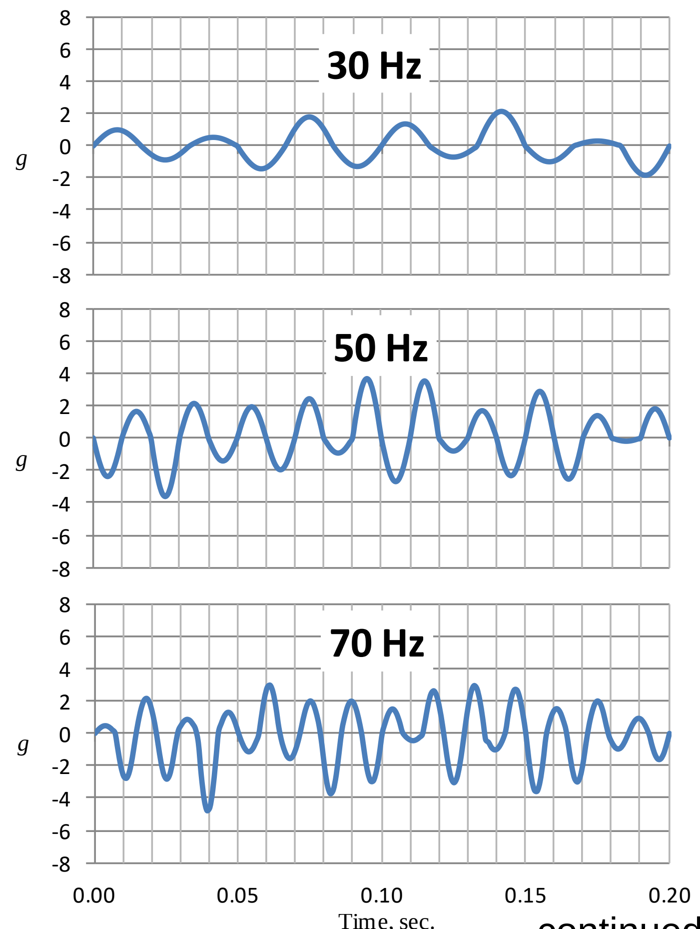
Power spectral density can be used for processing of any random signal that varies with time. The mean-square value of any random signal is referred to as the ***power*** of that signal.

Example Problem 2-c: Understanding Random Vibration

Say we record acceleration over a 0.2-sec interval:



Then we break the signal down into three sine functions with random amplitude

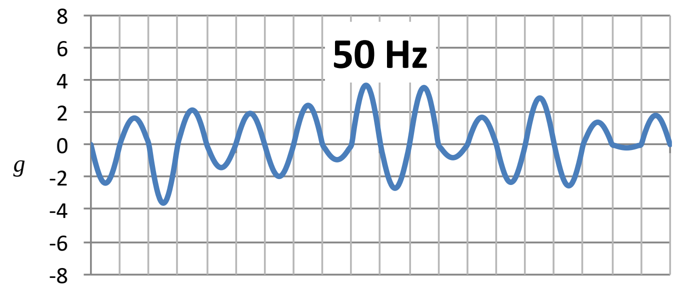


Here, the frequency resolution (bandwidth) is 20 Hz, and the bands are 20 – 40 Hz, 40 – 60 Hz, and 60 – 80 Hz.

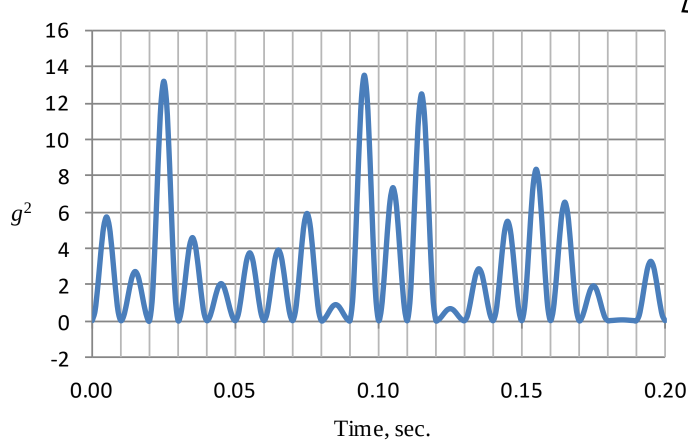
continued

Example Problem 2-c, continued

We'd like to characterize each random sine function, but calculating the average acceleration is of little use; we know the average is zero.



Squaring the acceleration makes all values positive:



We calculate the average squared (mean square) acceleration by taking the area under the curve and dividing it by the duration, T (0.20 sec):

$$\bar{a}^2_{40Hz} = \frac{1}{T}\int_0^T a^2 dt$$

We then divide this value by the frequency bandwidth (20 Hz in this example) to get the PSD in g^2/Hz.

continued

Example Problem 2-c, continued: Calculating PSD and RMS

Overall acceleration (g) — 30-Hz acceleration — 50-Hz acceleration — 70-Hz acceleration

t (sec)	a	a_1	a_1^2	a_1^2 dt	a_2	a_2^2	a_2^2 dt	a_3	a_3^2	a_3^2 dt
0.000	0.00	0.00	0.00	0.00000	0.00	0.00	0.00000	0.00	0.00	0.00000
0.001	-0.34	0.19	0.04	0.00004	-0.74	0.54	0.00054	0.21	0.04	0.00004
0.002	-0.66	0.37	0.14	0.00014	-1.40	1.97	0.00197	0.37	0.14	0.00014
0.003	-0.92	0.54	0.30	0.00030	-1.93	3.73	0.00373	0.47	0.22	0.00022
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
0.197	-1.10	-0.98	0.95	0.00095	1.46	2.13	0.00213	-1.59	2.51	0.00251
0.198	-0.87	-0.67	0.45	0.00045	1.06	1.13	0.00113	-1.26	1.59	0.00159
0.199	-0.48	-0.34	0.12	0.00012	0.56	0.31	0.00031	-0.70	0.49	0.00049
0.200	0.00	0.00	0.00	0.00000	0.00	0.00	0.00000	0.00	0.00	0.00000
Sum (area under the curve), g^2-sec				0.17108			0.52371			0.58094
Mean square (sum ÷ 0.2 sec), g^2				0.86			2.62			2.90
PSD, (mean square ÷ 20 Hz), g^2/Hz				0.043			0.131			0.145

Overall, the mean square acceleration is 0.86 + 2.62 + 2.90 = 6.38 g^2

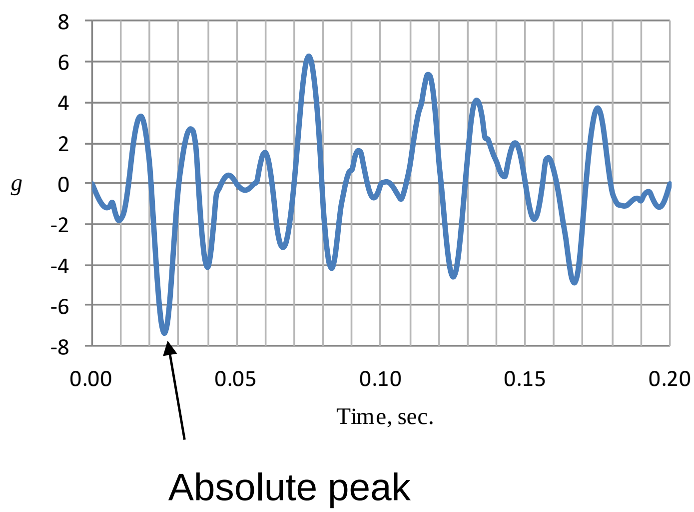
The square root of this value is the ***root-mean-square*** (RMS) acceleration: a_{rms} = 2.53 g, which can be written as 2.53 g_{rms} to indicate the acceleration is calculated as an RMS.

The RMS acceleration is the standard deviation, σ, of the random acceleration.

continued

Example Problem 2-a, conclusion: Peak Acceleration

From the original time history, we see the absolute peak acceleration is 7.35 *g.*



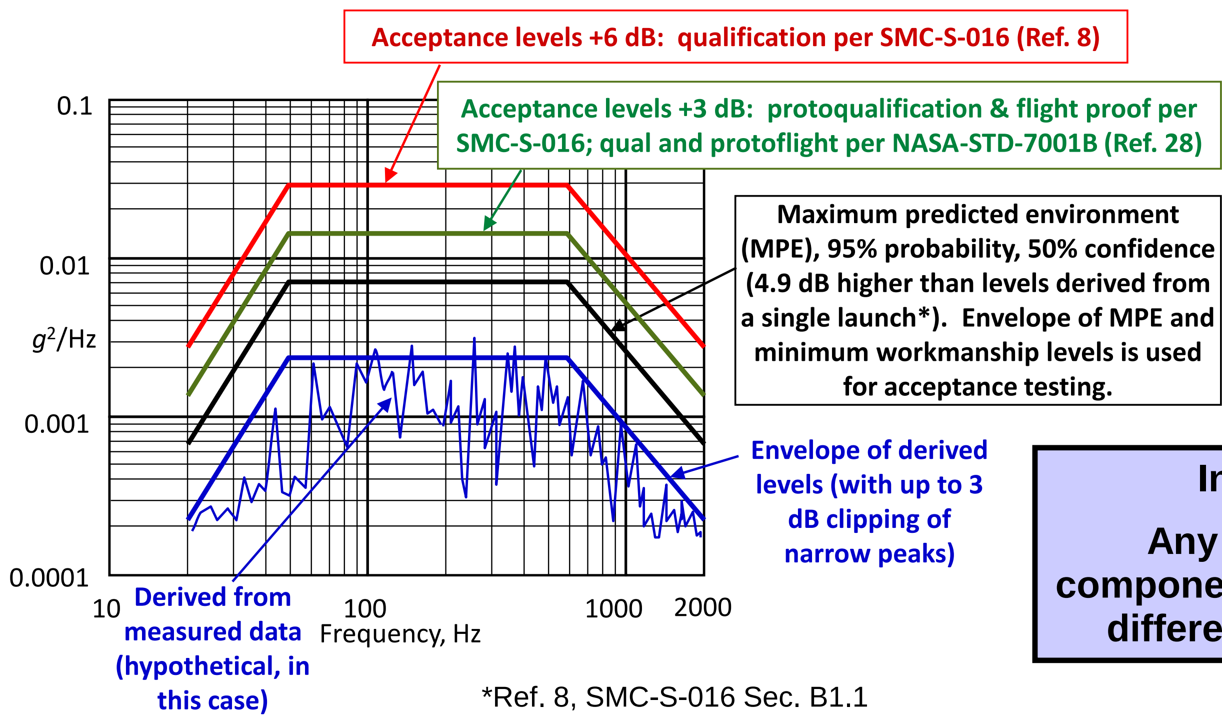
- With the standard deviation, σ = 2.53 *g*, the 7.35*g* peak is a 2.9σ value.
- This peak occurred in just 0.20 sec.
- If we sample a longer time span—say, the full 60 sec of a test—we would expect the peak value to be considerably higher.

It's often assumed that the peak load is 3σ, but the actual peak load during a random vibration event is usually well higher than that.

How Test Environments Are Derived from Launch Data

For easy control during test—and to account for uncertainty—we smooth out the PSD plot to envelop any significant peaks. Then we increase the levels to obtain 95% probability and 50% confidence, and add margin for test.

Example:



*Ref. 8, SMC-S-016 Sec. B1.1

The RMS (root mean square) acceleration is the square root of the area under the PSD curve and is equal to the standard deviation of random acceleration.

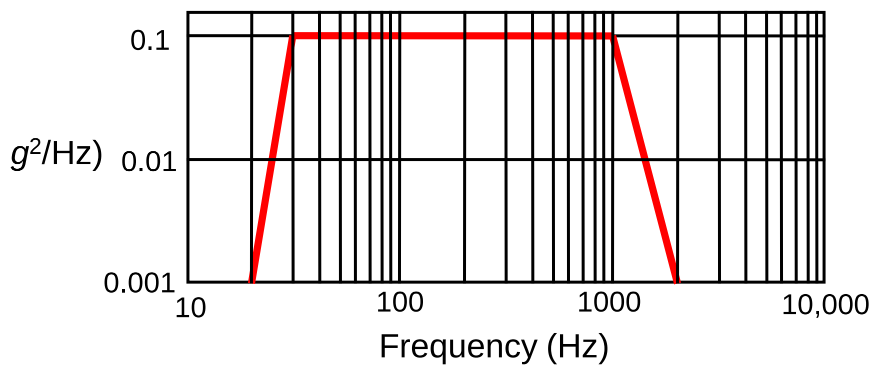
Input ≠ Response

Any point on a vibrating component will have its own PSD, different than the input PSD.

Example Problem 2-d:
Estimating Response to Base-Driven Random Vibration

An electronics box will be subjected to a single-axis random vibration test.

Test environment: 10.7 g_{rms}



SSAM Example 5.3

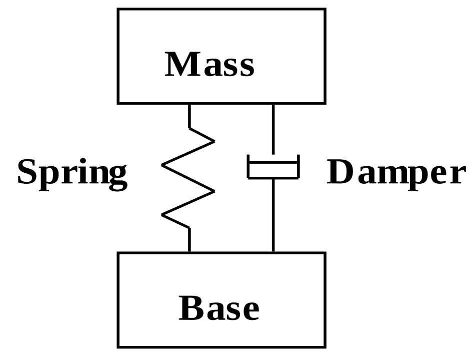
Problem statement: Derive a limit load for sizing the housing and the mounting bolts.

continued

Example Problem 2-d, continued

Solution:

Idealize the structure as a base-driven single-DOF system.



- We are assuming that the housing and mounting bolts are stressed mostly by the component's fundamental vibration mode, in which most of the mass moves in the same direction.
- Let's say we predict a fundamental frequency, f_n, of 43.6 Hz.
- We will assume the damping for this mode is 5% of critical (a reasonable estimate for an electronics box).

continued

Example Problem 2-d, continued

We will use a frequency-domain solution process: ***Miles' equation***, an approximation based on the assumption of ***white noise*** input (constant PSD from zero to infinity Hz).

Input PSD at the natural frequency f_n

RMS response acceleration, $$a_{rms} \cong \sqrt{\frac{\pi f_n W}{4\zeta}} \cong \sqrt{\frac{\pi Q f_n W}{2}}$$

Damping ratio

$$= \sqrt{\frac{\pi(43.6)(0.100)}{4(0.05)}} = 8.28\, g_{\text{rms}}$$

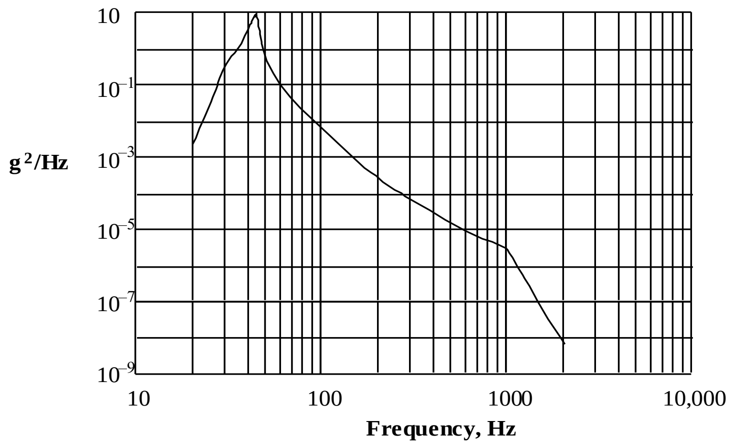
This is the standard deviation of the random response.

Note: The response would be 18.5g_{rms} if damping were 1% rather than 5%.

continued

Example Problem 2-d, conclusion

Here is the computed response PSD based on the Fourier transform, which is the exact solution if the model is accurate:



From numerical integration of this curve, the RMS response acceleration is

$$\ddot{x}_{1_{rms}} = \sqrt{\int_0^{\infty} W_{\ddot{x}_1}(f)df} = 8.07g_{\text{rms}}$$

vs. 8.28 g_{rms} from Miles' equation

Remember: This is the standard deviation, σ, of random response.

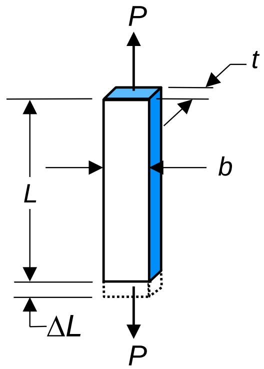
Miles' equation works pretty well when …

- **there is a single dominant response mode;**
- **the specified PSD is relatively constant near the item's fundamental frequency;**
- **the specified PSD does not exceed, at any frequency, the value used in the equation;**
- **and damping < 10% of critical.**

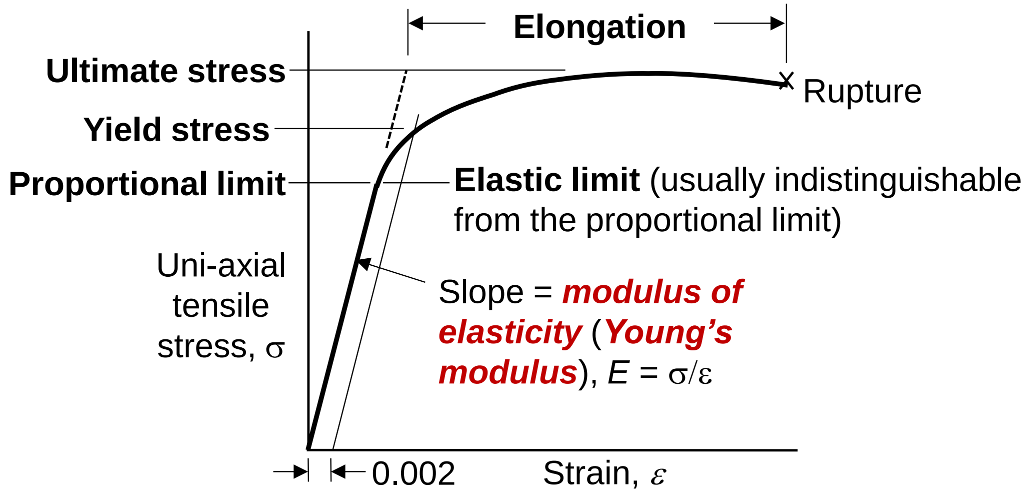
3. Mechanics of Materials

- Stress and Strain
- Combined State of Stress
- Principal Stresses and Mohr's Circle
- Beams and Bending Stress
- Unsymmetrical Bending
- Torsion and the Effects of Warping Constraint
- Thermal Effects

Stress and Strain—Tensile Loading



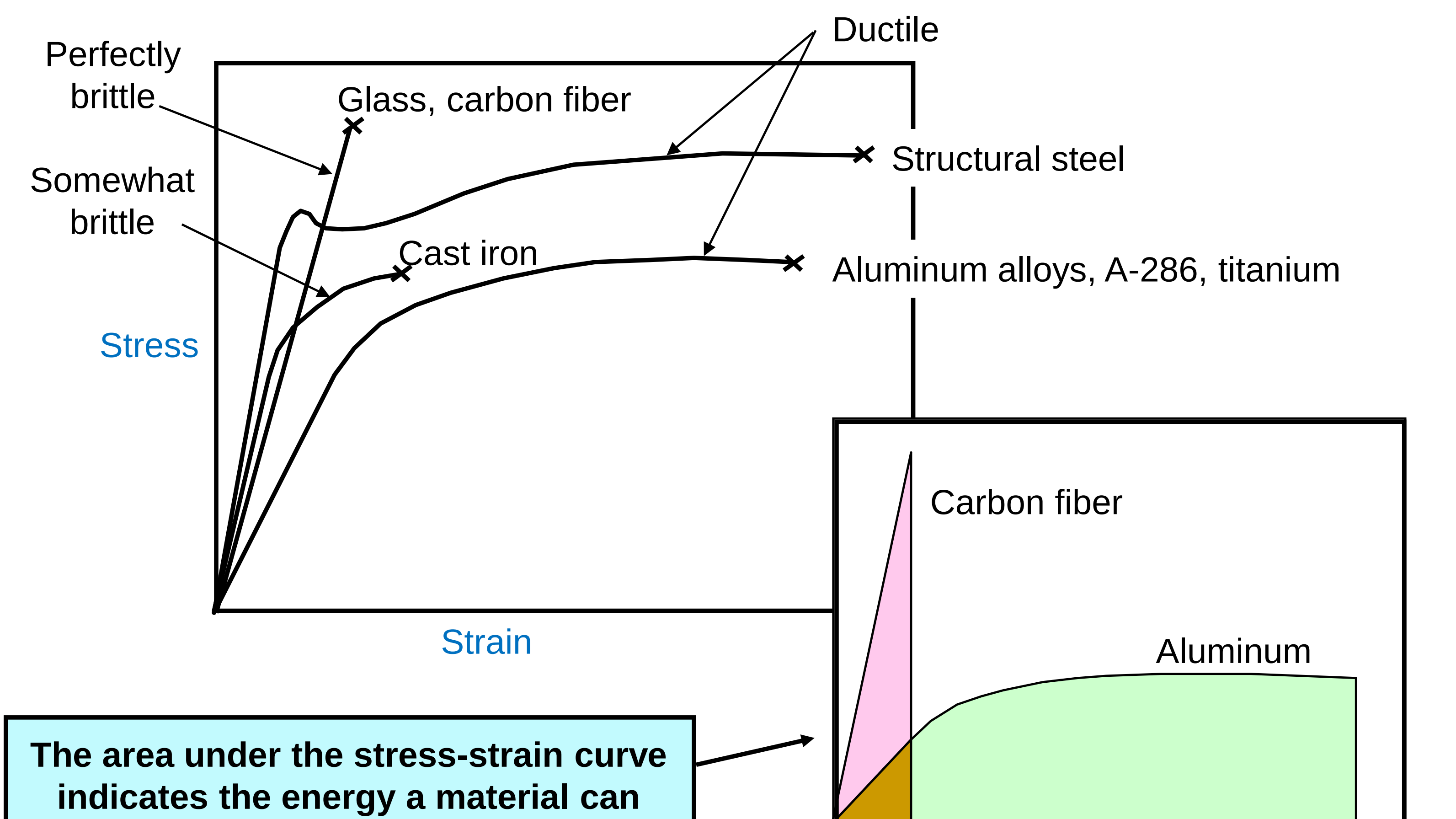
Stress, $\sigma = \dfrac{P}{bt}$ ***Strain***, $\varepsilon = \dfrac{\Delta L}{L}$



- Up to the ***proportional limit***, a material acts like a linear-elastic spring.
- Stress above the ***elastic limit*** will cause permanent strain.
- For materials with stress-strain curves of the above shape, the tensile ***yield stress*** (yield strength) is traditionally defined as the stress that causes 0.2% permanent strain.
- The ***ultimate stress*** (ultimate strength) is the highest stress the material can withstand.
- ***Elongation*** (plastic strain to rupture) is a measure of the material's ductility.

Where stress concentrates, such as in fastened joints, we want to use materials with good (> 10%) elongation so that loads and stresses can redistribute before rupture occurs.

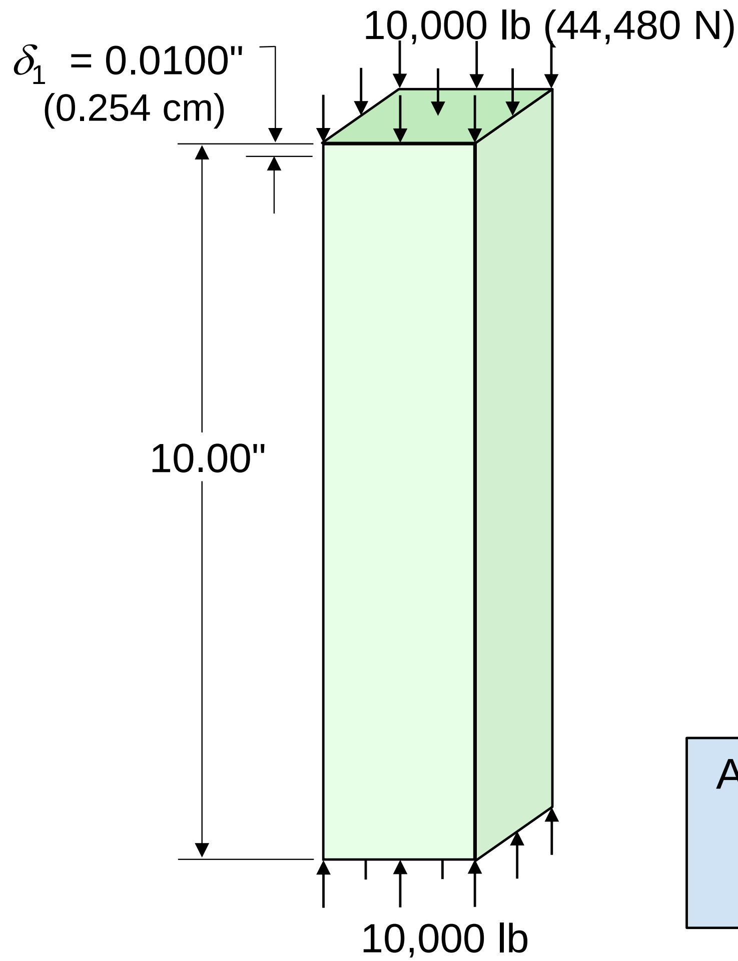
Forms of Stress-Strain Curves



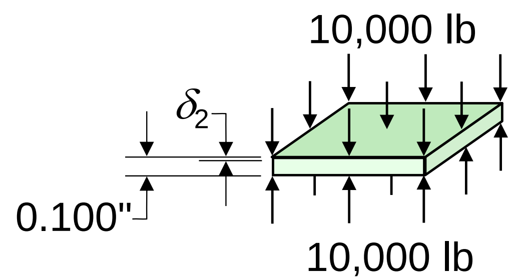
The area under the stress-strain curve indicates the energy a material can absorb before rupturing.

Energy absorption: a key benefit of ductility

Two Specimens in a Compression Test Machine



Same material, same cross section, 1/100th as long:

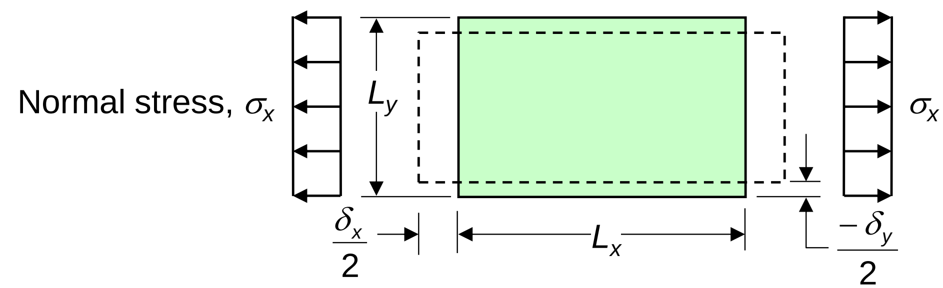


What is the deflection, δ_2?

Answer: Less than 0.0001" because friction from the test machine constrains lateral bulging (***Poisson effect***)

Poisson's Ratio

A material will stretch in the direction of applied tension and shrink in the other two directions—if unconstrained in those directions.



SSAM (Ref. 1) Fig. 6.3

Poisson's ratio, $\nu = \dfrac{-\varepsilon_y}{\varepsilon_x} = \dfrac{-\delta_y / L_y}{\delta_x / L_x}$ ≈ 0.3 for most metals (0.33 for aluminum alloys)

(Eq. 3.3)

Stress/strain curves apply directly only to uniaxial stress.

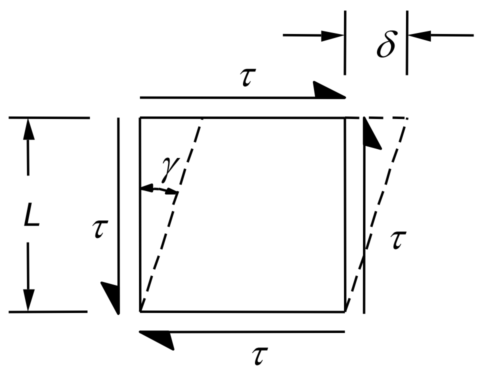
If a material in tension is constrained from shrinking in the two lateral directions, it will not be in uniaxial stress. It won't stretch as far as the stress/strain curve says.

Shear

Shear load, V, is load acting parallel to a surface.

Shear stress, $\tau = V/A$, where A = area **(Eq. 3.4)**

Deformation of a material in pure shear:

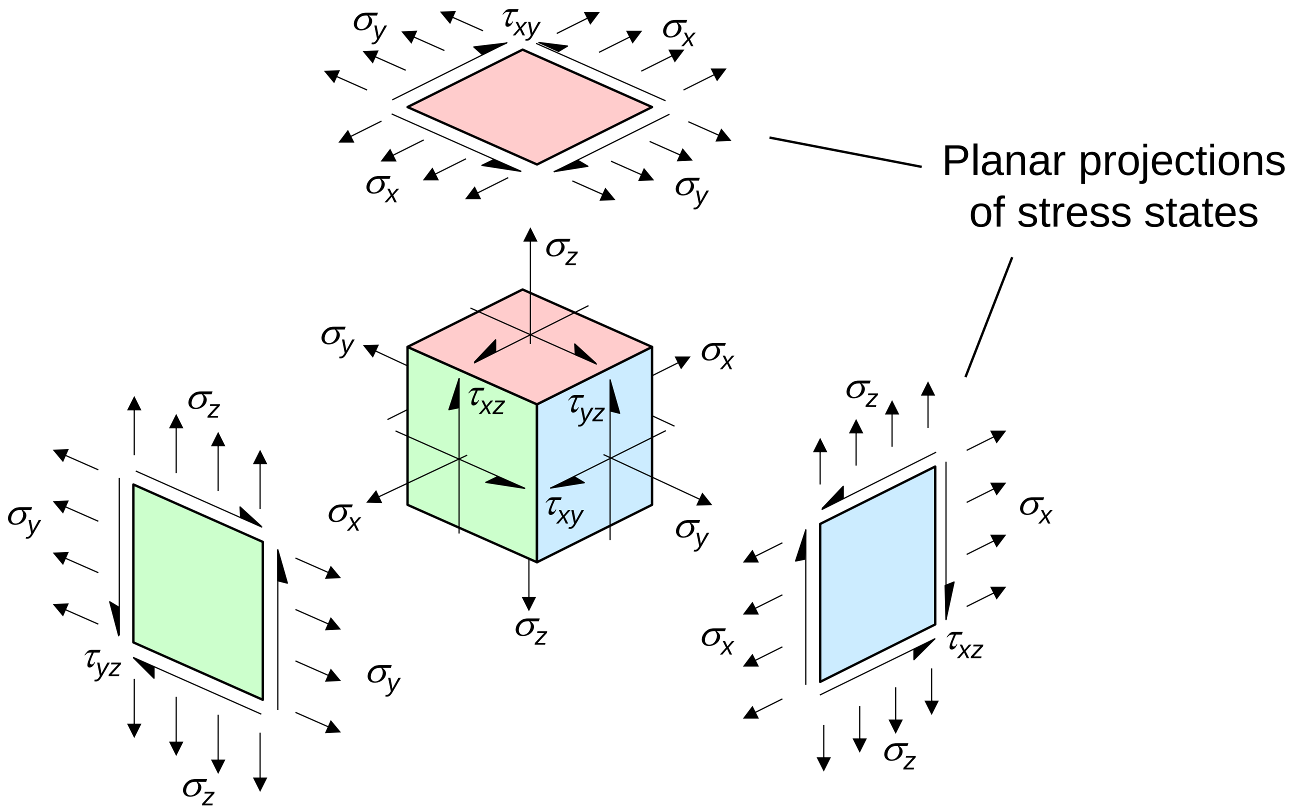


Shear strain, $\gamma = \dfrac{\delta}{L} = \dfrac{\tau}{G}$ **(Eq. 3.5)**

Shear modulus, $G = \dfrac{E}{2(1+\nu)}$ **(Eq. 3.6)**

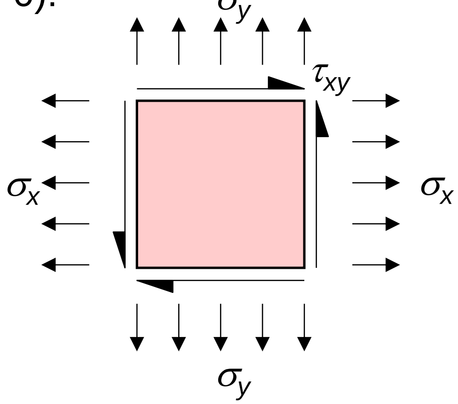
E = Young's modulus
ν = Poisson's ratio

The Three-dimensional State of Stress

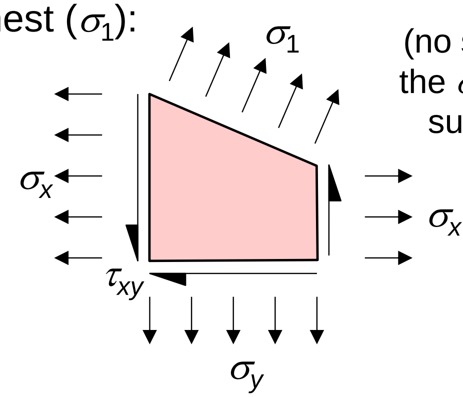


Principal Stresses for 2-dimensional Stress (*Plane Stress*)

1. Given a two-dimensional state of stress ($\sigma_z = \tau_{yz} = \tau_{xz} = 0$):

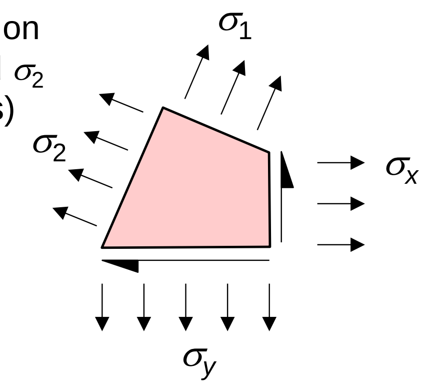


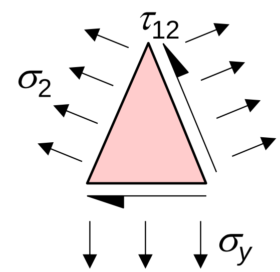
2. Based on geometry, at some angle the normal stress is highest (σ_1):



(no shear on the σ_1 and σ_2 surfaces)

3. 90˚ away from σ_1, the normal stress is lowest (σ_2).



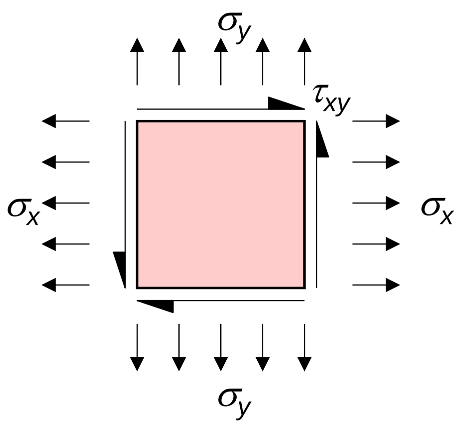


4. At 45˚ from either of those cuts, the shear stress peaks (τ_{12}).

Principal shear stress: $$\tau_{12} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \textbf{(Eq. 3.7)}$$

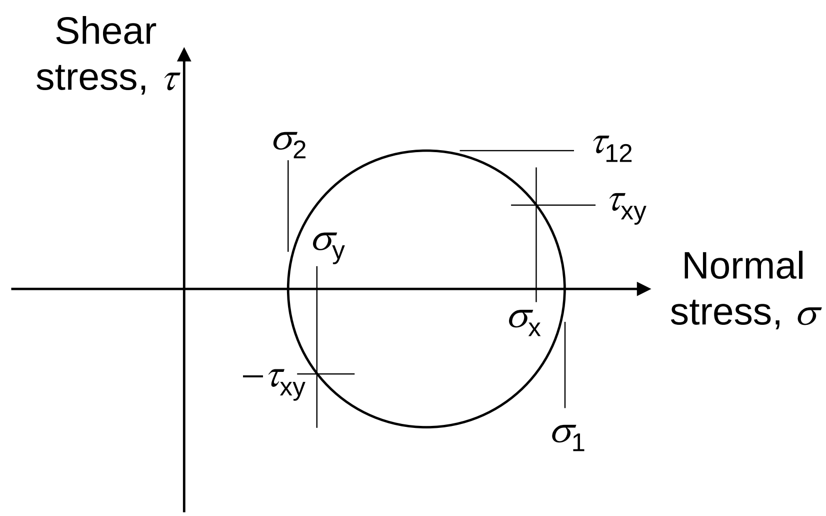
Principal normal stresses: $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \tau_{12} \quad \textbf{(Eq. 3.8)}$$

Mohr's Circle for Plane Stress



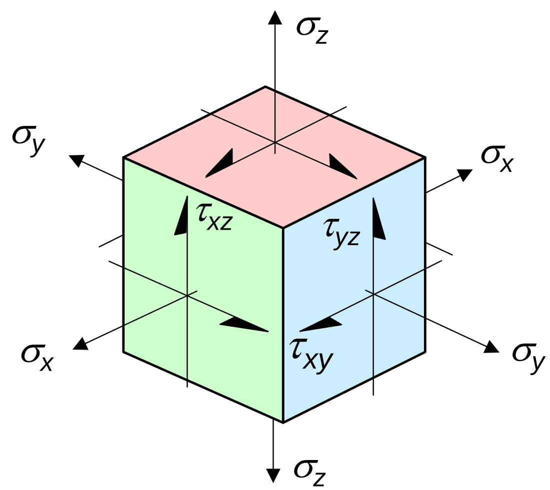
$\sigma_z = \tau_{xz} = \tau_{yz} = 0$

The two plotted points (σ_x, τ_{xy}) and (σ_y, -τ_{xy}) define a circle (***Mohr's circle***). The extremities of this circle are the principal stresses.

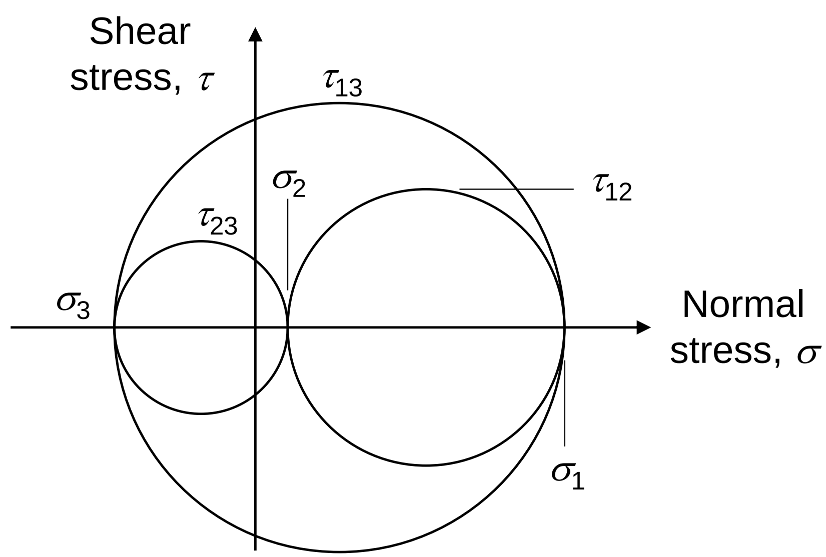


Principal normal stresses are 90° apart; thus, 90° in a material translates to 180° in Mohr's Circle.

Mohr's Circles for 3-dimensional Stress



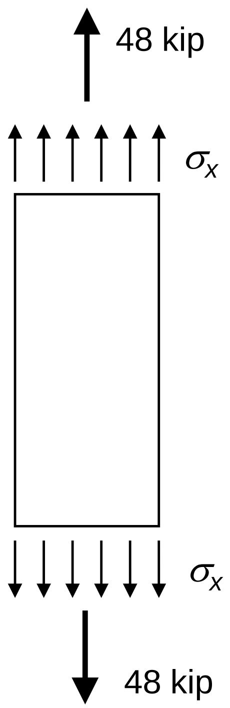
- Calculating the principal stresses for a 3-dimensional stress state is more complicated*.
- The three principal normal stresses are orthogonal.
- Example of Mohr's circles:



*For details, see Ref. 3.

Class Problem 3-1: Calculating Stresses in a Tensile Specimen

A test specimen with a cross-sectional area of 0.75 in^2, loaded in uni-axial tension, withstands a maximum load of 48,000 lb before the onset of yielding.



a. Calculate the principal normal stresses and principal shear stress at maximum load.

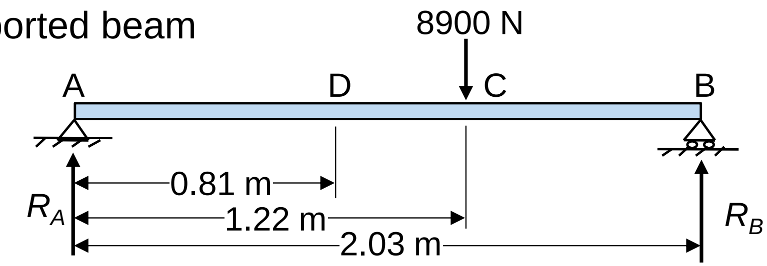
b. At what angle from the applied tensile force did the shear stress peak?

c. Draw a free-body diagram that shows the applied tensile force at one end and, at the other end, the forces acting on the surface on which the principal shear stress acts. Calculate the shear and normal forces on that surface needed for static equilibrium.

d. Calculate the shear stress from the shear force in part c, and see if it agrees with the principal shear stress calculated in part a.

Carrying Transverse Load with Beams

Example:
simply supported beam

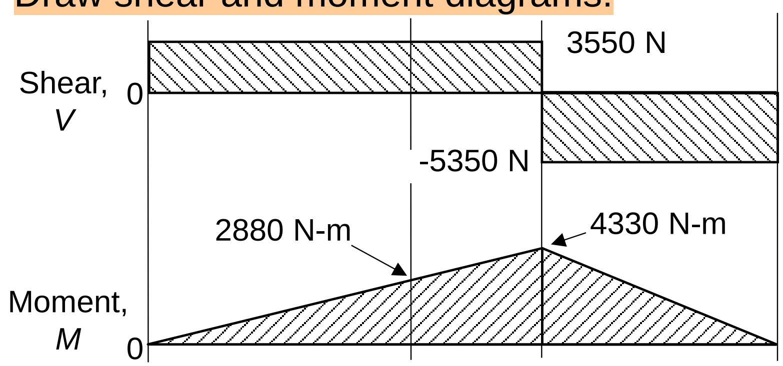


Use statics to solve for reactions:

$$\Sigma M_A = 0 = 122(8900) - 2.03 R_B;\quad R_B = 5350\ \text{N}$$

$$\Sigma F = 0 = -8900 + 5350 + R_A;\quad R_A = 3550\ \text{N}$$

Draw shear and moment diagrams:



Bending moment at points C and D:

$$M_C = (1.22)(3550) = 4330\ \text{N} \cdot \text{m}$$

$$M_D = (0.81)(3550) = 2880\ \text{N} \cdot \text{m}$$

Moment peaks where the shear diagram crosses zero (shear changes sign).

When shear is constant, moment builds linearly.

Example of When Drawing a Moment Diagram Would Have Helped

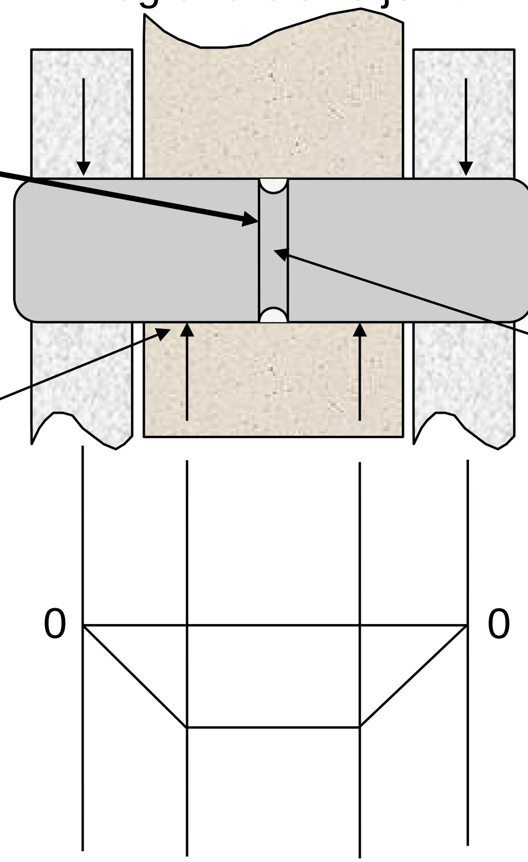
Lug-and-clevis joint

The pin was grooved to accommodate a retention device.

The engineer checked the pin for bending here, at the full cross section.

Assumed free-body diagram shown

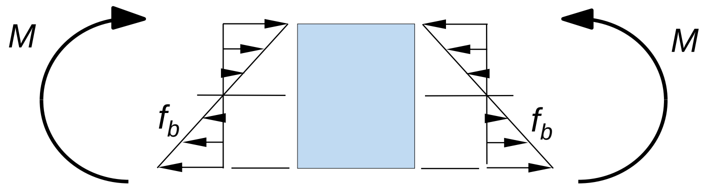
But the pin failed in bending at this cross section.



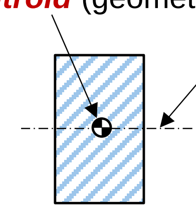
Moment diagram for the assumed loading:

Taking time to draw the moment diagram might have prevented this failure.

Linear-Elastic Bending Stress

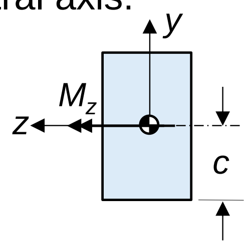


Beam segment under bending moment



Cross section

For a symmetrical section, discounting local effects from any discontinuities, bending stress, f_b, up to the material's proportional limit, is proportional to the distance from the neutral axis:



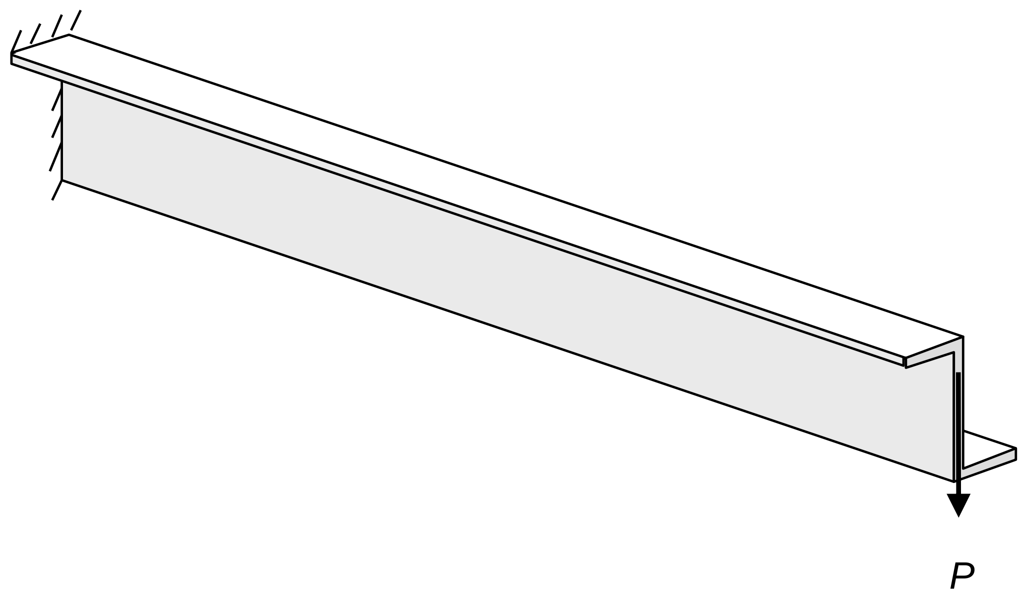
$$f_b = \frac{-M_z y}{I_z}$$ **(Eq. 3.9)** (minus sign implies compression)

Peak stress: $$f_{b\max} = \frac{M_z c}{I_z}$$ **(Eq. 3.10)**

Area moment of inertia, (moment of inertia of the cross section) $$I_z = \int_A y^2 dA$$ **(Eq. 3.11)**

Caution: These equations for bending stress apply only if the section is symmetrical about either the moment axis or the axis perpendicular to the moment axis.

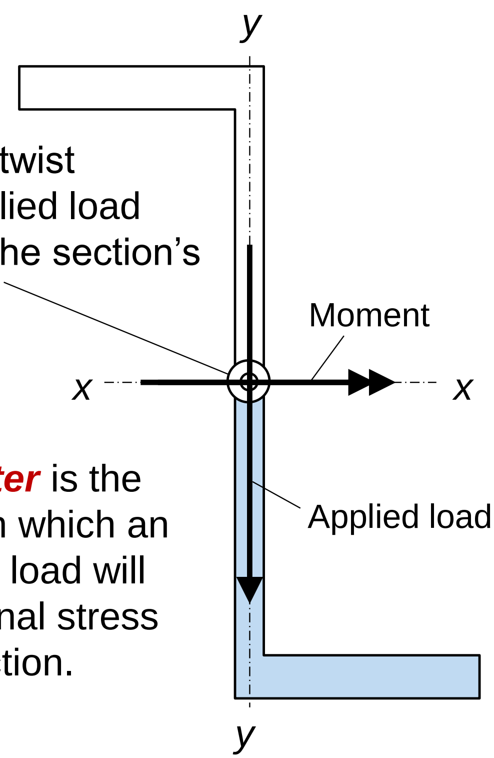
How Will This Beam Deflect?



Intentionally Left Blank

The Beam Will Not Twist; It Will Deflect Down and Sideways

View looking towards fixed end:



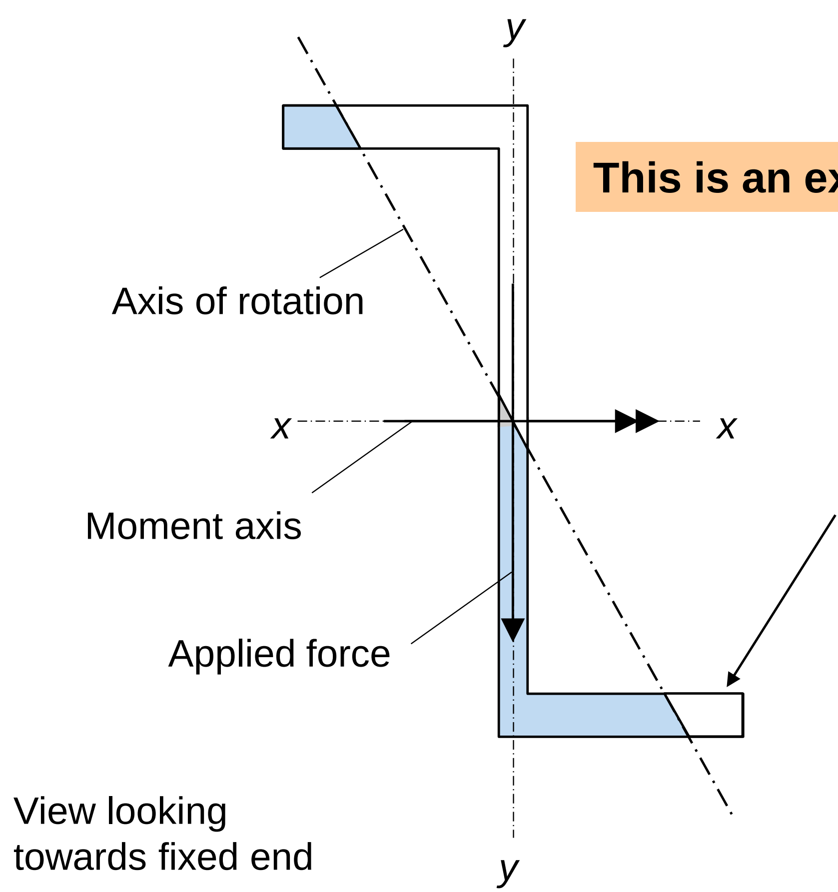
The beam won't twist because the applied load passes through the section's shear center.

The ***shear center*** is the location through which an applied transfer load will cause no torsional stress in the cross section.

- This section can't bend solely about the moment axis, *x-x*, because that would cause all of the shaded material to be in compression and all the unshaded material to be in tension.
- Such loading on the cross-section would cause a moment about the *y-y* axis, which would violate equilibrium. (The applied load is aligned with the *y-y* axis and thus cannot generate a moment about the *y-y* axis.)
- Thus, the neutral axis must be at some angle relative to the *x-x* axis, which means the beam will deflect laterally as well as vertically.

So, will the beam deflect to the left or the right under the loads shown?

The Only Axis of Rotation that Satisfies Equilibrium



This is an example of *unsymmetrical bending*

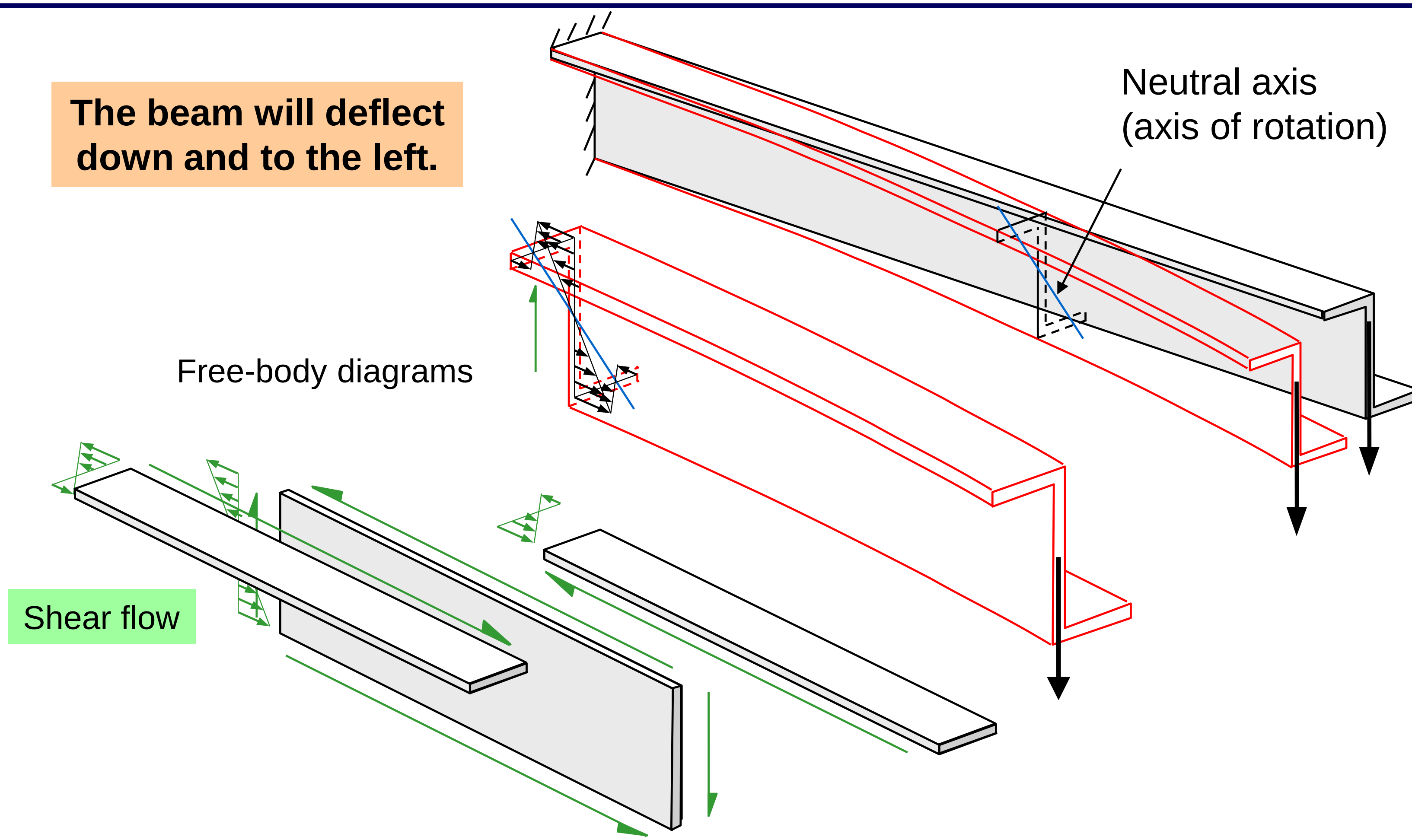
To keep the moment about the *y-y* axis equal to zero, a small tensile force is needed at the tip of the Z section to balance the moment about the *y-y* axis caused by the compressive force in the shaded section.

For an unsymmetrical cross section, we need to calculate the product of inertia, I_{xy}, in addition to the moments of inertia about the *x* and *y* axes.

(See SSAM, Ref. 1, Sec. 6.4.)

Unsymmetrical Bending of a Z-section Beam

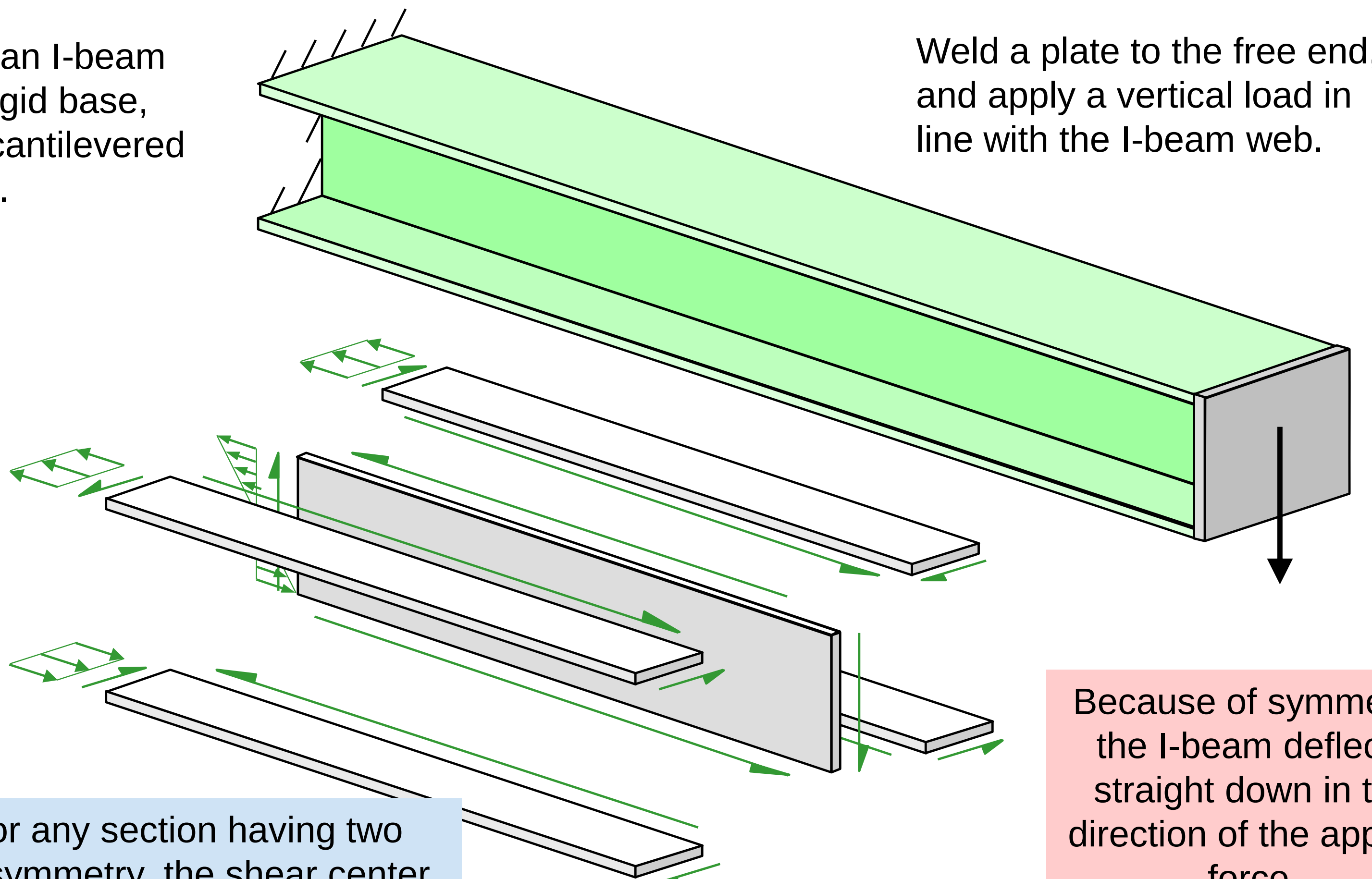
The beam will deflect down and to the left.



Shear Flow in an I-beam

Weld an I-beam to a rigid base, as a cantilevered beam.

Weld a plate to the free end, and apply a vertical load in line with the I-beam web.



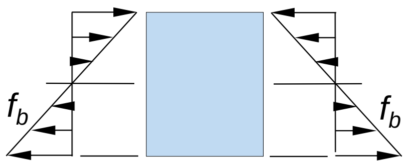
Because of symmetry, the I-beam deflects straight down in the direction of the applied force.

Note: For any section having two axes of symmetry, the shear center coincides with the centroid.

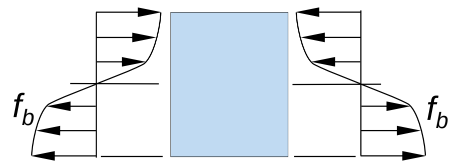
Plastic Bending

As noted earlier, the Eq. 3-10 for peak bending stress, $f_{b\max} = \dfrac{M_z c}{I_z}$, applies only for symmetrical sections.

It also applies only for stress up to the material's proportional limit. Bending <u>strain</u> is proportional to distance from neutral axis; <u>stress</u> is also as long as stress is proportional to strain.



Beam segment under bending moment, with stress ≤ the proportional limit



Plastic bending: The stress distribution takes a shape similar to the stress-strain curve

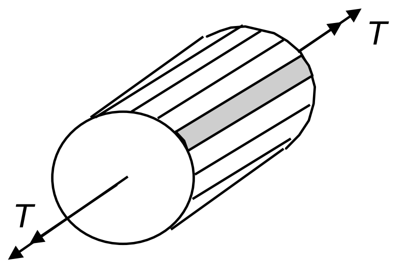
As the result of plastic bending, a solid cross section, when local buckling (discussed in Sec. 4) does not occur, can carry more bending moment than linear-elastic theory predicts.

The ***bending modulus of rupture***, a.k.a. ***plastic bending allowable***, F_{bu}, is a fictitious stress that (for a ductile material) is higher than the material's allowable tensile stress, F_{tu}, for comparison with linearly calculated bending stress (Eq. 3-10).

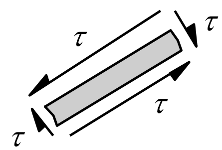
See SSAM Chap. 8 for detailed discussion of plastic bending, along with a method of estimating F_{bu} for different materials and cross sections. More accurate would be to use a stress-strain curve approximated by the Ramberg-Osgood equation. (See Rev. 5a Sec. 1.4.4.7.)

Torsion in a Solid Circular Cross Section

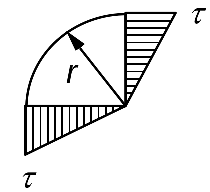
SSAM Fig. 6.24



Twisting of a rod in torsion



The shaded strip is in pure shear.



Shear stress is proportional to radius.

For a solid circular section in torsion, the peak shear stress, τ_{max}, is

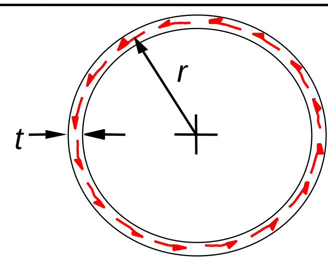
$$\tau_{max} = \frac{Tr}{J} = \frac{2T}{\pi r^3} \quad \textbf{(Eq. 3.12)}$$

Polar moment of inertia, $J = \dfrac{\pi r^4}{2}$ **(Eq. 3.13)**

But these equations apply only to axi-symmetric (solid or closed circular) cross sections.

Torsion In Closed Vs. Open Sections

Closed ⇨

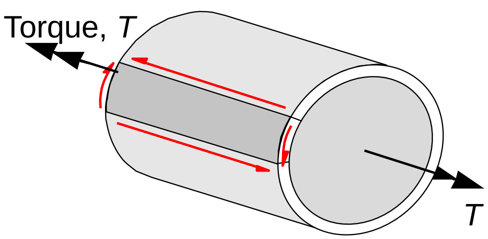


Shear stress,

$$\tau \cong \frac{T}{2\pi r^2 t}$$

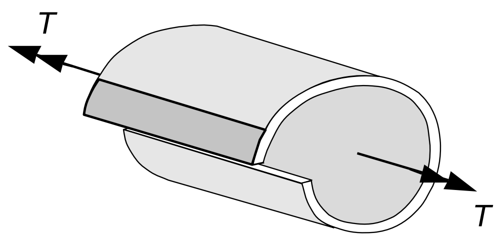
(Eq. 3.14)

a. Torsional shear flow in a thin-walled circular tube

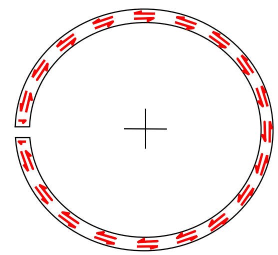


b. The tube wall is in pure shear.

Open ⇨

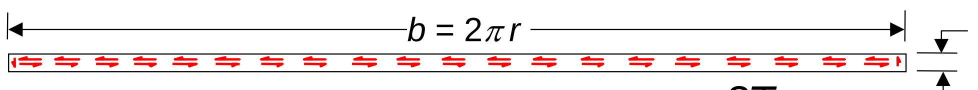


c. If the tube is cut down its length, torque causes the section to warp out of plane.



d. For an open (slit) tube, shear flows as shown.

SSAM
Fig. 6.26



e. The shear flow, torsional stiffness, and peak shear stress are the same as for a thin rectangular section:

$$\tau_{max} \cong \frac{3T}{bt^2}$$

(Eq. 3.15)

Closed sections are much stronger and stiffer in torsion than open sections.

Torsional Stiffness and Stress for Rectangular Sections

General equation for linear-elastic rotational displacement (angle of twist) under torsion:

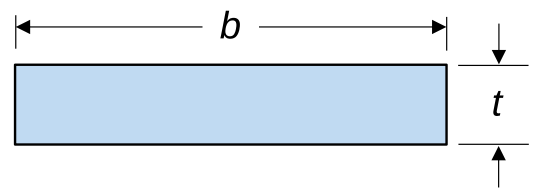
$$\theta = \frac{TL}{GK} \quad \textbf{(Eq. 3.16)}$$

T = torque
L = length
G = shear modulus
K = torsion constant

For a solid or closed circular cross section, $K = J$, the polar moment of inertia.

For a rectangular cross section,

$$K = \beta b t^3 \quad \textbf{(Eq. 3.17)}$$



where

$$\beta \approx \frac{1}{3} - \frac{0.21t}{b}\left[1 - \frac{1}{12}\left(\frac{t}{b}\right)^4\right]$$

(Eq. 3.18)

(from SSAM sec. 6.5)

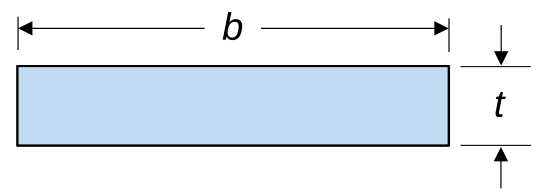
Torsional shear stress for this section:

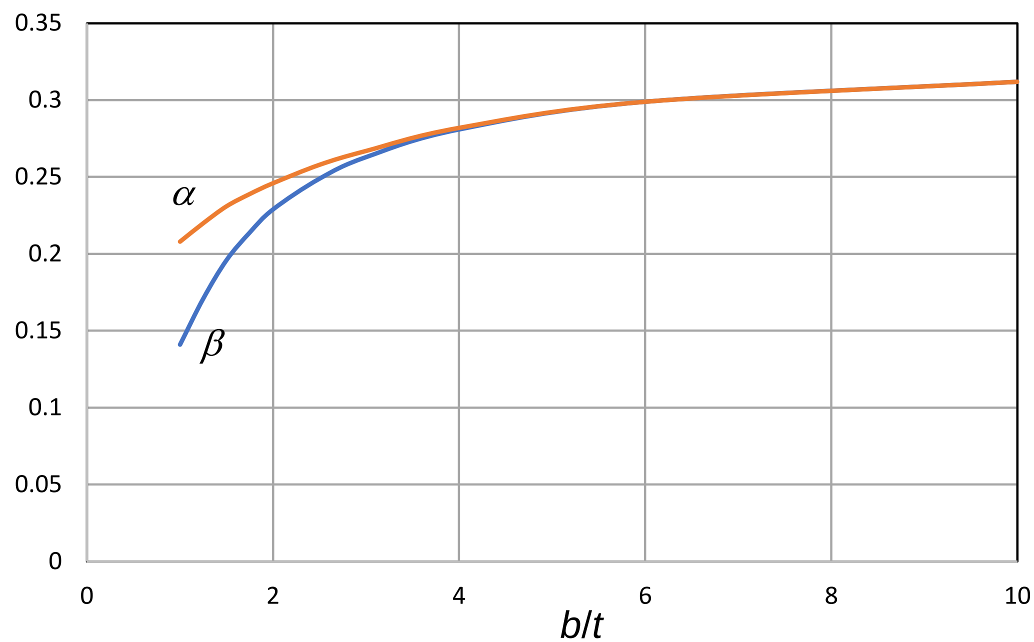
$$\tau_{max} = \frac{T}{\alpha b t^2} \quad \textbf{(Eq. 3.19)}$$

where $\alpha \approx \beta\left[1 - \frac{0.811}{\cosh(1.57b/t)}\right]^{-1}$ **(Eq. 3.20)**

Tabulated and Plotted Values of β and α

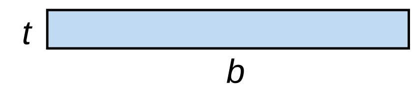
b/t	β	α
1	0.141	0.208
1.25	0.171	0.220
1.5	0.196	0.231
1.75	0.214	0.239
2	0.229	0.246
2.5	0.249	0.258
3	0.263	0.267
4	0.281	0.282
6	0.299	0.299
10	0.312	0.312
30	0.326	0.326
100	0.331	0.331





Calculating an Open Section's Torsion Constant

Referring back to Eq. 3.16, $\theta = \dfrac{TL}{GK}$ and Eq. 3.17 for a rectangular section, $K = \beta b t^3$



For an open section that can be decomposed into rectangular elements, <u>when the ends of the beam are free to warp out of plane:</u>

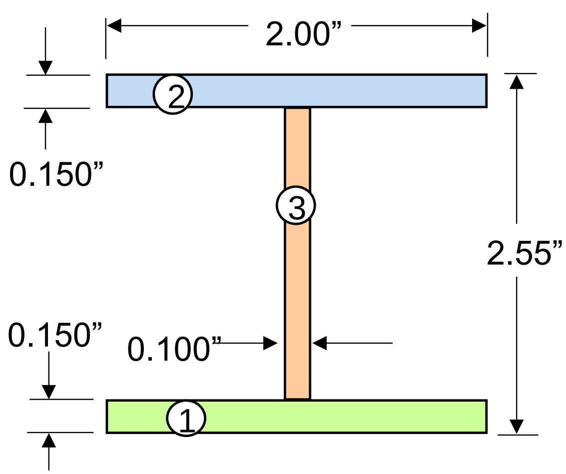
$$K \cong \sum_{i=1}^{n} \beta_i b_i t_i^3$$ **(Eq. 3.21)** (SSAM Eq. 6.52)

and

$$\tau_{max} \approx T t_{max} \left[\sum_{i=1}^{n} \alpha_i b_i t_i^3 \right]^{-1}$$ **(Eq. 3.22)** (SSAM Eq. 6.53)

Applied torque — Max thickness

Example:



Element	b	t	b/t	β	α	βbt^3	αbt^3
1	2.00	0.15	13.3	0.318	0.318	0.00214	0.00214
2	2.00	0.15	13.3	0.318	0.318	0.00214	0.00214
3	2.25	0.10	22.5	0.324	0.324	0.00073	0.00073
Total						0.00502	0.00502

K

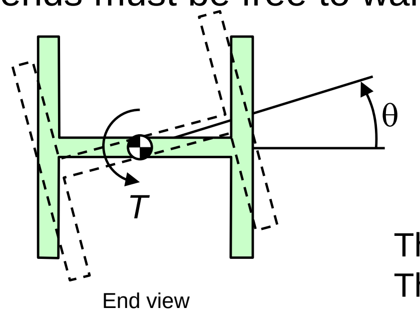
$$\tau_{max} \approx \frac{0.150T}{0.00502} = 29.9T$$

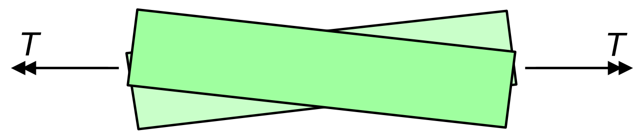
This approach ignores the effects of fillet radii in an open section (see Ref. 4 (Roark), Table 10.1 cases 23 & 26)

The Effect of Warping Constraint: Differential Bending

For an open section to twist according to the expression $\theta = \dfrac{TL}{GK}$ the beam ends must be free to warp out of plane:

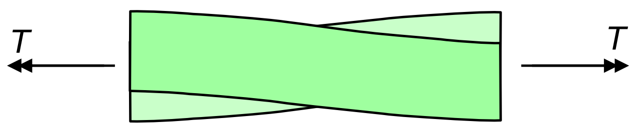
T = Torque
L = Length
G = Shear modulus
K = Torsion constant



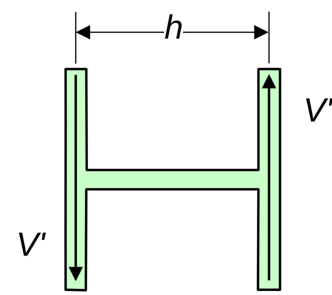


The flanges rotate nearly as rigid bodies in this side view. The ends warp, and torque is carried solely by torsional shear stress in the cross section.

Adapted from SSAM Fig. 6.28



If the ends are constrained against warping, the torsional stiffness is much greater. Torque is carried mainly by equal and opposite flexural shear forces, *V'*, in the flanges, which bend like fixed-guided beams.



This second load path can make an open-section much stiffer in torsion.

SSAM Table 6.7 provides equations for calculating equivalent torsion constants that account for warping constraint for beams with different boundary conditions.

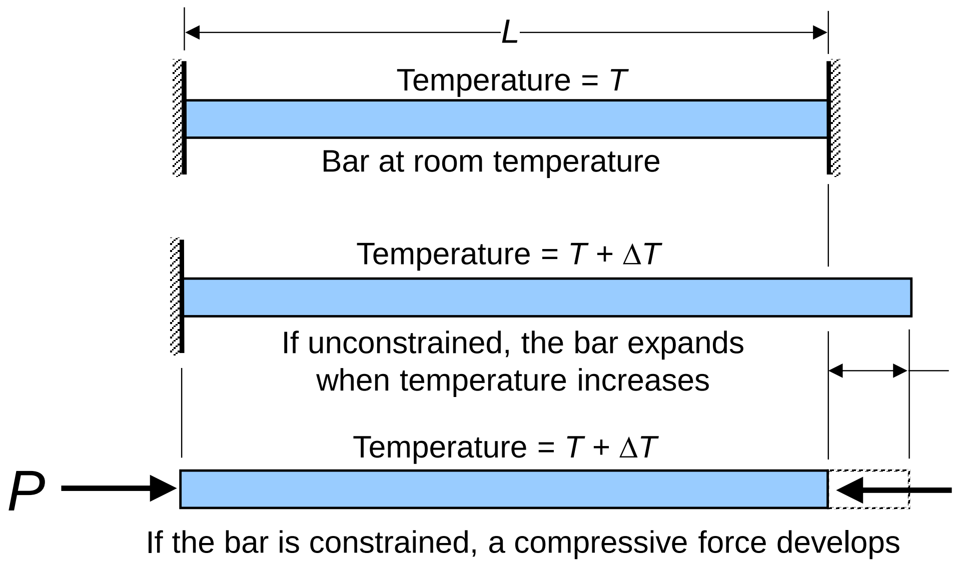
Thermal Effects

- Temperature affects material properties (strength, ductility, modulus, etc.).
  - Viscoelastic materials are particularly sensitive: properties vary widely with small changes in temperature.
- Materials expand and contract with changing temperatures.
  - Can misalign critical sensors
  - Can cause high stresses as materials in joined parts want to expand or contract different amounts
  - Can cause ***thermoelastic shock*** (sudden release of thermoelastic strain energy, such as when friction is overcome at sliding surfaces), also known as ***thermal snap***
    - This is a serious potential problem for spacecraft, as it causes on-orbit vibration that can impair performance.

Thermal Expansion and Contraction

A material freely expands and contracts according to its **coefficient of thermal expansion (CTE), α** :

Thermal strain, $\varepsilon_t = \alpha \Delta T$ **(Eq. 3.23)**



SSAM Fig. 10.5

$\Delta L = \alpha \Delta T L$ **(Eq. 3.24)**

$P = E \alpha \Delta T A$ **(Eq. 3.25)**

E = Young's modulus
A = Area

An unconstrained homogeneous material will expand or contract with changing temperature (and will bend under a linear gradient), but will experience no stress. Thermal stresses are caused when the structure is constrained or when the materials within a structure have different CTEs.

Key Points from This Section

- A material acts like a linear, elastic spring only when stressed up to its proportional limit, not its advertised yield stress.
- Ductility is a desirable material property, allowing materials to absorb energy and loads and stresses to redistribute before rupture.
- The simple equation we've all learned for bending stress, $f_b = Mc/I$, applies only for symmetrical cross sections up to the proportional limit.
- The equation most of us learned for torsional shear stress, $\tau = Tr/J$, applies only to axi-symmetrical (solid or closed circular) cross sections up to the proportional limit.
- Closed sections are much stronger and stiffer in torsion than open sections.
- Thermal expansion and contraction don't cause stress; it's the constraint of such expansion and contraction that causes stress.

4. Strength Analysis, Including Buckling

- An Important Thing to Understand
- Accounting for Variation in Material Strength: Allowable Stresses
- Revisiting the Margin of Safety
- Failure Theories for Materials
- Failure in Practice and the Benefits of Ductility
- Understanding Stress Analysis from the Engineer's Perspective
- Common Pitfalls
- An Effective Process for Strength Analysis
- Failure Modes for Fastened Joints
- Class Exercise: Identifying Potential Failure Modes
- Forms of Buckling
- Elastic Buckling of Columns
- Modified Johnson Parabola, Inelastic Buckling, and Eccentric Loading
- Buckling of Plates and Shells

Special thanks goes to Scott Malaznik for his significant contributions to this section, particularly regarding buckling.

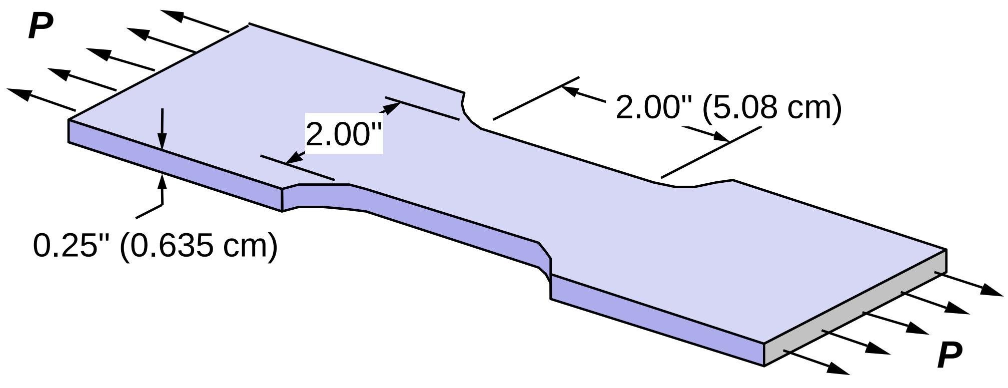
An Important Thing to Understand

Traditional stress analysis is not intended to predict failure.

It's used to establish confidence that a structure or mechanical assembly will withstand life-cycle environments and function as required, given many variables and uncertainties.

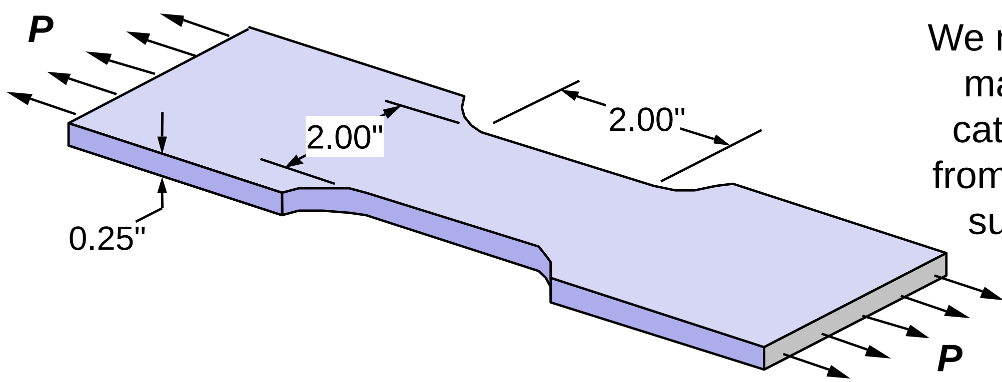
A program's structural design criteria—along with many of the assumptions we make in analysis—are aimed at staying on the safe side of these variables and uncertainties, including the possibility of human error.

Strength Varies Because of Random Defects



A tensile specimen sustains a maximum load *P* of 38,462 lb (170,080 N) before it breaks.

(Ultimate strength = 38,462 lb)

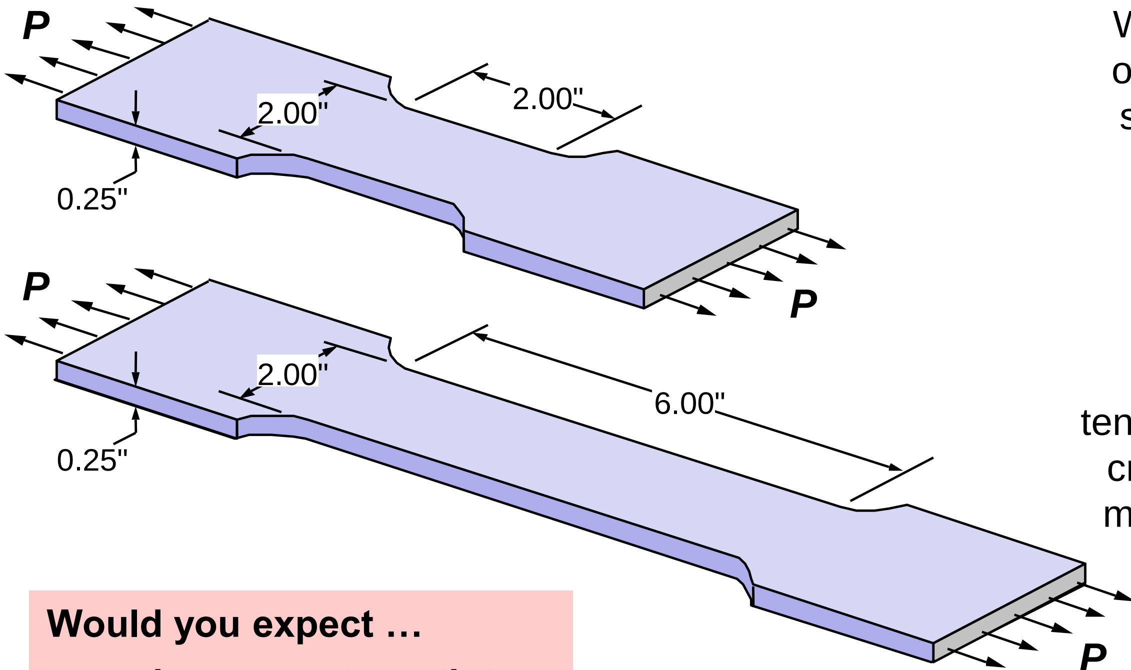


We make another specimen from the same material, per the same material specification and to the same dimensions—but from a different production lot or a different supplier—and we test it the same way.

What will its ultimate strength be?

Answer: Should be somewhat close to 38,462 lb. But "somewhat close" might mean a difference of 20% or more, depending on material and process controls.

Just for Grins: Does a Part's Length Affect Its Strength?



We make 100 tensile specimens out of the same material with the same dimensions and pull them to rupture.

Mean ult. strength = 34,297 lb,

Standard deviation = 1,324 lb.

We then make 100 longer tensile specimens with the same cross section out of the same material (same production lot) and pull them to rupture.

Would you expect …

… the mean strength to be equal to 34,297 lb? Higher? Lower?

… the standard deviation to be higher or lower than 1,324 lb?

Answer: Lower mean strength (greater chance of a weak section because of random defects) and lower standard deviation.

These effects may not be noticeable for ductile alloys but can be significant for brittle materials.

Standards for Strength Testing of Materials

For most strength analyses, calculated stresses are compared to ***allowable stresses*** (not-to-exceed stresses for design), a.k.a. ***allowables***.

– Allowables are minimum strength values, usually at a designated statistical basis.

– Ref. 5 (MMPDS) provides allowables—but refers to them as “design stresses”, unfortunately*—for many metal alloys that meet designated specifications.

The tensile allowables for most materials in the MMPDS were derived by testing of specimens per ASTM E8, with a standard ***gage length*** (length over which displacement is measured, applicable to the portion of the specimen having uniform cross section, away from stress concentrations).

– For example, a specimen taken from sheet metal looks similar to those shown on the previous page herein and has a gage length of 2 inches.

Similarly, the other types of allowable stresses in the MMPDS were derived from tests following other ASTM standards.

*MIL-HDBK-5 used the term “allowable stress” through revision G, 1994. Starting with revision H, 1998, the term started to change to “design stress” for some reason (liability?), although the term “allowable stress” still appeared in various sections. To most engineers, “design stress” means stress calculated at the design load (limit load times factor of safety).

Statistical Basis for Allowable Stresses

The MMPDS provides allowables at three levels of statistical assurance:

- ***A-Basis***: At least 99% of all specimens will exceed the A-basis allowable, with 95% confidence.
- ***B-Basis***: At least 90% of all specimens will exceed the B-basis allowable, with 95% confidence.
- ***S Basis***: Minimum value specified by a governing body; statistical basis unknown

A-basis is standard for metals; we'll discuss use of B-basis shortly.

Statistical ***confidence*** accounts for uncertainty in the conclusions drawn from a finite sample of data.

- We don't know how much the calculated sample mean (average) and unbiased sample standard deviation differ from the true mean and standard deviation of the population.
- A statistical confidence above 50% accounts for this uncertainty: Any value calculated at a certain probability is penalized (allowable stress reduced).
  - Example: At 95% confidence, there's only a 5% chance the derived allowable stress is on the unsafe side of the stated probability.

Deriving A- and B-basis Allowables from Test Data

MMPDS Sec. 9 defines a rigorous process for deriving allowables for inclusion in MMPDS.

When deriving allowables for materials not in MMPDS or for joints, for internal use only, one simple approach is to assume the strength data follow a normal distribution* and to use the one-sided tolerance limit factors, based on sample size, *n*, and the statistical distribution.

A-basis allowable load or stress = $\bar{Y} - k_{99} s_Y$ **(Eq. 4.1)**

B-basis allowable load or stress = $\bar{Y} - k_{90} s_Y$ **(Eq. 4.2)**

where

$\bar{Y} =$ sample mean $= \dfrac{1}{n}\sum_{i=1}^{n} Y_i$ **(Eq. 4.3)**

$s_Y =$ unbiased sample standard deviation $= \sqrt{\dfrac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}$ **(Eq. 4.4)**

n = sample size (number of data points (strength values from test specimens))

k_{99} and k_{90} are the one-sided tolerance limit factors for A-basis and B-basis allowables, respectively (see next page)

*MMPDS Sec. 9 provides methods of assessing data for fit with normal and Weibull distributions.

Deriving A- and B-basis Allowables from Test Data, continued

When the data follow a normal distribution, the one-sided tolerance limit factors, k_{99} and k_{90} are calculated as …

$$k_{99} = 2.326 + \exp\left[1.34 - 0.522\ln(n) + \frac{3.87}{n}\right] \quad \textbf{(Eq. 4.5)}$$

$$k_{90} = 1.282 + \exp\left[0.958 - 0.520\ln(n) + \frac{3.19}{n}\right] \quad \textbf{(Eq. 4.6)}$$

Ref. MMPDS Table 9.10.1

n	k_{99}	k_{90}
6	5.18	3.03
8	4.42	2.60
10	4.02	2.36
12	3.77	2.22
15	3.53	2.07
20	3.30	1.93
30	3.06	1.78
40	2.94	1.70
50	2.86	1.65
75	2.75	1.57
100	2.68	1.53
200	2.57	1.45
300	2.52	1.42
∞	2.33	1.28

Example Problem 4-a: Deriving A- and B-basis Allowables from Test Data

Given the hypothetical test data below for bonded-joint specimens, calculate the A- and B-basis allowable loads for the joint, using the one-sided tolerance limit factors for a normal distribution.

Test no., i	Strength, Y_i, lb	$(Y_i - \bar{Y})^2$
1	3873	1239
2	3920	6757
3	4212	140026
4	3683	23963
5	3733	10983
6	3816	475
7	3981	20506
8	3158	462128
9	4076	56739
10	3926	7779
Sum	38378	730596
$\bar{Y}$ (Eq. 4.3)	3838 (sample mean)	

Solution:

Unbiased sample standard deviation per Eq. 4.4:

$$s_Y = \sqrt{\frac{730{,}596}{10-1}} = 285 \text{ lb}$$

One-sided tolerance limit factors for n = 10:

$$k_{99} = 4.02, \quad k_{90} = 2.36$$

A- and B-basis allowables per Eqs. 4.1 and 4.2:

$$P_A = 3838 - 4.02(285) = 2690 \text{ lb (70\% of sample mean)}$$

$$P_B = 3838 - 2.36(285) = 3170 \text{ lb (82\% of sample mean)}$$

Note that the derived B-basis allowable for this hypothetical data sample is greater than the minimum strength found in test, 3158 lb.

Allowables such as these are statistically based and thus are not the minimum possible values. A-basis allowables provide more statistical assurance.

Allowable Stresses for Composites and Bonded Joints

For composites, other brittle materials, and bonded joints, it's often not practical to derive A-basis allowables, as we do for metals produced with characterized, well-controlled processes.

- When an aerospace program's production volume is low—especially just one!—it's too costly to build and test many specimens to quantify a strength distribution.
- With a small sample size and a material with high variation in strength (typically the case for brittle materials), the A-basis allowable can be quite low—possibly negative.

As a result, many aerospace programs design with internally derived B-basis allowables for composites and bonded joints.

- thus accepting risk during design
- but reducing risk by planning to test each region of each flight article (***proof test***)

If we can't afford the test program needed to fully quantify strength variation (such as with statistical process control), we should proof test each part and each joint of each flight structure.

To minimize impact of potential proof-test failure, try to design the structure so that any needed proof testing can be done at low levels of assembly.

Revisiting the Margin of Safety

Recall from Sec. 1: The margin of safety is a measure of how much the design load can increase before the criteria are not satisfied, as calculated with Eq. 1-1:

$$MS = \frac{\text{Allowable applied load}}{\text{Limit load} \times FS} - 1$$

Factor of safety

Can we substitute stress for load in the above equation?

Answer: Yes, but only if stress is proportional to the applied load.

If not, using stress gives a misleading margin of safety.

Examples: preloaded bolts, eccentrically loaded columns (as we'll see later in this section), membranes

The margin of safety should tell us how much the <u>applied load</u> can increase before the structure can no longer meet the strength criteria (or must decrease to just satisfy the criteria).

The margin of safety is <u>not</u> intended to be a comparison of the predicted stress to the allowable stress!

A Commonly Used Process for Strength Analysis … Unfortunately!

1. Generate a detailed finite element model.
2. Subject the model to the design loads.
3. Identify the peak stress.
4. Calculate a margin of safety by comparing the peak von Mises stress (including the appropriate factor of safety) to the allowable stress.

What's wrong with this process?

- **There's no recognition of requirements (e.g., maintain alignment). No difference in process for different types of structures or requirements.**
- **Depending on modeling assumptions, the analysis may not identify the critical mode of failure.**
- **Margin of safety is based on stress rather than load. Stress is often not proportional to load, especially after yielding begins.**
- **von Mises stress applies only to onset of yielding, not detrimental yielding or rupture.**
- **No assurance that the allowable stress applies to the calculated stress.**

The above process is rarely meaningful!

How Does the State of Stress Relate to Failure?

- In Sec. 3 we reviewed how to calculate principal stresses and how to draw Mohr's circle.
- But we didn't address how principal stress relates to failure.
- In Class Problem 3-1, we concluded that, for a uniaxial tensile test specimen, the principal shear stress is half the principal normal stress (uniaxial tensile stress).
  - If the material is ductile, yielding occurs before rupture.
  - <u>It's the shear stress that causes yielding, not the normal stress.</u>
- So, is shear stress more important to understand for strength analysis than normal stress?

Over the centuries—particularly from the late 1700s to the early 1900s (Coulomb, Maxwell, Tresca, Huber, von Mises, etc.)—a great deal of research has been done to understand how to predict failure from the state of stress.

Some failure theories have held up better than others, but none have perfectly agreed with empirical data.

Failure Theories for Onset of Yielding (Ductile Materials)

Two theories have compared well with test results for onset of yielding:

Maximum Shear Stress Theory: Onset of yielding predicted when $\tau_{12} = \dfrac{\sigma_m}{2}$ **(Eq. 4.7)**
(a.k.a. ***Tresca theory***)

τ_{12} = principal shear stress
σ_m = uniaxial tensile stress that causes the onset of yielding

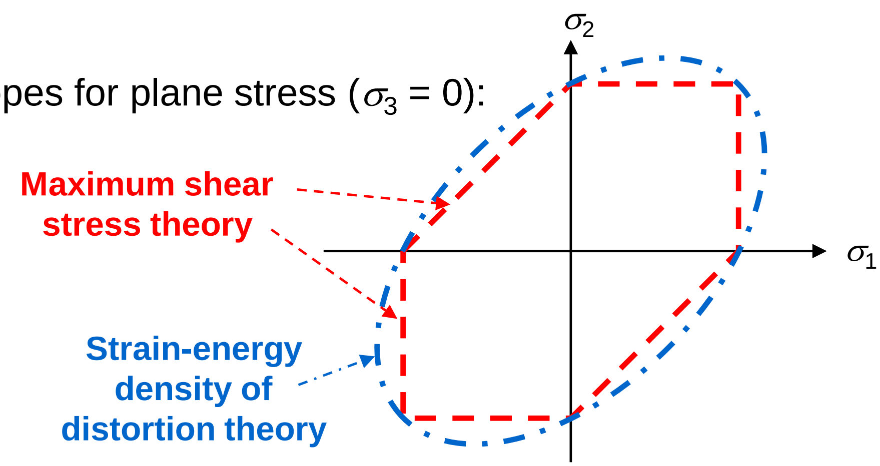
Strain-Energy Density of Distortion Theory (***von Mises criterion***)

von Mises stress, $\sigma_{vm} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3\right)}$ **(Eq. 4.8)**

Principal normal stresses

Onset of yielding predicted when $\sigma_{vm} = \sigma_m$

Theory envelopes for plane stress (σ_3 = 0):

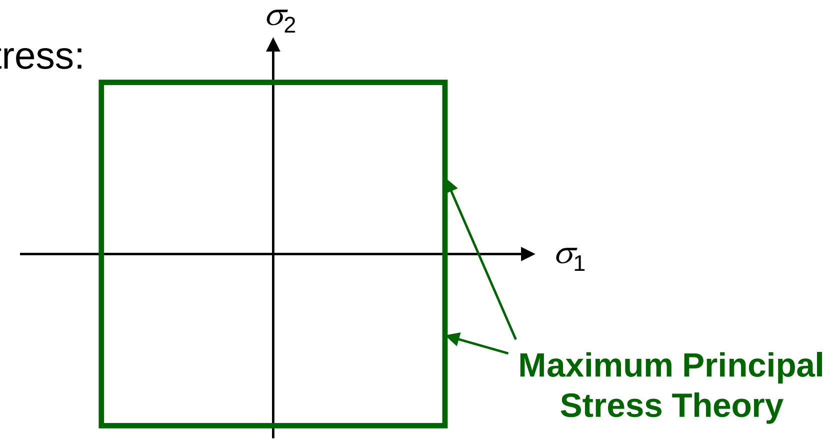


Failure Theories for Rupture (Ultimate Failure)

Rupture and ultimate strength of brittle materials:

The ***Maximum Principal Normal Stress Theory*** (a.k.a ***Rankine's Theory***) tends to agree best with test data: failure when the principal tensile stress equals the ultimate tensile stress found from single-axis tensile testing.

Plane stress:



Rupture and ultimate strength of ductile materials:

Ductile materials most often rupture as a result of fatigue (Sec. 5 herein).

For ductile materials under static or one-time loading, there is no failure theory for ultimate strength that applies to various geometries and stress states.

Why is that? Let's explore this topic (next page) …

What Failure Theory Applies to Ductile Rupture?

The von Mises criterion is based on linear theory and thus applies only to the onset of yielding, not rupture.

The Maximum Shear Stress Theory doesn't match most test data for rupture, either.

– Ultimate shear strength of ductile materials is usually about 55% to 65% (not 50%) of ultimate tensile strength, depending on material, orientation of shear stress, and test method.

No failure theory actually applies to rupture of ductile materials under static loading in regions of changing geometry. Once the material yields, the stress state changes, as internal loads seek the stiffest path, depending on the part's geometry.

Tensile specimen: cup-cone failure

Near the surface, the material is primarily in a state of plane stress (σ_z = 0), and rupture tends to be at 45 degrees. In the center region, the material is under a triaxial state of stress and tends to fail along a surface that is perpendicular to the applied load.

Failure in Practice

Most structural failures occur in regions of discontinuity, such as in joints, where ductile materials are most often used.

In regions of changing geometry and stress concentration, ductility invalidates linear-elastic analysis for ultimate strength!

Local yielding in high-stress regions causes loads to redistribute and stress states to change, usually increasing strength beyond what we would expect.

Most structural failures occur in joints, where geometry is changing; so, in most cases, linear-elastic FEA does not apply up to rupture for ductile materials.

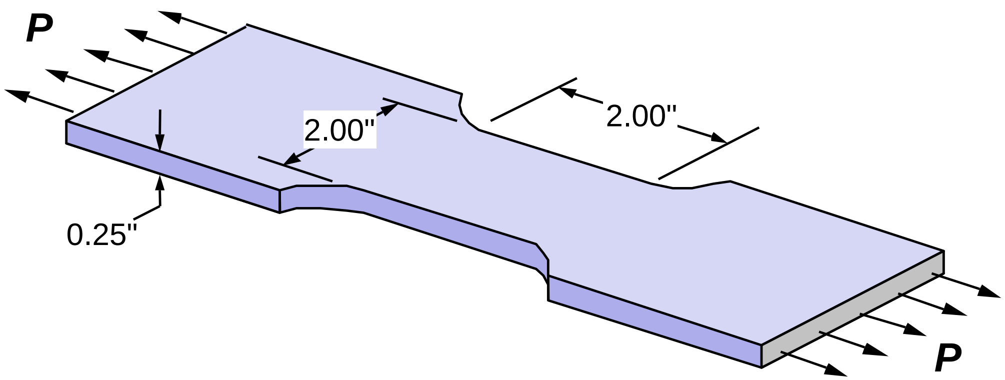
Even inelastic FEA is of limited use for assessing ductile rupture. How does the calculated stress state—and the von Mises stress—relate to failure?

How the state of stress changes and relates to rupture for a given material depends on geometry!

The best approach to strength analysis of joints and other regions of discontinuity—for metals and composites—is to use methods that are based on tests of specimens that are similar to the designs being assessed.

Find and use empirical and semi-empirical methods.

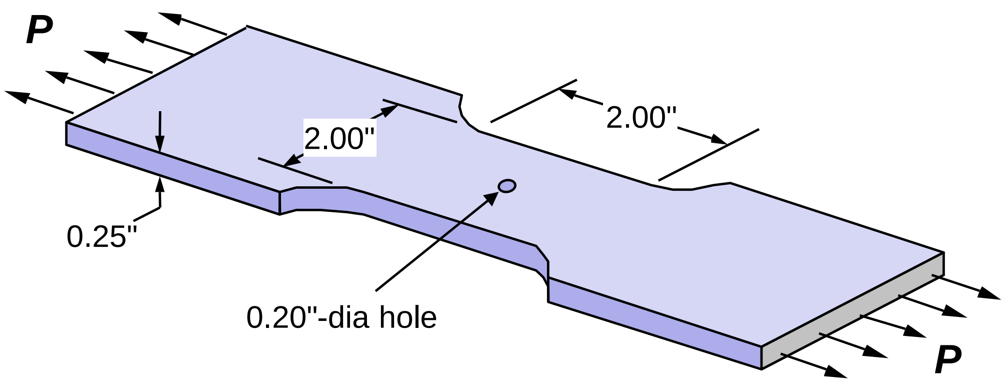
Example of the Effects of Ductility



We make 100 tensile specimens out of the same material with the same dimensions and pull them to rupture.

Mean ult. strength = 34,297 lb,

Standard deviation = 1324 lb.



We then drill a 0.20”-dia hole through each of 100 otherwise identical specimens of the same material and pull them to failure.

What would you expect the mean strength to be?

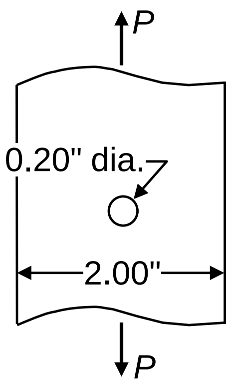
Answer: (a) brittle material: strength ≈ 33% of 34,297, according to the stress concentration factor of approximately 3. (b) ductile material: strength ≈ 90% of 34,297 lb because 90% of the cross-sectional area remains (net section).

A Key Benefit of Ductility

If a structural part is made of a sufficiently ductile material, the part's strength is relatively unaffected by many stress concentrations, which occur in regions of changing geometry.

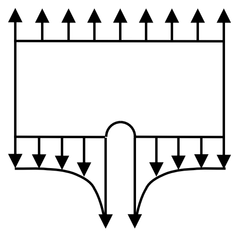
Example—flat plate made of an alloy with 6% elongation, with drilled hole, under tension:

Plate thickness, t = 0.25" (0.635 cm)



Gross stress:

$$\sigma_g = \frac{P}{0.25(2.00)} = 2P$$



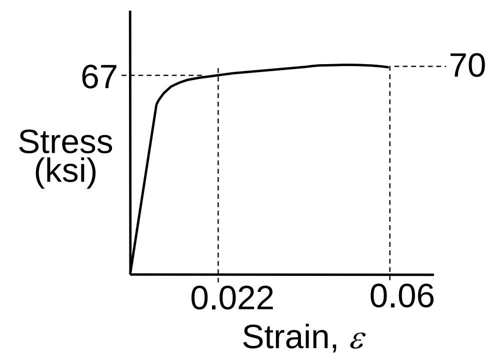
Peak stress (elastic):

$$\sigma_{\max} \approx 3\sigma_g = 6P$$

Net-section stress:

$$\sigma_n = \frac{P}{0.25(2.00 - 0.20)} = 2.22P$$

Note: 1 ksi = 1000 psi = 6.895 MPa



When the edge of the hole is at a rupture strain of 0.06, the strain at the edge of the part is 0.022, and the state of stress is nearly uniform over the net section.

Adapted from SSAM Fig. 8.1

Stress concentrations such as these do not significantly reduce the ultimate strength of parts made of sufficiently ductile materials.

Such stress concentrations do, however, greatly reduce fatigue life (Sec. 5 herein).

Understanding Stress Analysis from the Engineer's Perspective

from Instar's course "Space Mission Structures, from Concept to Launch"

To the engineer, stress analysis is not about predicting stresses.
It's about making designs efficient and avoiding failure!

Stress analysis:

- Often thought of as the process of predicting stresses caused by applied loads
- Actually, to the engineer, it is a process of relating applied loads to allowable loads (or to allowable stresses, more often) in an apples-to-apples comparison
- Allowable stresses are derived from tests, but the stresses actually are not measured, and neither are the strain values.
  - Load is measured and then converted (by some process) to stress.

An apples-to-apples comparison means two things:

1. **The structural design and failure mode of concern correspond to those that were tested to derive the allowables.**
2. **We use a method of converting load to stress that is consistent with the way in which the allowable stress was derived.**

Apples to oranges is unavoidable at times; use an uncertainty factor in such cases.
When it's apples to elephants, we won't know what factor to use!

Common Pitfalls in Stress Analysis

- Not recognizing the critical failure mode
- Using an allowable load or stress that does not correspond to the design, the manufacturing process, or the failure mode
- Using an allowable that does not correspond to the structural requirement
- Not identifying the critical loading event in the product's life cycle
- Not accounting for dimensional tolerances and potential misalignment
- Not accounting for inelastic effects in compression members
- Inadequately representing a part of the structure with a finite element model
- Making a math error, or an input error when using analysis software

This is why we test!

Candidates for strength verification by analysis without a test:

Simple structures made of ductile materials, with failure modes that are well characterized in tests of similar designs

An Effective Process for Strength Analysis

1. Start with a drawing or solid model of the structure.
   – You're assessing a design, not a finite element model!
2. Identify the structural requirements.
   – How much can the structure permanently deform without adversely affecting function (e.g., alignment)?
3. Draw a free-body diagram.
   – Take time to understand the load paths.
4. Identify potential failure modes.
   – ***Ultimate failure*** is rupture or collapse.
   – ***Yield failure*** is defined from step 2, above.
5. Identify or calculate appropriate allowables.
   – Allowable loads or stresses that correspond to the identified potential failure modes; empirically based.
   – Development testing may be necessary.
6. Predict design loads or stresses that correspond to the allowables.
   – Finite element analysis might be best.
7. Calculate margins of safety.

Improve design

Don't waste time analyzing a bad design!

Key steps!

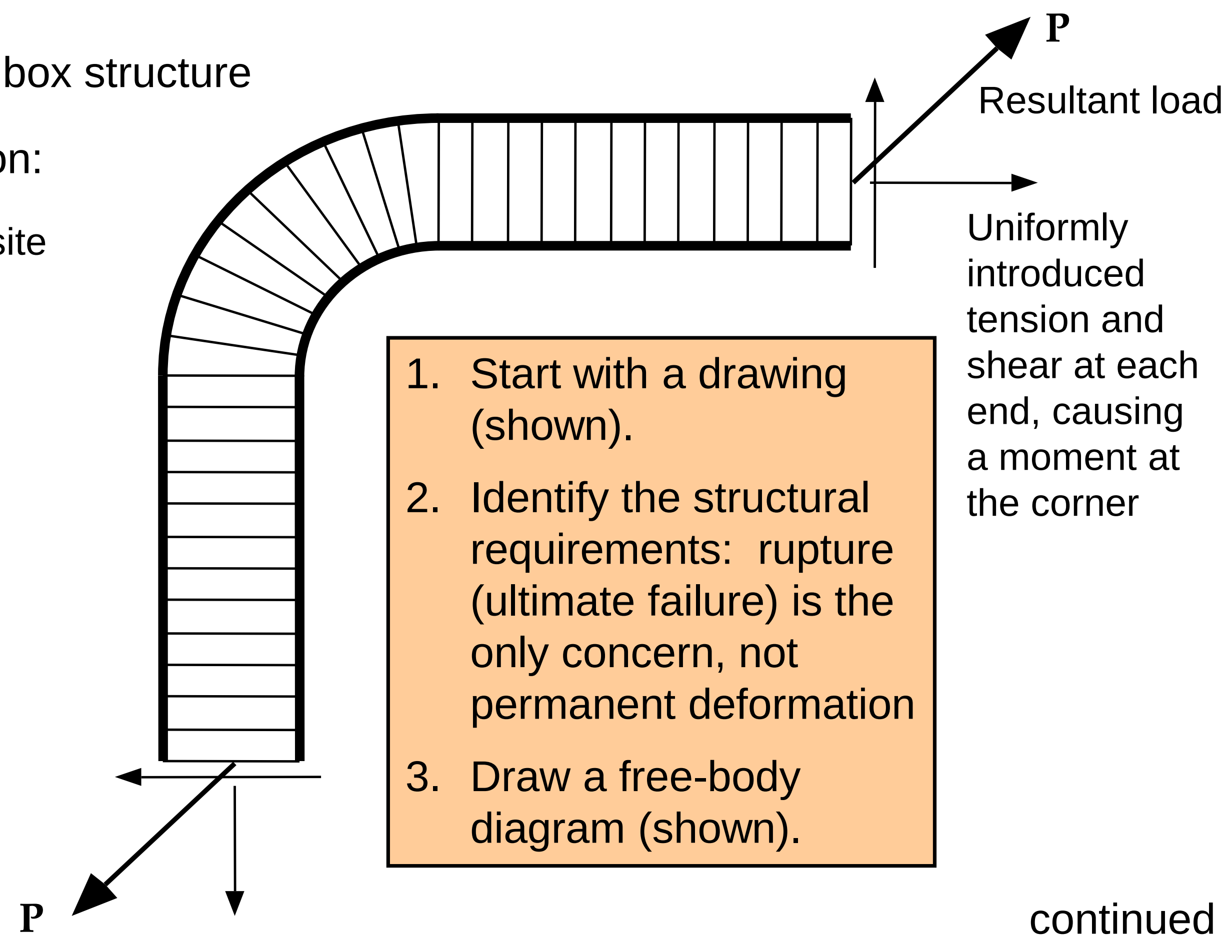
Finite element analysis will predict loads and stresses, but the engineer must identify potential failure modes and appropriate allowables.

Example Problem 4-b: Applying the Strength-Analysis Process

Rounded corner in a box structure

Sandwich construction:

- Carbon-fiber composite laminate face sheets
- Flexible aluminum honeycomb core



1. Start with a drawing (shown).
2. Identify the structural requirements: rupture (ultimate failure) is the only concern, not permanent deformation
3. Draw a free-body diagram (shown).

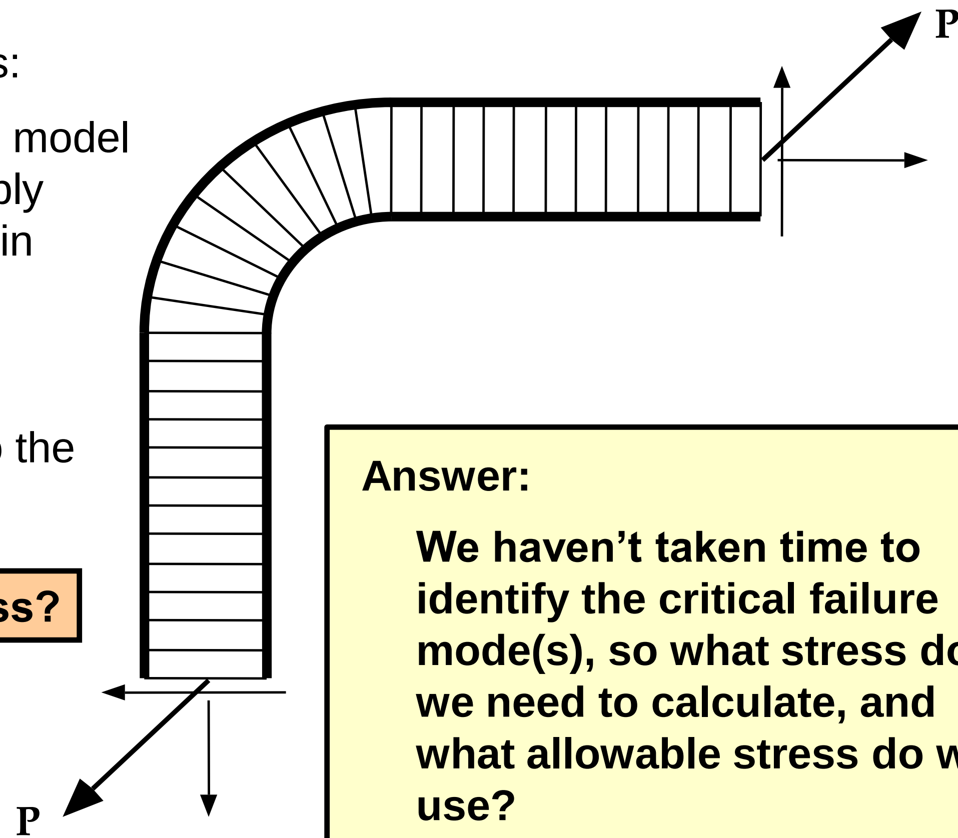
continued

Example Problem 4-b, continued

One way to do the stress analysis:

- Make a detailed finite element model (FEM): shell elements, each ply modeled separately (PCOMP in Nastran).
- Calculate stresses.
- Compare the peak stresses to the allowable stresses for a ply.

What's wrong with this process?



Answer:

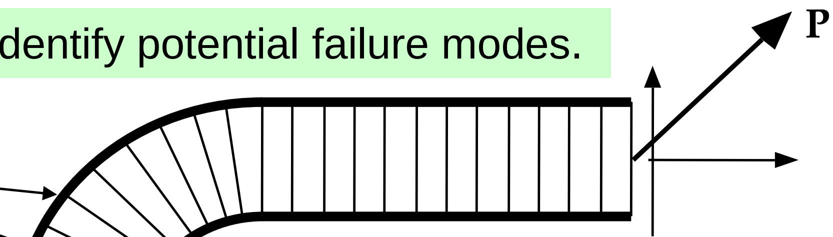
We haven't taken time to identify the critical failure mode(s), so what stress do we need to calculate, and what allowable stress do we use?

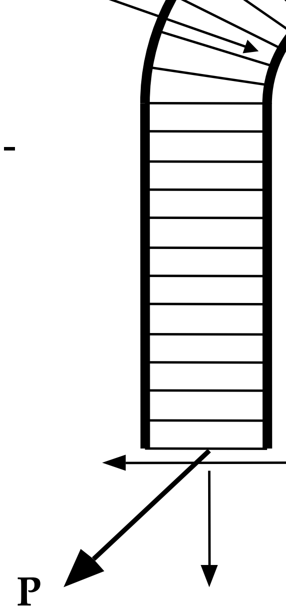
continued

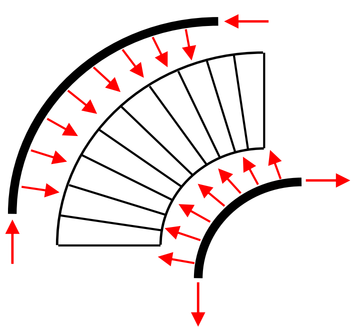
Example Problem 4-b, continued

Better approach: Step 4. Identify potential failure modes.

Critical failure mode: debonding or delamination at the corner caused by interlaminar (through-thickness) tensile stress







FBDs of face sheets

Now we know what type of FEM we need: Shell elements are two dimensional and thus don't calculate interlaminar tensile stress.

- Model each ply with solid elements rather than shell elements.
- Calculate the peak through-thickness stress.
- Compare the peak stress to the allowable stress established from flat-wise tensile (out-of-plane) tests of flat coupons.

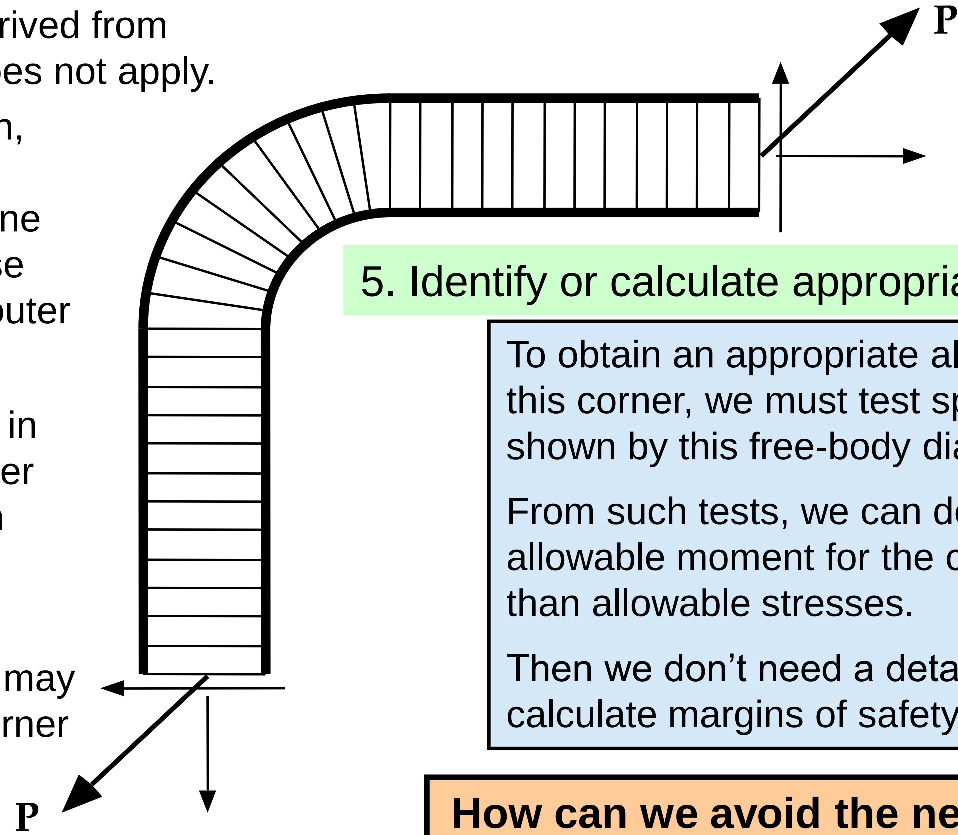
What's wrong with this process?

continued

Example Problem 4-b, continued

The allowable stress derived from flat-wise tensile tests does not apply.

- For the corner shown, interlaminar tension combines with in-plane compression to cause delamination in the outer face sheet.
- The flex core results in less effective bond per square inch between core and outer face sheet.
- The curing pressure may be different at the corner than for the flat test specimens.



5. Identify or calculate appropriate allowables.

To obtain an appropriate allowable for this corner, we must test specimens as shown by this free-body diagram.

From such tests, we can derive an allowable moment for the corner rather than allowable stresses.

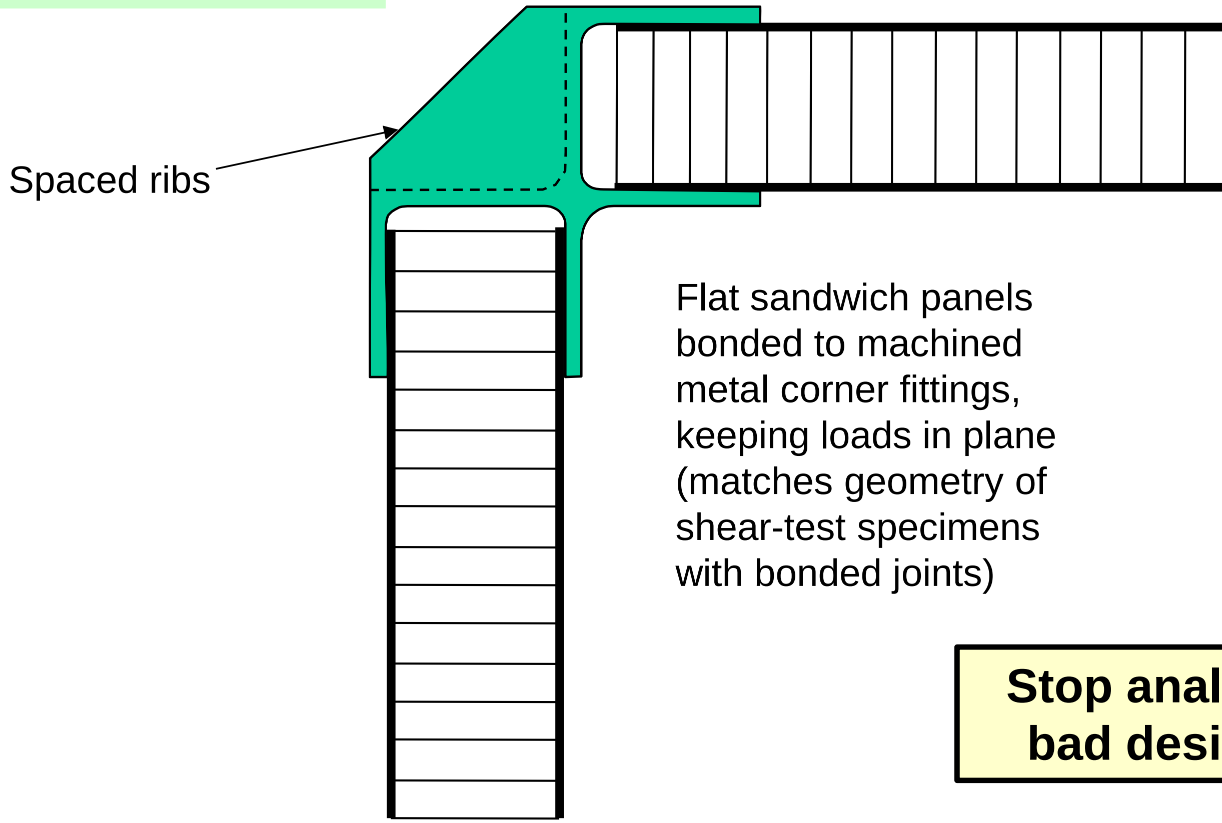
Then we don't need a detailed FEM to calculate margins of safety.

How can we avoid the need for such a development test program?

continued

Example Problem 4-b, conclusion

Improve the design!



Flat sandwich panels bonded to machined metal corner fittings, keeping loads in plane (matches geometry of shear-test specimens with bonded joints)

Stop analyzing bad designs!

The Most Common Modes of Structural Failure

These failures are assessed with <u>strength</u> analysis.

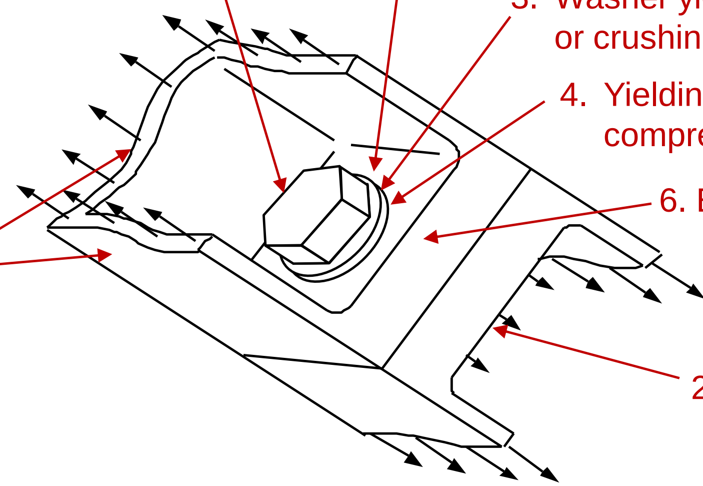
- Yielding or rupturing at joints
  - It's hard to predict how loads and stresses will distribute.
  - Because of abrupt changes in geometry, joints often rupture without much plastic deformation, and thus little opportunity for loads to redistribute, even with ductile materials.
- Buckling
  - Many potential modes of instability
  - Buckling often occurs at a lower load than elastic theory predicts because of …
    - Imperfections in geometry
    - Inelastic effects (exceeding the material's proportional limit)

A structural member hardly ever ruptures away from a joint or other region of discontinuity.

Addressed by fatigue analysis and fracture control. (See Sec. 5)

- Fatigue (usually in joints)
  - By far the most common mode of metallic structural failure in all industries combined
  - The most common mode of failure during random vibration or acoustic testing
  - Rare for the primary structure of a launch vehicle or a large (> 500 lb or so) spacecraft when either is used for only one mission

Potential Failure Modes for Bolted Tension Joints

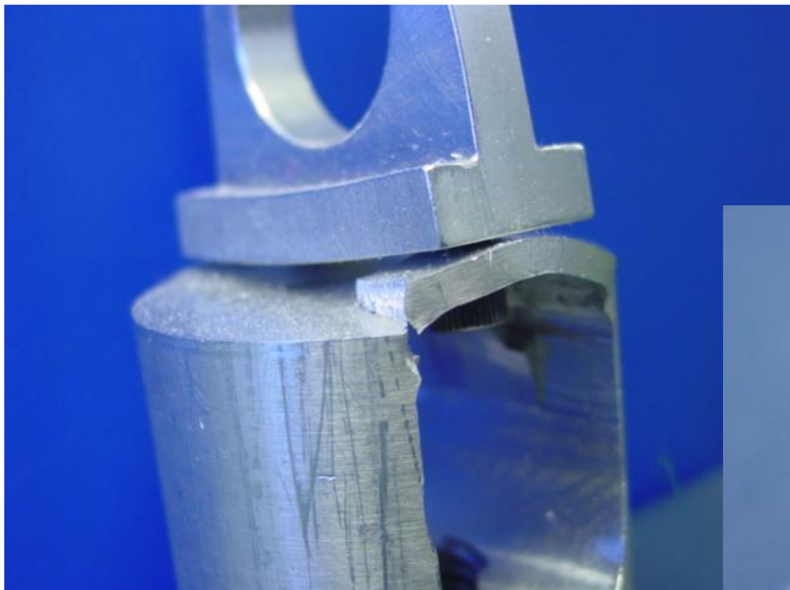


Design guidelines:

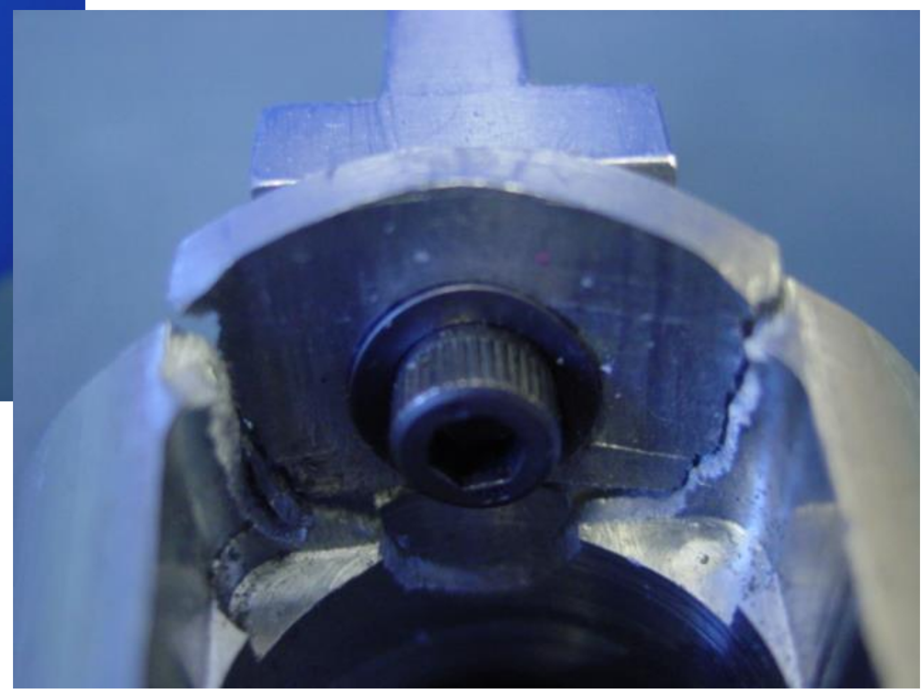
- Select a nut or an insert that is at least as strong as the bolt (reduces risk of relatively brittle failure or of the analyst forgetting to check the nut or insert).
- Make the end pads thick enough to avoid bending and shear failures.
- Tuck the bolt close to the tension walls to minimize end-pad thickness and make the joint stiff.

From my course “Design and Analysis of Bolted Joints” (DABJ), Ref. 33

End-pad Bending Yield and Shear Ultimate Failures



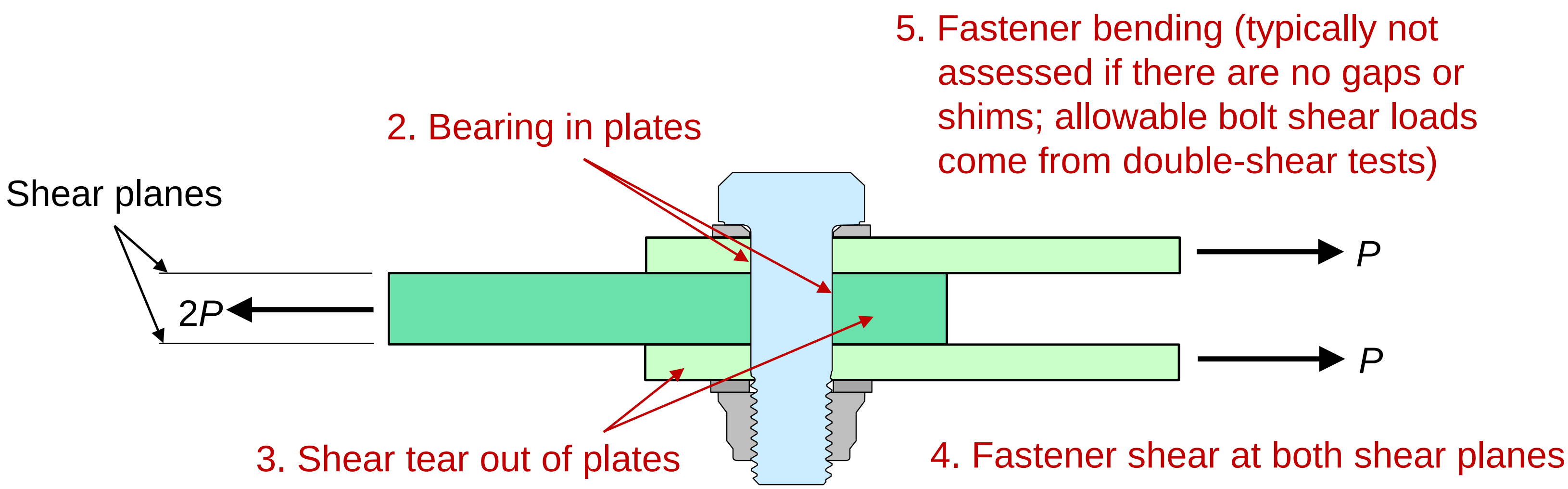
Plastic deformation prior to rupture, so linear-elastic theory does not apply. Use empirical methods (or clearly conservative methods) for fastened joints.



Material: 6061-T6 aluminum alloy

From DABJ

Potential Failure Modes for Shear Joints



1. Net tension in plates (tensile failure where the plate's cross section is at a minimum, at the hole)—account for stress concentration: see the semi-empirical lug analysis method in Sec. D1.11 of Bruhn or Sec. B2.1.0 of the Astronautic Structures Manual

Joint slip under the design yield load may be considered failure as well if friction is being counted on (e.g., to maintain alignment of critical interfaces)

From DABJ, Ref. 33

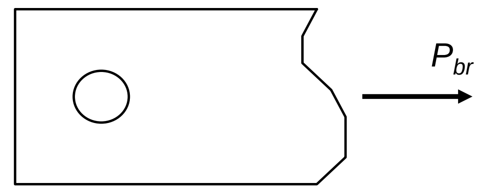
Bearing Stress

For a double-shear joint without shims, ***bearing stress***, f_{br}, is calculated as the average contact stress between the fastener body and the surface of the hole:

$$f_{br} = \frac{P_{br}}{Dt} \quad \textbf{(Eq. 4.9)}$$

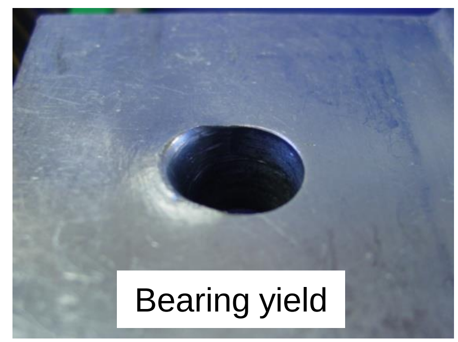
P_{br} ← Bearing load acting on a plate

D = bolt or pin diameter
t = plate thickness

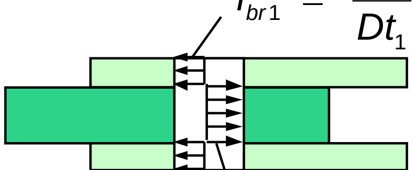


Given an allowable bearing ultimate stress, F_{bru}, the allowable bearing ultimate load is

$$P_{bru-allow} = F_{bru}Dt$$

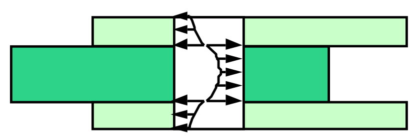


$$f_{br1} = \frac{P_{br1}}{Dt_1}$$



$$f_{br2} = \frac{2P_{br1}}{Dt_2}$$

Assumed distribution of contact pressure



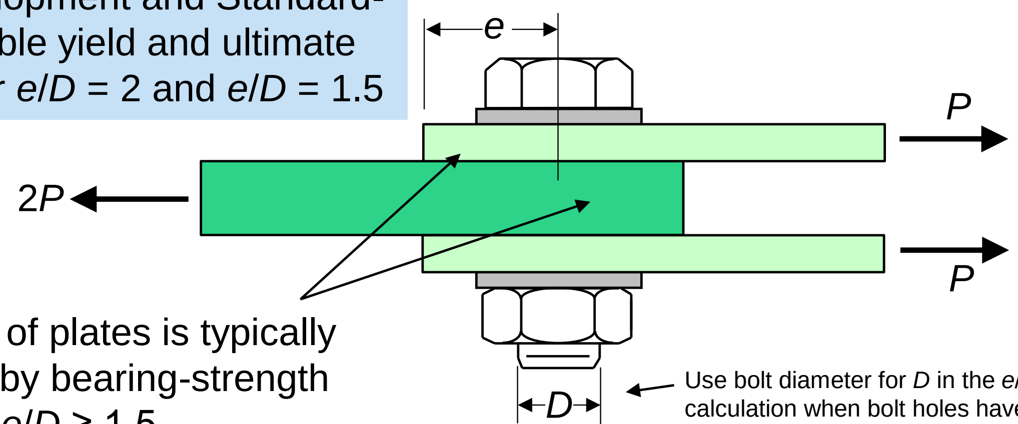
Actual distribution of contact pressure

From DABJ, Ref. 33

Bearing and Shear Tear Out

Metallic Material Properties Development and Standardization (MMPDS) provides allowable yield and ultimate bearing stresses, F_{bry} and F_{bru}, for e/D = 2 and e/D = 1.5

- For 1.5 < e/D < 2, interpolate to find the allowable (per MMPDS).



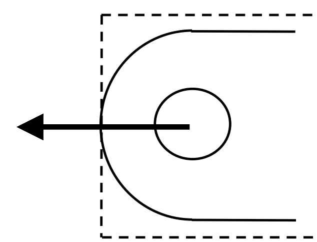
Shear tear-out of plates is typically encompassed by bearing-strength analysis when $e/D \geq 1.5$

Use bolt diameter for *D* in the *e*/*D* calculation when bolt holes have only a few thousandths clearance; otherwise use bolt-hole diameter.

- For e/D < 1.5, the following methods (both considered conservative when using a 1.15 fitting factor) have been used:

  1. Define an equivalent lug, as shown at right, and analyze it using the semi-empirical lug-analysis method in Bruhn (ref. 8) Sec. D1.11 and the Astronautic Structures Manual (ref. 9) Sec. B2.0.0.

  2. Derive an allowable bearing stress by interpolating between the e/D = 1.5 allowable and zero at e/D = 0.5.



From DABJ, Ref. 33

Bearing Ultimate Failure for Ductile Materials

The material can undergo very large plastic deformation, elongating the hole.

Ultimate failure is typically shear tear out.

From DABJ, Ref. 33

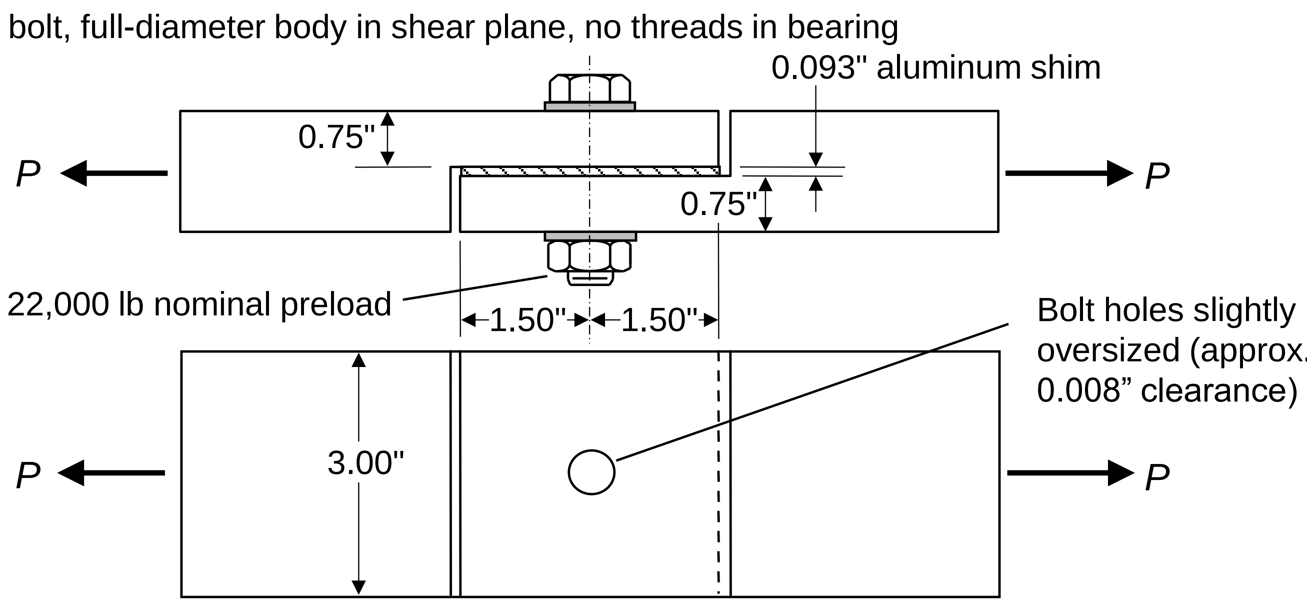
Applicability of MMPDS Bearing Allowables

- Bearing allowables apply only for double-shear joints.
  - Testing per ASTM E238 puts the pin in double shear.
  - Higher peak bearing stress occurs in single-shear joints.
- Bearing allowables from MMPDS-08 (Ref. 5a) apply only when the ratio of plate thickness to bolt diameter (*t*/*D*) is between 0.25 and 0.50.
  - The limitation used to be $0.18 \le t/D \le 1.00$ (per MIL-HDBK-5H, 1998).
  - It presumably changed in MMPDS as a result of additional test data.
- MMPDS-08 Sec. 1.4.7 says "Due to differences in results obtained between dry-pin and wet-pin tests, designers are encouraged to consider using a reduction factor with published bearing stresses for use in design."
  - Tests are done per ASTM E238, with specially cleaned ("dry") pins.
  - Tests with pins not cleaned per this process ("wet" pins) "can show bearing (strengths) at least 10% lower" (MMPDS-08 Sec. 1.4.7).
- The tests are conducted without shims, which can concentrate bearing stresses at the mating surfaces and reduce joint strength.

From DABJ, Ref. 33

Example Problem 4-c:
Allowable Loads for a Single-shear Joint with Shim

Calculate the allowable yield load ($P_{y\text{-allow}}$) and the allowable ultimate load ($P_{u\text{-allow}}$).



Fastener material: A-286 alloy F_{ty} = 120 ksi F_{su} = 95 ksi
F_{tu} = 160 ksi

Edge-distance ratio

Fitting material: Aluminum alloy; F_{ty} = 49 ksi F_{bry} = 94 ksi (for e/D = 2)
F_{tu} = 62 ksi F_{bru} = 123 ksi (for e/D = 2)

A-basis allowable bearing stresses

From DABJ, Ref. 33

continued

Example Problem 4-c, continued

This joint violates all the conditions for which allowable bearing stresses apply:

- Single shear rather than double shear
- Shim
- Plates thicker than half the bolt diameter

In classes taught between 1996 and 2010, I asked approximately 1000 aerospace structural engineers to calculate allowable loads for this joint.

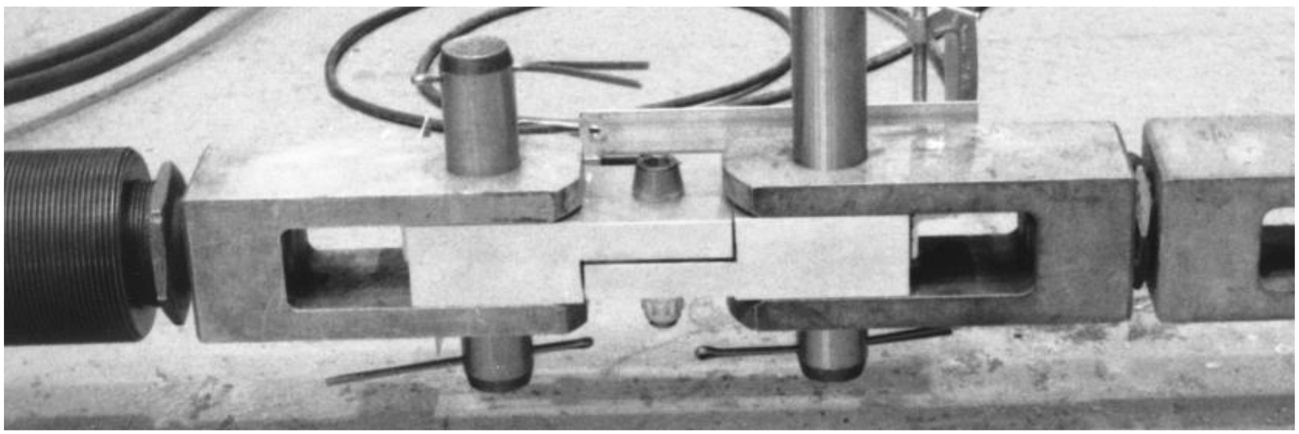
Answers ranged from 2000 lb to 90,000 lb!

We can reliably assess a joint only if the analysis is based on meaningful test data!

continued

Example Problem 4-c, continued: Single-shear Test Setup

This joint was tested, but not with enough specimens to establish statistically appropriate allowable loads or to establish an empirical method of analysis.

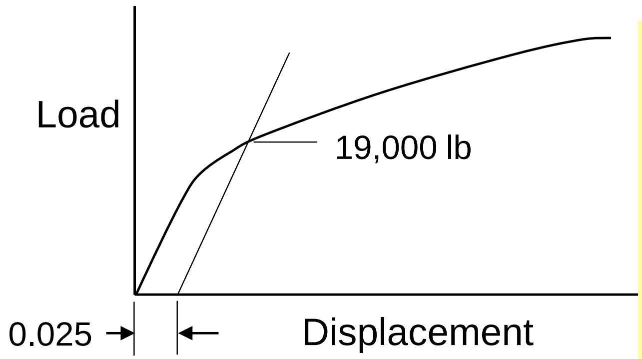


Not shown (far side): LVDT to measure displacement from one part to the other

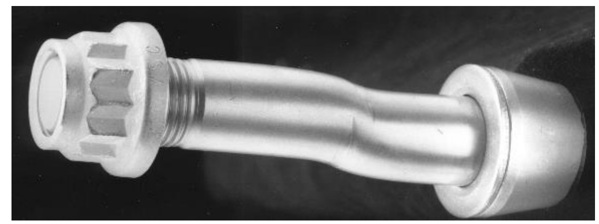
LVDT = linear variable differential transformer

continued

Example Problem 4-c, conclusion: Test Results



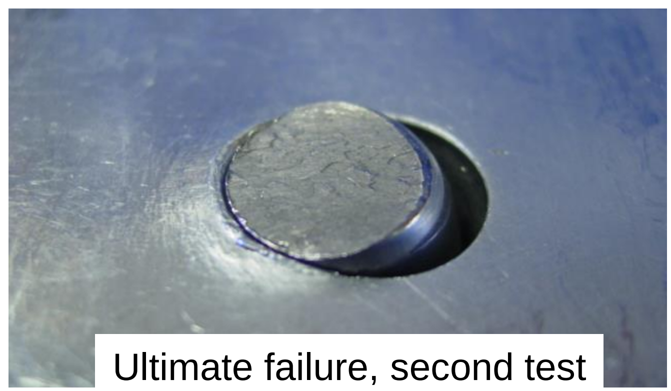
The joint saw an onset of yielding at about 14,000 lb and suffered yield failure (per MMPDS definition) at about 19,000 lb.



Bent bolt from first test

The second test was run to failure. The joint carried an ultimate load of 35,500 lb.

As compared with 29,100 lb allowable shear load for the bolt!



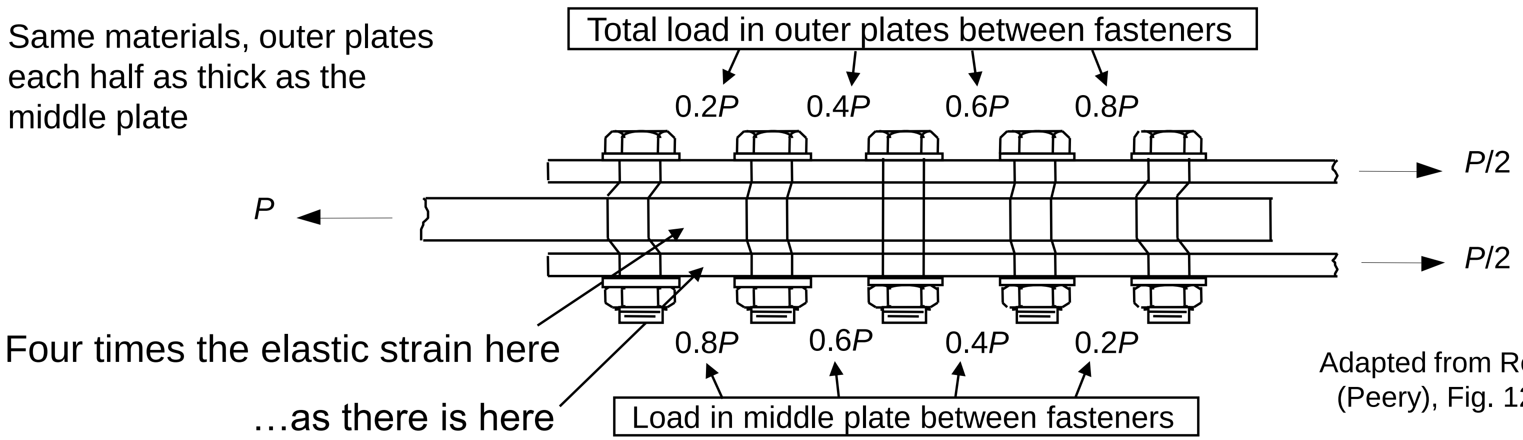
Ultimate failure, second test

See DABJ Sec. 9 for a proposed method of assessing shimmed single-shear joints with A-286 bolts, based on tests at Marshall Space Flight Center.

Fasteners Don't All Carry the Same Shear Load

Even if a joint has tight bolt holes (or interference-fit shear pins or rivets), the fasteners may not carry the same load.

Example: If the bolts shown below each carry 20% of the applied load, …



Adapted from Ref. 20 (Peery), Fig. 12.18

In this joint, the two end fasteners want to take all the load so the inner and outer plates will have consistent deformation.

The internal fasteners carry load only to the extent caused by deformation of the outer bolts and holes (bearing).

From DABJ, Ref. 33

When the plates are made of ductile materials,

Design Shear Joints to Be Bearing Critical

***Shear-critical joint*:** one that has a lower margin of safety for fastener shear than for bearing.

– Can have little plastic deformation before ultimate failure.

– The highest-loaded fasteners can fail before the others take their share of load.

– Result: the joint can "unzip", as each fastener next to a failed one becomes overloaded and then also fails.

***Bearing-critical joint*:** one that has a lower margin of safety for bearing than for fastener shear.

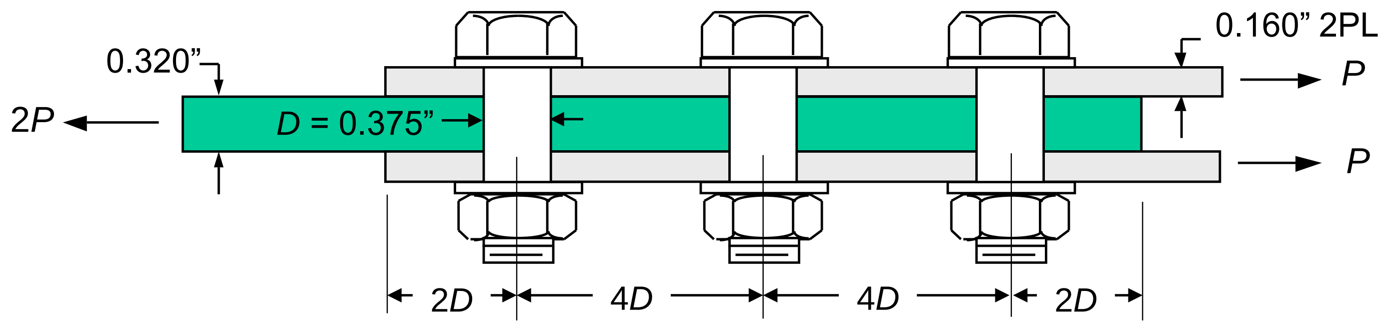
– Bearing failure in ductile materials exhibits more plastic deformation than bolt shear failure, so the bolts share load before the joint fails.

– Bearing failure in composite materials shows little or no deformation prior to rupture because the materials are brittle, so fasteners don't share load as well.

With metal plates, design the joint to be bearing critical by increasing fastener diameter relative to plate thickness.

Even if the joint is bearing critical, avoid long strings of fasteners in a splice, such as the one shown on the previous page.

Example Problem 4-d: Is this joint bearing critical or shear critical?



Bolt: A-286 alloy F_{su} = 95 ksi

Plates: Aluminum alloy F_{bry} = 94 ksi (for e/D = 2)

F_{bru} = 123 ksi (for e/D = 2)

Solution:

1. Calculate the allowable shear ultimate load, $P_{su\text{-allow}}$, per bolt and per shear plane:

Shear area, $$A_s = \frac{\pi(0.375)^2}{4} = 0.1104\,\text{in}^2 \qquad P_{su\text{-allow}} = F_{su}A_s = 95{,}000(0.1104) = 10{,}500\text{ lb}$$

Total allowable shear ultimate load per bolt with two shear planes = $2P_{su\text{-allow}}$ = 21,000 lb

2. Calculate the allowable bearing ultimate load, $P_{bru\text{-allow}}$, per bolt:

The middle plate has twice the load and is twice as thick as the outer plates:

$$P_{bru\text{-allow}} = F_{bru}Dt = 123{,}000(0.375)(0.320) = 14{,}800\text{ lb} < 21{,}000\text{ lb}$$

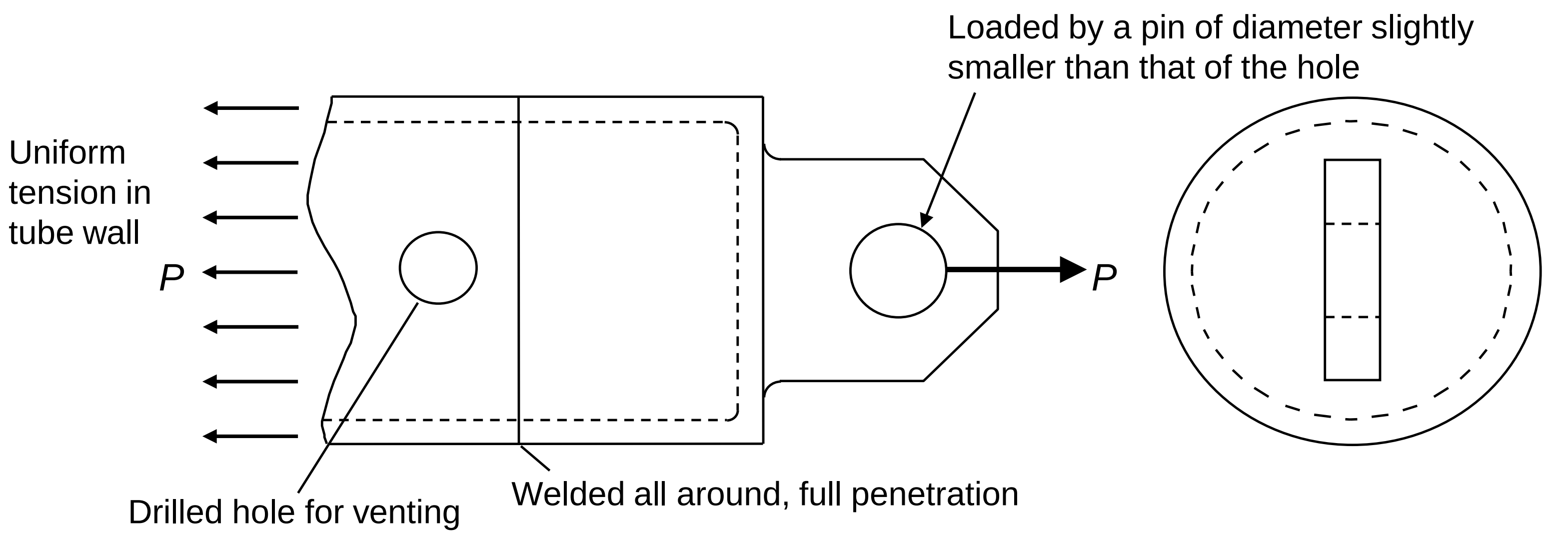
Therefore, the joint is bearing critical

From DABJ, Ref. 33

Class Problem 4-1: Recognizing Potential Failure Modes

A. Identify all potential ultimate failure modes of concern.

B. How would you improve the design?

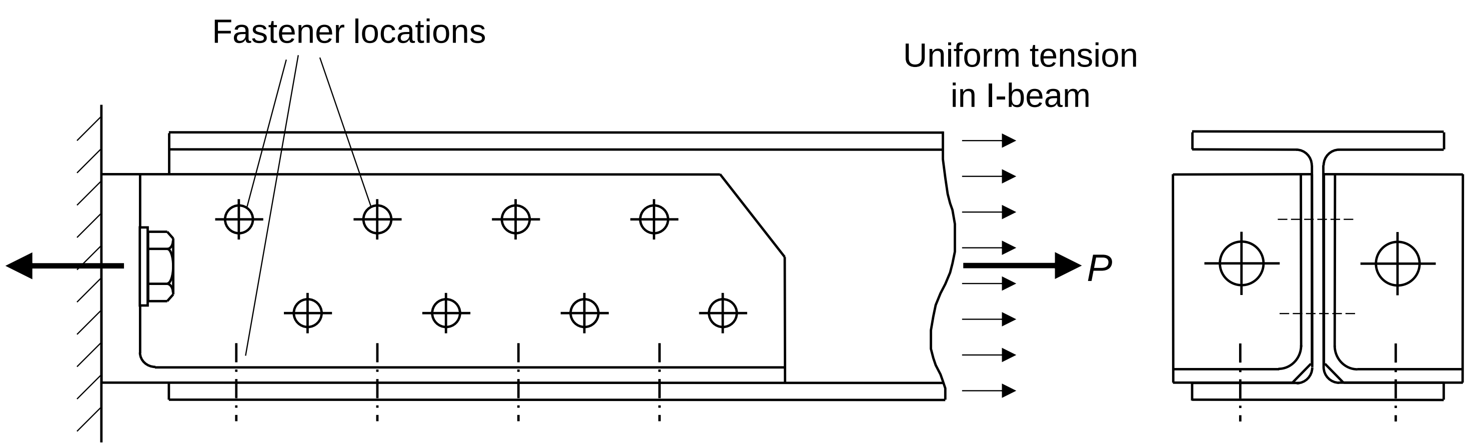


Material: 6061-T6 aluminum alloy

Class Problem 4-2: Recognizing Potential Failure Modes

A. Identify all potential ultimate failure modes of concern.

B. How would you improve the design?



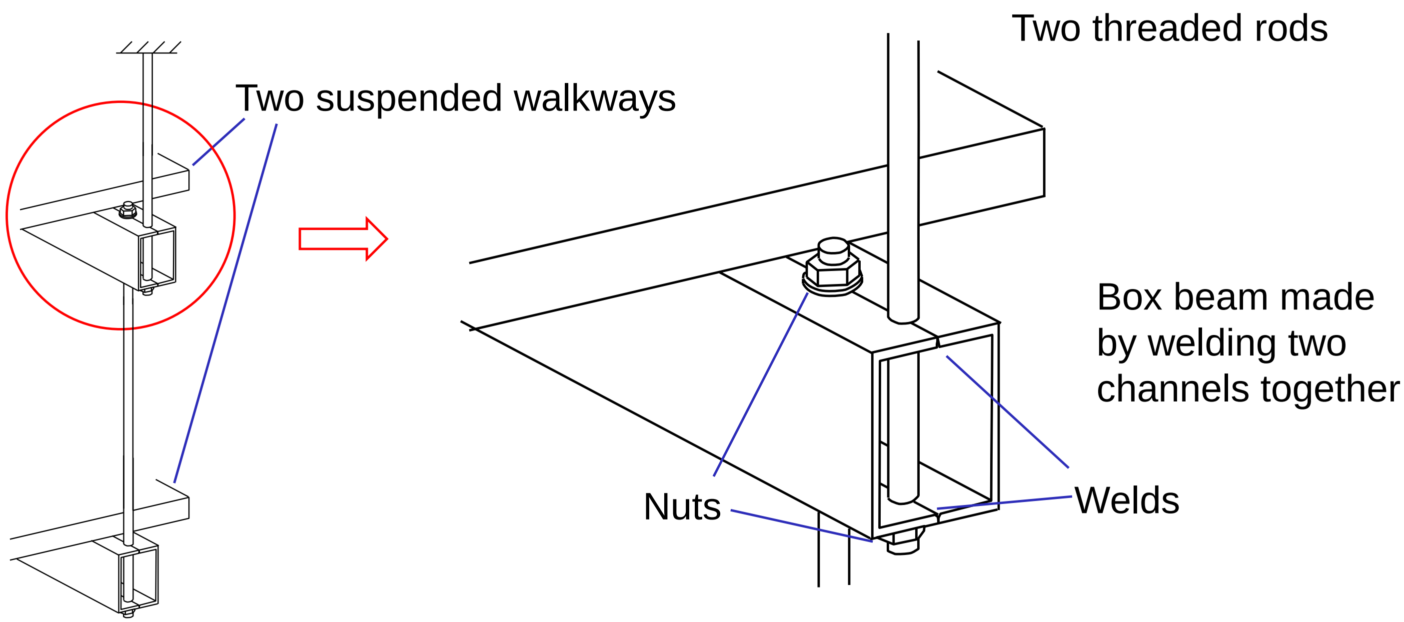
Materials: Aluminum alloy for I-beam and channels

From DABJ, Ref. 33

Class Problem 4-3: Recognizing Potential Failure Modes

A. Identify all potential ultimate failure modes of concern.

B. How would you improve the design?

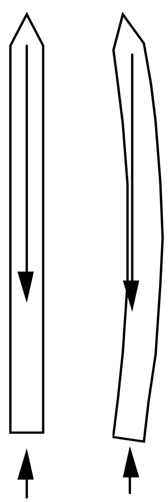


Materials: structural steel

Forms of Buckling

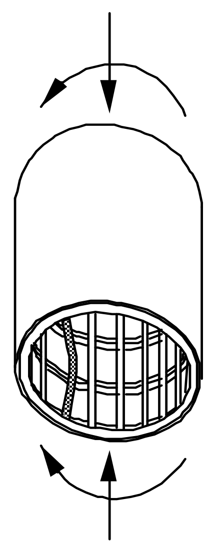
Instability: the state in which any small disturbance to a structure under compression or shear can cause it to ***buckle*** (suddenly bow laterally or twist)

The entire structure can buckle:



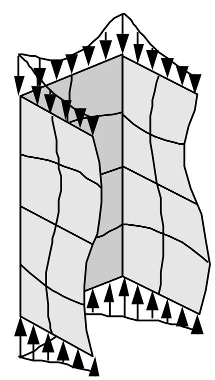
Usually catastrophic

An individual member in the structure can buckle:



Usually catastrophic

A flange or a web can buckle (***local buckling***):



Sometimes catastrophic

Other types of shells can buckle:

Usually catastrophic

The elastic-buckling force depends on geometry and the material's modulus of elasticity.

How Columns Fail in Compression

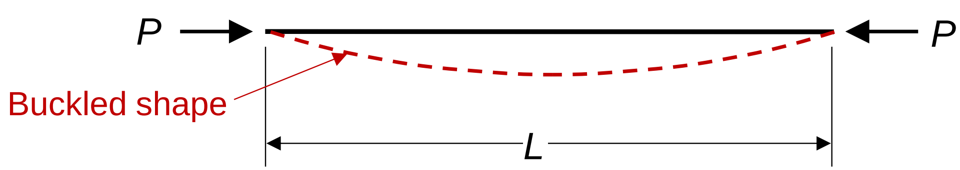
Length \ Cross Section	Compact, Thick-Walled	Thin-Walled
Long	Elastic column buckling	Elastic column buckling
Short	Compressive yielding and crushing	***Crippling*** (crushing after local buckling of flanges or webs)
Intermediate	Inelastic column buckling—compressive stress exceeds proportional limit	• Local buckling, followed by column buckling • ***Torsional instability*** (sudden twisting followed by collapse)

After local buckling occurs, a short compression member can carry more load until crippling occurs (shown at right), when the corners (junctions of flanges and webs) crush under concentrated stress.

Local buckling is <u>not</u> the same as crippling.

Elastic Column Buckling

A column becomes unstable when the applied load, *P*, reaches the column's ***critical load*** (aka ***buckling load***), P_{cr}:



Note: When the topic is buckling in this course, "*E*" is the compressive modulus of elasticity, referred to as "E_c" in the MMPDS.

With linear theory, for a pinned-end column (no moment fixity, as shown above),

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \textbf{(Eq. 4.10)}$$

I = moment of inertia

(Euler column elastic buckling equation)

And the ***buckling stress*** (a.k.a. ***column critical stress***), F_{cr}, is

$$F_{cr} = \frac{\pi^2 E}{\left(\frac{L}{\rho}\right)^2} \quad \textbf{(Eq. 4.11)}$$

Radius of gyration, $\rho = \sqrt{\frac{I}{A}}$

I = area moment of inertia
A = cross-sectional area

(Eq. 4.12)

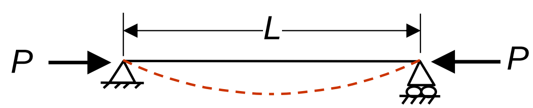
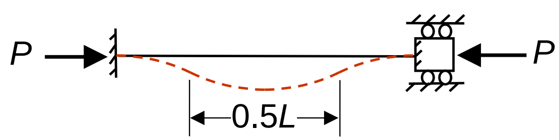
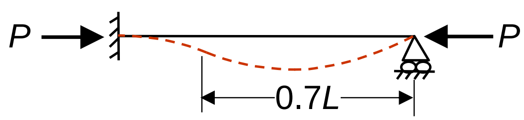
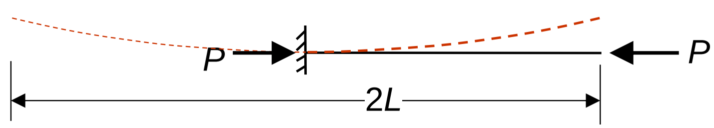
How End Fixity Affects a Column's Elastic Buckling Stress

For columns with different end fixity, the buckling stress is …

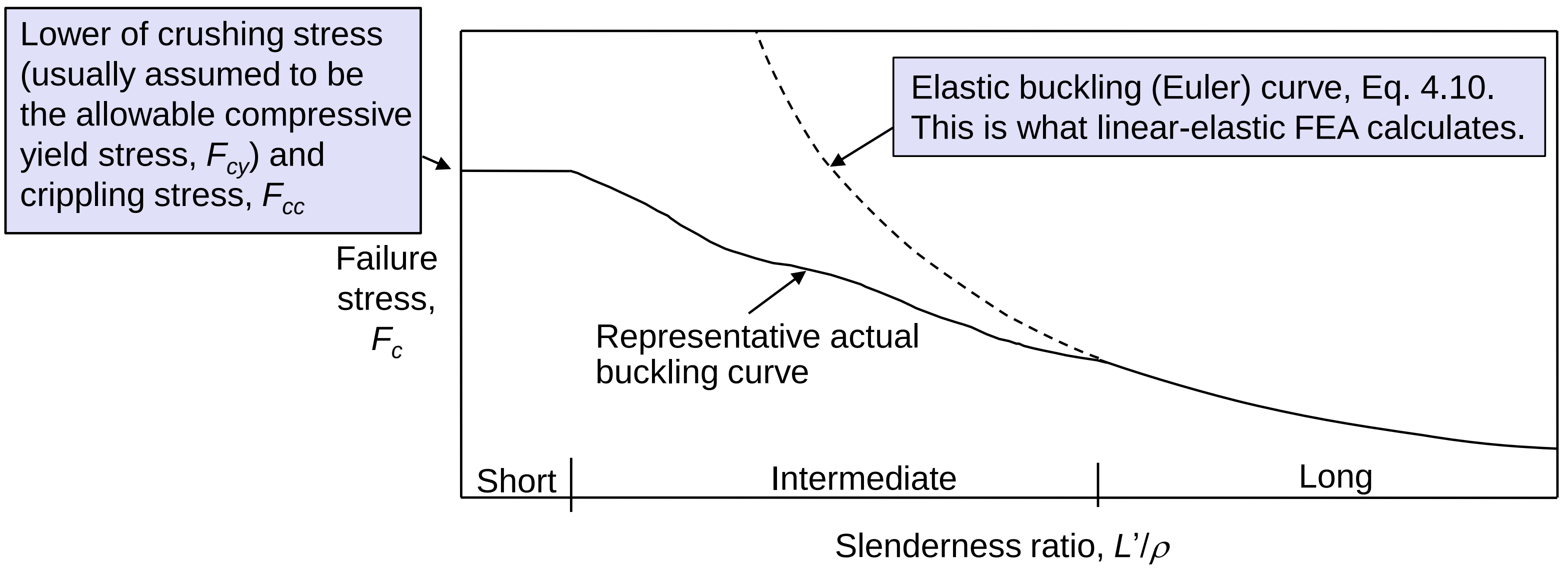
$$F_{cr} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2} = \frac{c_f \pi^2 E}{\left(\frac{L}{\rho}\right)^2}$$ **(Eq. 4.13)** (SSAM Eq. 8.37)

$\frac{L'}{\rho}$ is the ***slenderness ratio***

SSAM Table 8.6

Boundary Conditions		Effective length, L'	End-fixity coefficient, c_f
Pinned-Pinned:		L	1.0
Fixed-Guided:		0.5L	4.0
Fixed-Pinned:		0.7L	2.05
Fixed-Free:		2L	0.25

Relationship Between Length and Failure Stress for a Column



As a column gets shorter relative to its cross section's radius of gyration, the actual buckling stress tends to fall further below the linear-elastic buckling stress.

This is for a number of reasons: inelastic buckling, imperfect geometry, and local buckling of flanges or webs in the cross section.

Modified Johnson Parabola (for Torsionally Stable Columns)

The original Johnson Parabola was derived by J. B. Johnson circa 1900 to match test results, with the failure stress set equal to the compressive yield stress, F_{cy}, at $L'/\rho = 0$ and the parabola merging with the elastic Euler equation at a stress equal to $F_{cy}/2$. The equation was subsequently modified so that the limiting stress at $L'/\rho = 0$, F', is the lesser of F_{cy} and the crippling stress, F_{cc}. (Use F_{cc} for thin-walled sections and F_{cy} for compact sections.)

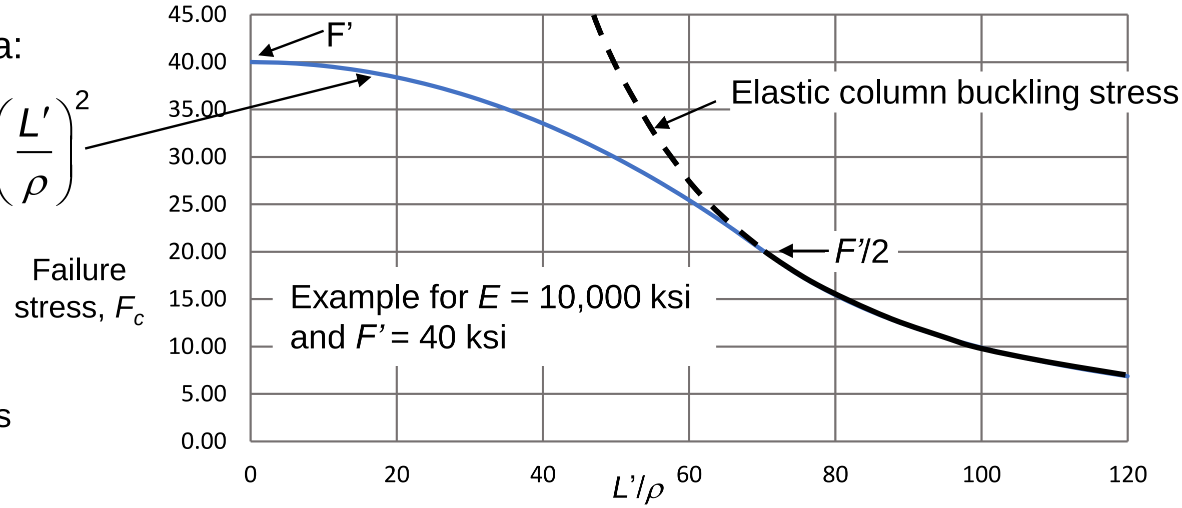
Modified Johnson Parabola:

$$F_c = F' - \frac{0.02533\left(F'\right)^2}{E}\left(\frac{L'}{\rho}\right)^2$$

(Eq. 4.14)

Similar to SSAM Eq. 8.58

where F' = lower of yield stress and crippling stress



This method is intended to account for the effects of inelastic buckling, local buckling, and dimensional imperfection—<u>but it does so only to some extent</u>.

Let's look at its limitations …

Limitations of the Modified Johnson Parabola

- The equation (4.14) uses *F'*, which is the lower of the allowable compressive yield stress, F_{cy}, and the crippling stress, F_{cc} .
- But inelastic-nonlinear behavior starts at the proportional limit, F_{pl} , not F_{cy} .
- And buckling of an intermediate-length column depends more on the local buckling stress (lowest stress at which a flange or web in the cross section buckles), $F_{cr\text{-loc}}$, than on F_{cc} .
- Thus, how well the Modified Johnson Parabola would match or conservatively envelop test results for columns made of different materials and different cross sections depends on the ratios F_{pl} / F_{cy} and $F_{cr\text{-loc}}$ / F_{cc} .
- The compressive failure stress also depends on how well we control dimensional tolerances.

The Modified Johnson Parabola is an approximation, and it may not be on the safe side for certain situations.

I prefer to dig deeper, which we'll do in the coming charts …

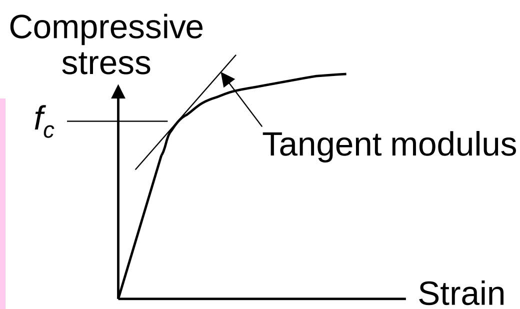
Inelastic Column Buckling Analysis

Equations for elastic buckling apply only if the compressive stress does not exceed the material's proportional limit, which is often well below the yield stress.

If the column's compressive stress exceeds the proportional limit, use the equation below for inelastic column buckling:

$$F_{cr} = \frac{\pi^2 E_t}{\left(L'/\rho\right)^2} \quad \textbf{(Eq. 4.15)}$$

where E_t = ***tangent modulus*** (tangent slope of stress/strain curve) at the stress level F_{cr}



This equation requires the material's stress/strain curve or, better yet, a curve showing tangent modulus vs. stress. (See upcoming example.) It also requires iteration because it has two unknowns: F_{cr} and E_t

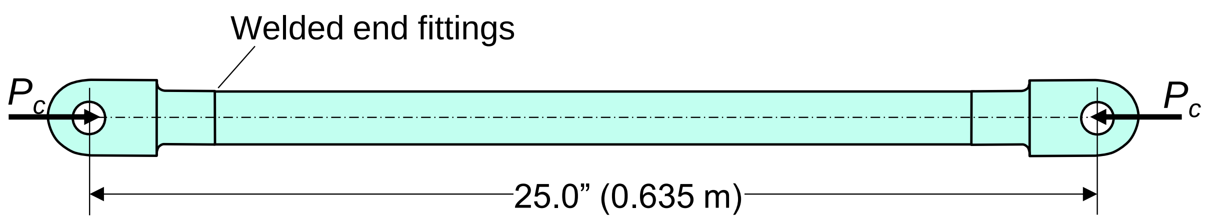
Any buckling equation that uses *E*, the elastic modulus, applies only up to the material's proportional limit.

In preliminary design, avoid the need for nonlinear buckling analysis by keeping the design ultimate compressive stress at or below the material's minimum compressive proportional limit, F_{pl}.

For aluminum alloys, you might start by assuming $F_{pl} = 0.7F_{cy}$ (allowable compressive yield stress). (The compressive proportional limit is between 60-80% of the compressive yield stress for most aluminum alloys.)

Example Problem 4-e: Column Buckling Analysis

Problem statement: Determine the critical load for buckling, P_{cr} :



Given: The column has a solid, circular cross section with diameter d = 1.85” (0.0470 m).

The material is 6061-T6 aluminum alloy extrusion with the following properties:

A-basis allowable compressive yield stress,
F_{cy} = 34 ksi (234 MPa)

(Ref. 5, MMPDS)

Young’s modulus in compression,
E = 10.1E+6 psi = 10,100 ksi (69,600 MPa)

continued

Example Problem 4-e: Buckling, continued

Solution: Area, $A = \dfrac{\pi d^2}{4} = \dfrac{\pi (1.85)^2}{4} = 2.69 \text{ in}^2$

Moment of inertia, $I = \dfrac{\pi d^4}{64} = 0.575 \text{ in}^4$

Radius of gyration, $\rho = \sqrt{I / A} = 0.462 \text{ in.}$

$L' / \rho = 25.0 / 0.462 = 54.1$

The column has pinned ends, so L' = L

First, find the elastic solution with Eq. 4.12, assuming the load is perfectly aligned:

$$F_{cr} = \frac{\pi^2 E}{(L' / \rho)^2} = \frac{\pi^2 E}{(54.1)^2} = 0.00337E = 0.00337(10{,}100) = 34.0 \text{ ksi}$$

Critical load: $P_{cr} = F_{cr} A = 91.7 \text{ kip}$ (407 kN)

However, the calculated buckling stress, F_{cr}, is equal to the allowable compressive yield stress, which means it's well above the proportional limit.

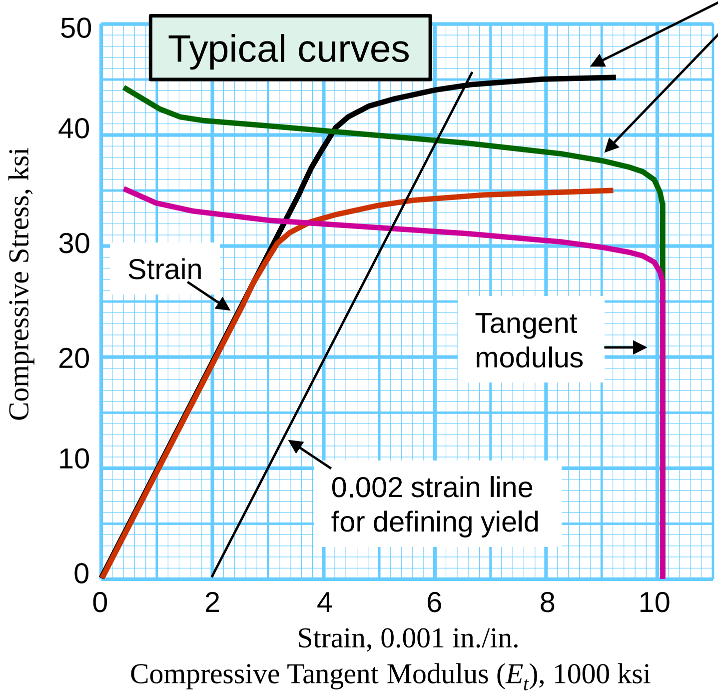
The above analysis is not valid. We need to account for inelastic buckling.

continued

Example Problem 4-e: Buckling, continued

Generating an A-basis-equivalent Tangent Modulus Curve

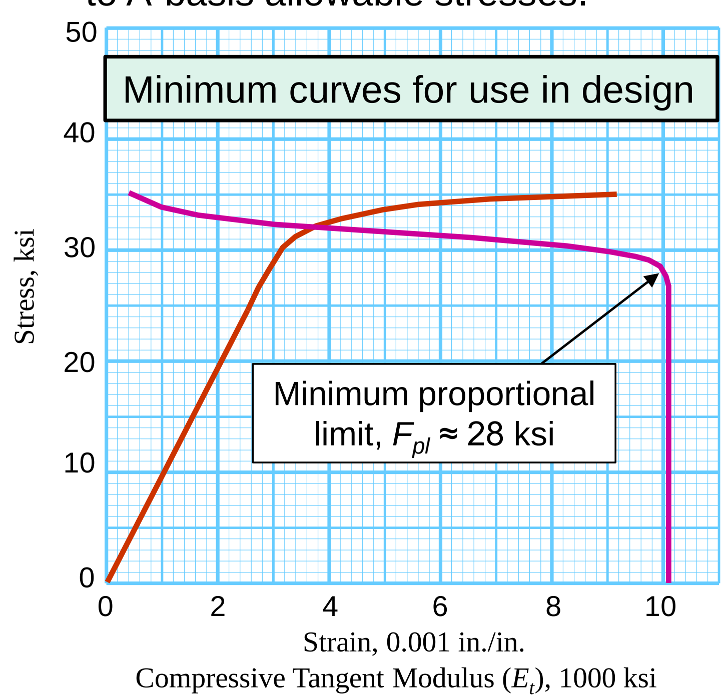
Start with MIL-HDBK-5J Fig. 3.6.2.2.6(i)* (Typical compressive stress-strain and tangent-modulus curves for 6061-T6 extrusion at room temperature, longitudinal direction)



Scale the curves down to agree with the A-basis allowable compressive yield stress of 34 ksi.

*Same figure and figure number in MMPDS-08

The resulting curves can be used for design with similar statistical assurance to A-basis allowable stresses.



Alternatively, use the Hill formulation of the Ramberg-Osgood equation to generate a design stress-strain curve. (See MMPDS Sec. 1.4.4.7.)

Example Problem 4-e: Buckling, continued

Compute Allowable Ultimate Compressive Load

Guess at F_{cr}, look up the corresponding tangent modulus, E_t, and then calculate F_{cr} with the inelastic buckling equation (Eq. 4.15) to see if it agrees with the assumption. Iterate as needed.

F_{cr}	E_t	F_{cr}	
29	9800	33.0	
31	7000	23.6	
30	8800	29.7	⇦ Close enough

Units: ksi

$$F_{cr} = \frac{\pi^2 E_t}{\left(L'/\rho\right)^2} \qquad \text{(Eq. 4.15)}$$

Thus, $\boxed{F_{cr} = 29.7 \text{ ksi}}$

and $\boxed{P_{cr} = AF_{cr} = 2.69(29.7) = 79.9 \text{ kip}}$ ← Inelastic column buckling load

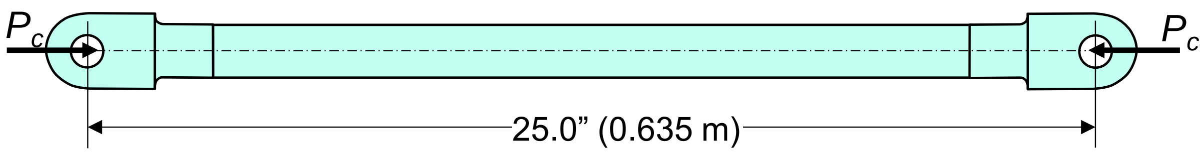
Note that if we had assumed the stress/strain relationship is linear up to the allowable yield stress, 34 ksi, we would have accepted the elastic solution.

But this strut may buckle at 29.7 ksi—87% of the allowable yield stress.

continued

Example Problem 4-e: Buckling, conclusion

Finite Element Analysis Results



A = 2.69 in^2
I = 0.575 in^4
E = 10.1 msi

Results of NEI Nastran linear-elastic buckling solution, with between 1 and 8 CBAR elements of identical properties:

Number of elements	P_{cr} (kip)
1	111.504
2	91.467
4	90.763
8	90.698

As compared with 91.7 kip from the Euler column equation and 79.9 kip from the inelastic buckling solution (previous page)

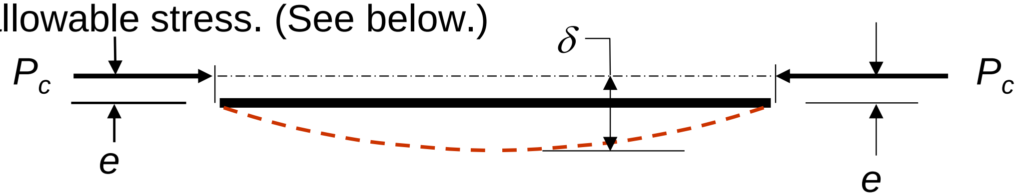
Conclusions:

1. **The FEA solution depends on model fidelity.**
2. **Generally, greater fidelity (more elements) leads to more accuracy, but for this example the answer is converging on something different than the theoretical solution.**
3. **The analyst must recognize when linear solutions don't apply.**
4. **Don't rely solely on FEA!**

Eccentrically Loaded Columns

Because of dimensional tolerances, a compressive load will not be perfectly aligned with the cross section's centroid.

To determine an allowable compressive load for a given eccentricity, calculate the compressive stress, f_c, caused by the combination of compressive load and bending moment, and compare that stress to the allowable stress. (See below.)



For a symmetrical section,

$$f_c = \frac{P_c}{A} + \frac{Mc}{I} = \frac{P_c}{A} + \frac{P_c \delta c}{I} \quad \textbf{(Eq. 4.16)}$$

$$\text{where} \quad \delta = e \cdot \sec\left(\frac{L}{2}\sqrt{\frac{P_c}{EI}}\right) \quad \textbf{(Eq. 4.17)}$$

(Ref. 10, McGinty; agrees well with the approximate equation in SSAM)

This quantity is in radians, not degrees

A = cross-sectional area
M = bending moment
c = distance from neutral axis to extreme edge
I = area moment of inertia
e = initial eccentricity
δ = final eccentricity after deformation

Equation 4.17 is based on linear theory, so it is valid only if the stress calculated with Eq. 4.16 does not exceed the material's compressive proportional limit.

Use the lower of the minimum proportional limit and the local elastic buckling stress (if applicable—see upcoming discussion) as the allowable ultimate stress.

Class Problem 4-4: Eccentrically Loaded Column

Problem statement:

For the strut used in the Example Problem 4-e, calculate the margin of safety for a limit compressive load, P_c, of 37,000 lb and an ultimate factor of safety, FS_u, of 1.4, assuming that, because of dimensional tolerances, the load can be misaligned by 0.030" and the strut can be initially bowed by 0.020" over its 25" length.

Approach:

Analyze the strut as an eccentrically loaded column, with an eccentricity, e, equal to 0.030 + 0.020 = 0.050".

To avoid the uncertainty associated with inelastic effects, use an allowable ultimate compressive stress, F_{cu}, of 28 ksi, which is the approximate minimum proportional limit for 6061-T6 extrusion, based on the tangent-modulus curves presented earlier.

This Page Intentionally Left Blank

Options for Redesigning the Strut in Class Problem 4-4

1. Change the material
   - Probably would increase cost, though
2. Increase the diameter of the strut
   - Increases weight
3. Change from a solid rod to a tube, with larger outer diameter
   - If there is room to do so
4. Reduce dimensional tolerances
   - Reducing the total tolerance from 0.050” to 0.040” would do the trick for this strut

Summary of Analysis Results for Our Example Column

		Allowable Load (kip)	
	Linear-elastic theory: Euler column equation	91.7	(100%)
	Linear-elastic FEA (8 elements)	90.7	(98.9%)
	Inelastic column analysis assuming perfect geometry	79.9	(87.1%)
⇨ Recommended	Eccentric column analysis with stress limited to the minimum proportional limit, accounting for dimensional tolerances	49.2	(53.6%)
	Modified Johnson Parabola (Eq. 4.14)	68.6	(74.8%)

Results would be similar for shell buckling, depending on whether you account for misalignments and inelasticity.

It's easy to overestimate compressive load capability with linear-elastic methods, which are commonly used in finite element buckling analysis.

In many cases, buckling analysis should not be based on linear-elastic theory, with or without use of FEA!

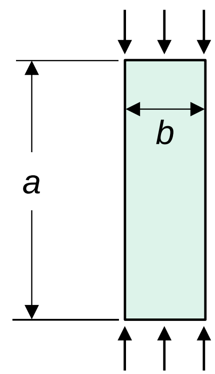
Be cautious in analysis … and test your structures!

Buckling of Flat Plates with Unsupported Edges

Consider a wide, thin plate made of a homogeneous, isotropic material with unsupported side (non-loaded) edges.

When such a plate bends about its weak axis, Poisson's effect is constrained in the width (b) direction, causing a transverse stress.

As a result, the Euler elastic column buckling equation (Eq. 4.10) does not apply. Instead, …



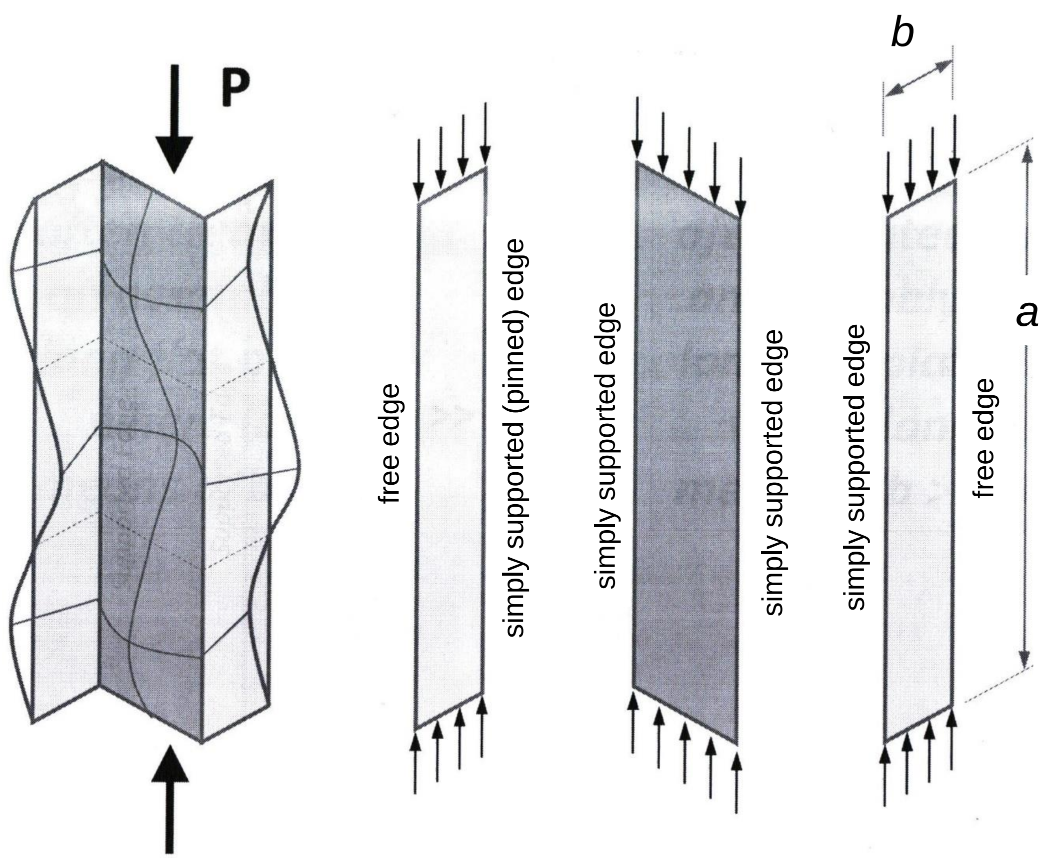
$$P_{cr} = \frac{\pi^2 EI}{\left(1-\nu^2\right)a^2} \quad \textbf{(Eq. 4.18)}$$

where P_{cr} = elastic buckling load
ν = Poisson's ratio
t = plate thickness

And the elastic buckling stress is …

$$F_{cr} = \frac{\pi^2 E t^2}{12\left(1-\nu^2\right)a^2} \quad \textbf{(Eq. 4.19)}$$

Local Buckling of Thin-Walled Sections



A column with a thin-walled section can be idealized as an assembly of flat plates with at least one long edge supported (constrained).

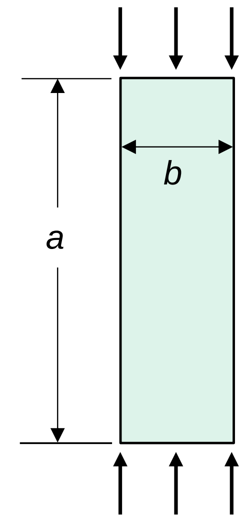
(image courtesy Scott Malaznik)

Buckling of Plates with Supported Edges

Elastic buckling stress, F_{cr}, for a flat plate made of a homogeneous, isotropic material, with constraints on one or more of the non-loaded edges:

$$F_{cr} = \frac{k\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$ **(Eq. 4.20)** (SSAM eq. 8.46)

where ν = Poisson's ratio
t = plate thickness
k = buckling coefficient

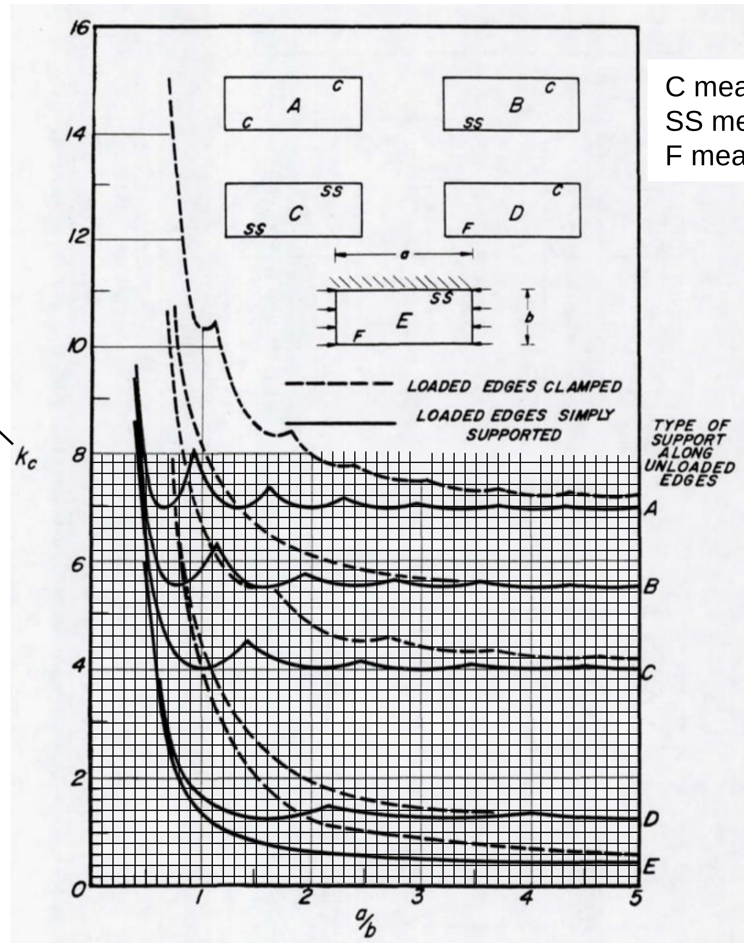


The buckling coefficient, k, depends on edge conditions and the ratio of length, a, to width, b. (See next page.)

For inelastic buckling, when the compressive stress exceeds the minimum proportional limit, include a plasticity reduction factor, η, in this equation. (See Refs. 9 and 35.)

Elastic Buckling Coefficient for Flat Plates under Compression

Buckling coefficient, *k*
(also referred to as k_c)



C means clamped (fixed)
SS means simply supported (pinned)
F means free

From ref. 11. Also appearing in the Astronautic Structures Manual, ref. 9, Fig. C2-5. (Fine grid added by Tom Sarafin.)

Ref. 9 also provides reduction factors for cladding. (A ***clad*** aluminum part is one whose outer layers are pure aluminum, for better corrosion resistance. Pure aluminum yields at a lower stress than the alloy, thus reducing buckling strength.)

Once the ratio *a*/*b* reaches about 4, the value of *k* becomes relatively constant for given side-edge conditions, and the effect of whether the loaded edges are fixed or pinned is nearly negligible.

Simplified Elastic Buckling Equation for Flat Plates

For infinitely long (approximated when a/b > 4) flat plates made of homogeneous (e.g., not clad), isotropic materials having a particular Poisson's ratio, ν, the equation for elastic buckling stress can be simplified:

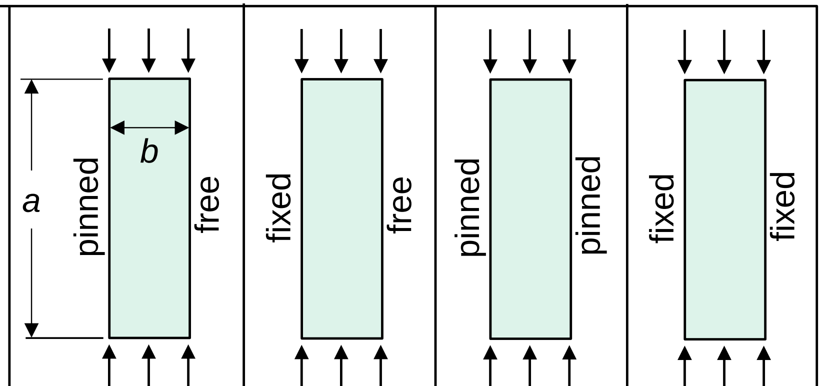
$$F_{cr} \approx k'E\left(\frac{t}{b}\right)^2$$

(Eq. 4.21a) (SSAM eq. 8.47) where k' is an adjusted buckling coefficient

Combining Eqs. 4.20 and 4.21a,

$$k' = \frac{k\pi^2}{12\left(1-\nu^2\right)}$$

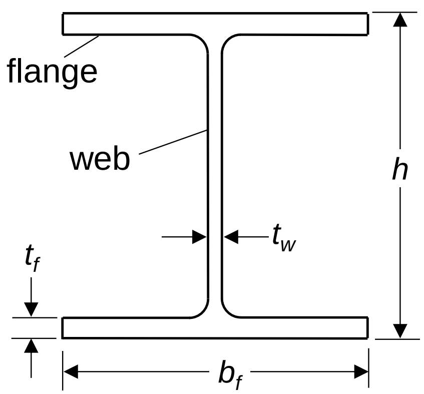
(Eq. 4.21b)



	Edge conditions	pinned / free	fixed / free	pinned / pinned	fixed / fixed
From Ref. 35 Table 7 ⇨	k	0.43	1.28	4.00	6.98
For ν = 0.33, per Eq. 4.21b ⇨	k'	0.40	1.18	3.69	6.44

Note: I recommend using the above k' values instead of those shown in SSAM Table 8.9, for which no reference is given.

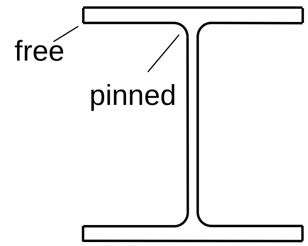
Example Problem 4-f, Local Buckling of I-section Flange and Web



Material: aluminum alloy, not clad
$E = 10 \times 10^6$ psi $= 10^4$ ksi
$\nu = 0.33$
F_{pl} = 38 ksi, minimum proportional limit
h = 3.00"
b_f = 2.75"
t_w = 0.080"
t_f = 0.100"

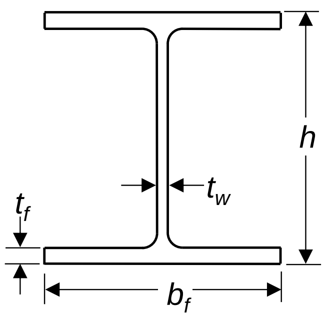
Problem statement: Calculate the flange and web buckling stresses for safe use in design.

Approach: Conservatively idealize a flange as a long, flat plate with pinned-free edge fixity and the web as a long, flat plate with pinned-pinned fixity.

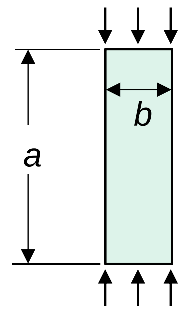


continued

Example Problem 4-f, solution



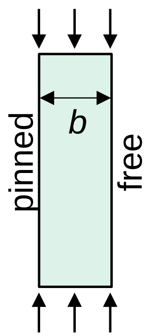
h = 3.00”
b_f = 2.75”
t_w = 0.080”
t_f = 0.100”



With the assumption that the plate is “long,” dimension *a* is not relevant for flange or web buckling stress.

Flange:

b = 2.75/2
= 1.375”



Elastic buckling stress per Eq. 4.21a is

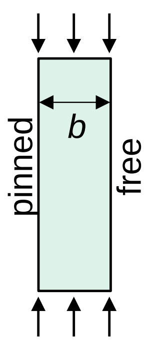
$$F_{cr} \approx k'E\left(\frac{t}{b}\right)^2$$ where k' = 0.40

$$= 0.40\left(10^4\right)\left(\frac{0.100}{1.375}\right)^2 = \boxed{21.2\text{ ksi}}$$

Because these stresses are below the 38 ksi minimum proportional limit, no plasticity correction is necessary.

Web:

b = 3.00 – 0.100
= 2.90”



$$F_{cr} \approx k'E\left(\frac{t}{b}\right)^2$$ where k' = 3.69

$$= 3.69\left(10^4\right)\left(\frac{0.080}{2.90}\right)^2 = \boxed{28.1\text{ ksi}}$$

To ensure no local buckling, uniform compressive stress in the I-section is limited to 21.2 ksi.

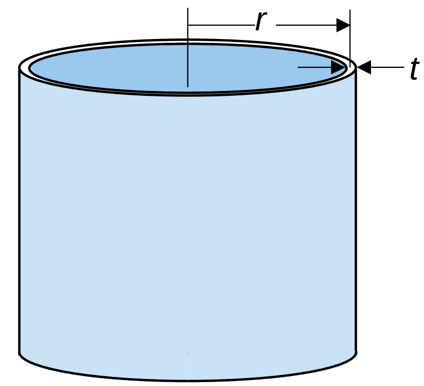
Reducing Conservatism when Needed

- When weight is critical, idealizing a cross-section’s flanges and webs as plates can be excessively conservative.
- Section C4 of Ref. 9 provides empirical data and methods for local buckling of various thin-walled cross sections. These plots also appear in Ref. 1 (SSAM) Sec. 8.5.
- For short columns, local buckling usually is not catastrophic. As explained earlier, if the member does not buckle as a column as a result of local buckling, compressive stress can increase up to the crippling stress.
- Section C1.3.1 of Ref. 9 provides a good treatment of crippling and how to predict the crippling stress.

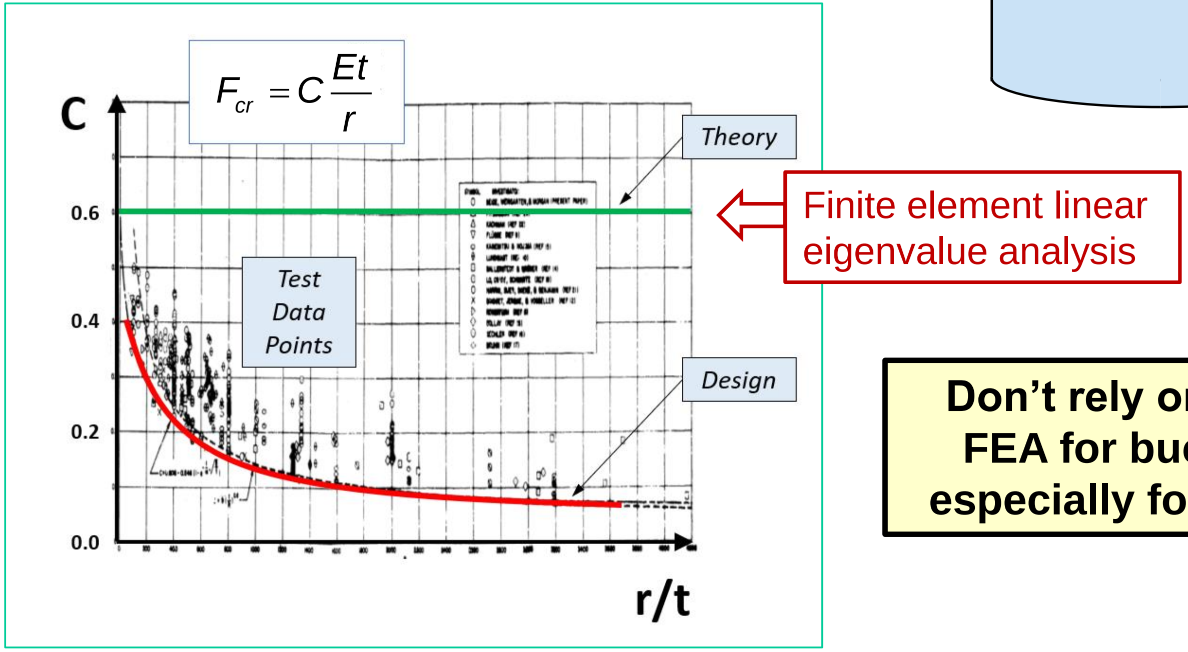
Buckling of Monocoque Cylinders

Theoretical elastic buckling stress of an axi-symmetric, thin-walled shell under uniform compression:

$$F_{cr} = 0.61\frac{Et}{r} \quad \textbf{(Eq. 4.22a)}$$



But shell buckling strength is very sensitive to geometric imperfections from dimensional tolerances:



Source: Ref. 36 (best copy available)

Don't rely on linear FEA for buckling, especially for shells!

Historical Knockdown Factors for Buckling of Monocoque Cylinders

Given all of the following:

- Monocoque cylinder (no stiffeners)
- Isotropic
- Poisson's ratio = 0.33 (aluminum alloys)
- Not pressurized
- Compressive loading (axial load or moment), without stress concentration
- Calculated buckling stress does not exceed the proportional limit

Elastic buckling stress,

$$F_{cr} = 0.61\gamma \frac{Et}{r}$$

(Eq. 4.22b)

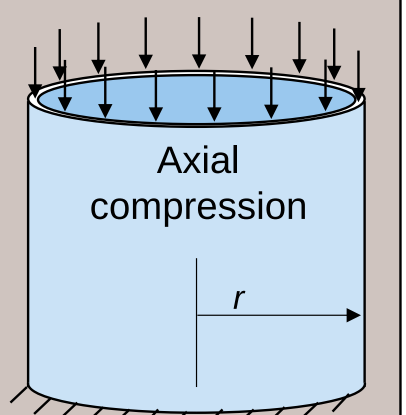
t = wall thickness
r = radius
γ = (see below)

(Derived from ref. 9 (ASM) Vol. 2, Sec. 3.1.1)

γ is a factor ("knockdown factor") used to correlate theory with lower-bound test results:

$$\gamma = 1 - 0.901\left(1 - e^{-\phi}\right)$$

(Eq. 4.23) (SSAM Eq. 8.61)



where

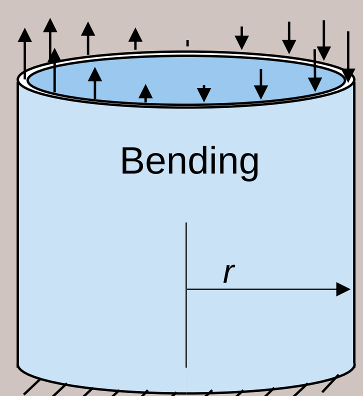
$$\phi = \frac{1}{16}\sqrt{\frac{r}{t}} \quad \text{for} \quad \frac{r}{t} < 1500$$

(Eq. 4.25) (SSAM Eq. 8.62)

We'll use these equations in a sizing example in Sec. 6.

$$\gamma = 1 - 0.731\left(1 - e^{-\phi}\right)$$

(Eq. 4.24) (SSAM Eq. 8.63)



In 2007, NASA started the Shell Buckling Knockdown Factor Project to derive less-conservative factors. I have not been able to find results.

Key Points from This Section

- Use statistically appropriate allowable stresses.
- Proof test your structures …
  – when strength varies widely
  – or when using B-basis allowables rather than A-basis allowables.
- Remember that comparing stresses calculated with FEA (or any other method) to allowable stresses is not valid without test substantiation.
  – Especially true for assessing ductile rupture, for which failure theories such as the von Mises criterion don't apply.
  – Find and use empirical and semi-empirical methods.
  – Try to make an apples-to-apples comparison.
  – If the process is apples to oranges, use a penalizing uncertainty factor.
  – If the best process you can find is apples to elephants, lobby to either change the design or do development tests to derive dependable methods or allowables.

(continued)

Key Points from This Section (continued)

- Stop analyzing bad designs! Improve the design instead.
- In buckling analysis, account for the following, and recognize that linear-elastic buckling solutions with FEA do not:
  - inelastic buckling
  - imperfect geometry and eccentric loading

  Use empirical or semi-empirical methods when available.
- Test structures of new design.
  - It's too easy to overlook the critical failure mode
  - and you may not be able to find applicable empirical methods of analysis.

5. Fatigue Assessment

This section contains material developed by Poti Doukas, Mike Browning, and John MaCoun.

- What Is Fatigue?
- Brief History of Fatigue Failures and Ensuing Research
- Stress Concentration Factor
- Terms Defining a Loading Cycle
- Fatigue Analysis Process
- Quantifying Fatigue Life by Test
- Presentation of Fatigue Data
- High-cycle vs. Low-cycle Fatigue
- Miner's Rule
- Life (Scatter) Factor and Fatigue Analysis Factor
- Fatigue Notch Factor and Notch Sensitivity Factor
- The Goodman Method and Equivalent Alternating Stress
- Linear-Elastic Fracture Mechanics and Fracture Control
- Generating a Loading Spectrum

What Is Fatigue?

Fatigue is the weakening of a material as a result of cyclic stress, which may lead to failure at a lower stress than the stress that causes failure from a single loading event.

– Fatigue is a phenomenon common to metals.

– Under cyclic stress, a crack can form and then, under repeated loading, grow to full rupture (***fracture***).

Fatigue is a cumulative process.

Fatigue damage remains in materials, even after years of inactivity.

When a ductile material fails under one-time application of stress, failure is normally ductile (plastic deformation prior to rupture).

When a ductile material fails from cyclic stress, failure is brittle (little to no plastic deformation).

Brief History of Fatigue Failures and Ensuing Research

1829—Wilhelm Albert (German mining engineer) observed and studied the failure of iron chains used in mines, caused by repeated loading. Wrote the first paper on metal fatigue in 1838, although he did not use the term “fatigue”.

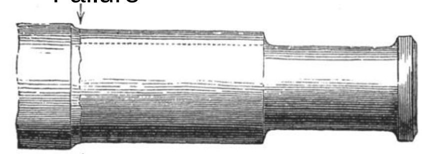
1839—Jean-Victor Poncelet (French engineer and mathematician) was the first to use the term “fatigue”, as he explained that cast iron axles fail with repeated use because they get tired.

1842, Versailles, France: A train axle failed, causing a crash that killed up to 200 people (estimated).

Painting of the Versailles train crash

(Images from Wikipedia)

William Rankine (Scottish engineer and physicist), based on studying the 1842 Versaille train crash and other axle failures, wrote a paper recognizing that such failures were fatigue failures, the result of crack formation and growth, normally in regions of stress concentration, at discontinuities.



continued

Fatigue Research Ramped Up as Metal Usage—and Failures—Increased

1867—August Wöhler (German railroad engineer), reporting on cyclic-load testing he had performed, presented the results in the form of stress (*S*) vs. number of cycles to failure (*N*). *S-N* curves followed and soon became known as "Wöhler curves".

- First to recognize the importance of both cyclic stress and mean stress
- Noted that the range of cyclic stress was more important to fatigue life than the peak stress

But fatigue research in the 1800s and early 1900s was mostly ignored by engineers.

Fatigue failures were typically addressed by increasing the factor of safety used in strength analysis.

As mentioned in Sec. 1 herein, …

In response to failures in connecting rods in locomotives as recently as 1910, a factor of safety of 18 was used for design! (ref. 12)

Into the 1900s

22 Pages Today

The Boston Post

EXTRA

HUGE MOLASSES TANK EXPLODES IN NORTH END; 11 DEAD, 50 HURT

Giant Wave of 2,300,000 Gallons of Molasses, 50 Feet High, Sweeps Everything Before It—100 Men, Women and Children Caught in Sticky Stream—Buildings, Vehicles and L Structure Crushed

35 STATES ON DRY LAW LIST

Amendment Ratified by Five Yesterday—One More Needed—Predict Nation Dry July 1

SECRECY IN PEACE CONGRESS

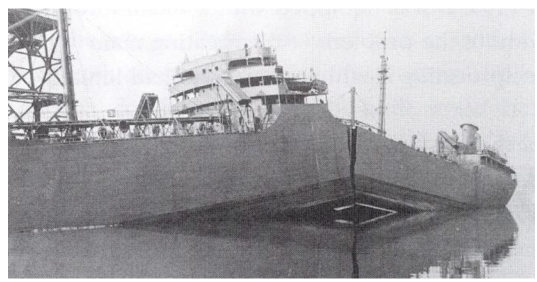
Search for More Victims During the Night

No Escape From Gigantic Wave of Fluid

1919, Boston: A huge steel tank holding 2.3M gallons of molasses exploded as a result of a fatigue crack.

1920—Alan Griffith originated the field of fracture mechanics as an extension to ongoing fatigue research.

1942: The SS Schenectady tanker broke in two from brittle fracture. A fatigue crack starting at the square corner of a hatch grew across the hull.



Multiple ships built to the original Liberty ship design failed in this manner and were lost at sea.

1945—M. A. Miner popularized a method of assessing fatigue damage that becomes known as the “Palmgren-Miner Linear Damage Rule”, or simply “Miner’s Rule”, based on an approach first proposed by A. Palmgren in 1924.

Modern Perspective

Even though most structural failures of metals are fatigue failures, strength analysis still is emphasized in engineering far more than fatigue analysis.

Fatigue education and engineering practices continue to lag.

Wikipedia image

1998 derailment of high-speed train near the village of Eschede, Germany, as a result of fatigue failure of a wheel, killing over 100 people

Image from https://www.dw.com/en/eschede-germanys-worst-train-disaster-remembered-20-years-on/a-44056391

Is Fatigue a Concern for Spacecraft, Launch Vehicles, or Missiles?

- Fatigue is clearly a concern for ships, aircraft, and trains that may be used for decades.
- But fatigue failures are not uncommon for single-mission flight vehicles, such as missiles, launch vehicles, and satellites:
  - Electronics components, avionics boxes, and even primary structures of small spacecraft experience many loading cycles from random vibration, especially during ground testing.
  - A satellite in low-Earth orbit sees many cycles of thermal stress from changing sun angles and shading; longer missions mean more thermal cycles.
  - Some metals that we may use because of certain attractive properties, such as beryllium (high modulus vs. density), are not very resistant to fatigue.
- Some vehicles are used for long-duration missions (International Space Station), and launch-vehicle reuse over multiple missions is becoming more popular.

Fatigue is typically not a concern for a structure that sees relatively few significant loading cycles, such as the primary structure of a large single-mission vehicle—but even then an extremely high concentration of stress can lead to fatigue failure.

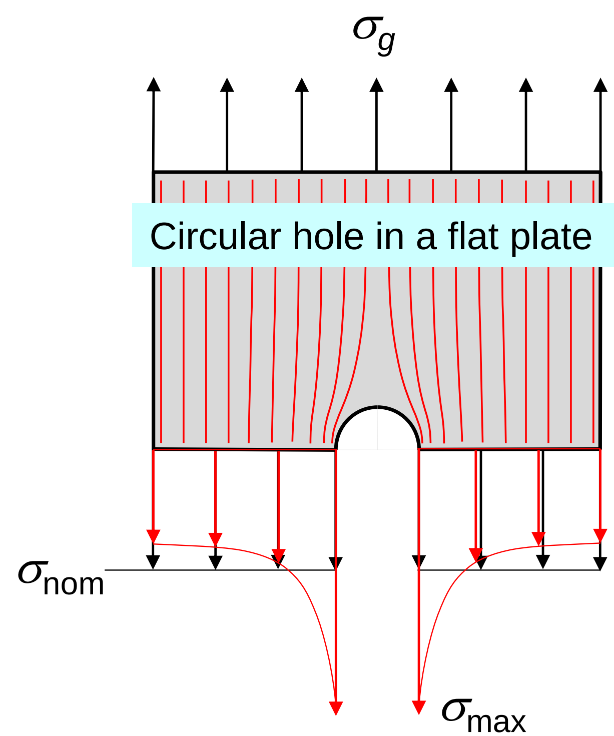
Fatigue is a concern whenever a part has either a high number of loading cycles or a high stress concentration.

Stress Concentration Factor, K_t

Stress concentrates in regions of changing geometry, such as holes, notches, and fillets.

The ***stress concentration factor, K_t***, is defined as the ratio of the maximum stress, σ_{max}, (when linear elastic) to the ***nominal stress***, σ_{nom}, which is the calculated stress when discounting the concentration.

For clarification, we can refer to K_{tn} when σ_{nom} is the net-section stress, as is the case on the right, and to K_{tg} when σ_{nom} is the gross-section stress, σ_g.

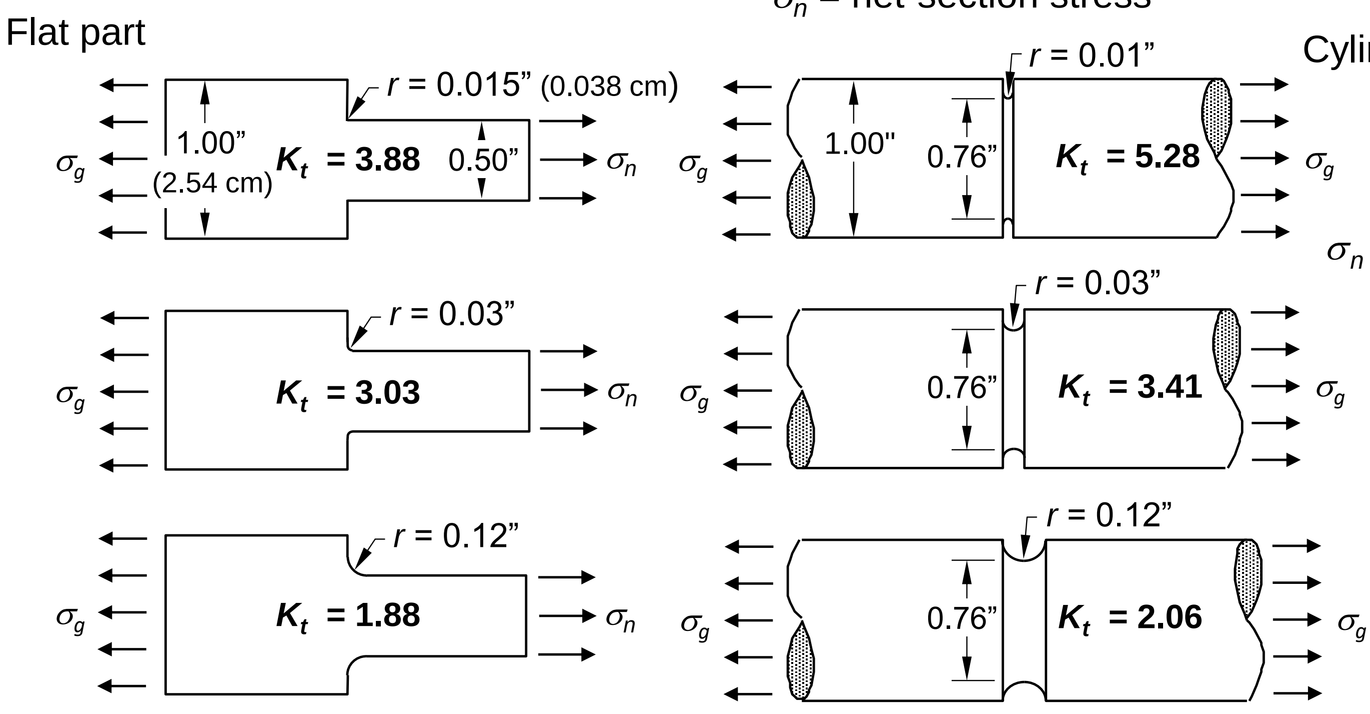


$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad \textbf{(Eq. 5.1)}$$

References 4 and 13 are good sources of stress concentration factors for different types of geometry.

Reducing Stress Concentrations

Peak linear-elastic stress = $K_t \sigma_n$ where K_t = stress concentration factor
σ_n = net-section stress



$$\sigma_n = \left(\frac{1.00}{0.76}\right)^2 \sigma_g$$

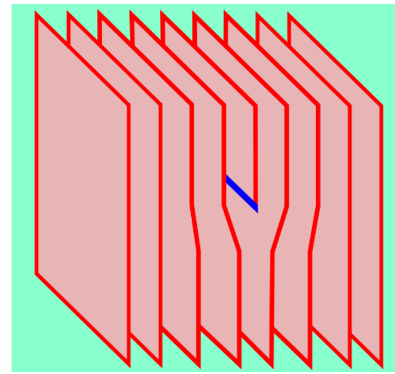
Reduce stress concentrations and improve fatigue life by changing the cross section gradually and increasing fillet radii.

Above stress concentration factors calculated using equations from Table 17 of Ref. 4 (Roark)

What Causes Fatigue?

- All materials have defects, and cracks form at defects, where stress concentrates.
- A ***dislocation*** is a linear defect in the arrangement of atoms.
- Under cyclic stress, dislocations can move along slip planes, as atoms slide over each other.
- A dislocation can move to the surface, where stress is usually highest, as atoms break their bonds and rebond with the atoms at the terminating edge.
- Cyclic slip of atom planes can cause a crack to form, most often at the part surface in regions of geometric stress concentration.
- Continued slip of atom planes from cyclic stress causes the crack to grow, slowly at first, until a critical flaw size is reached; then the crack grows rapidly to full fracture.

Other imperfections can occur from oxidation at the surface or an embedded foreign substance (***inclusion***).



Edge dislocation: One plane of atoms terminates, and the adjacent planes bend around it.

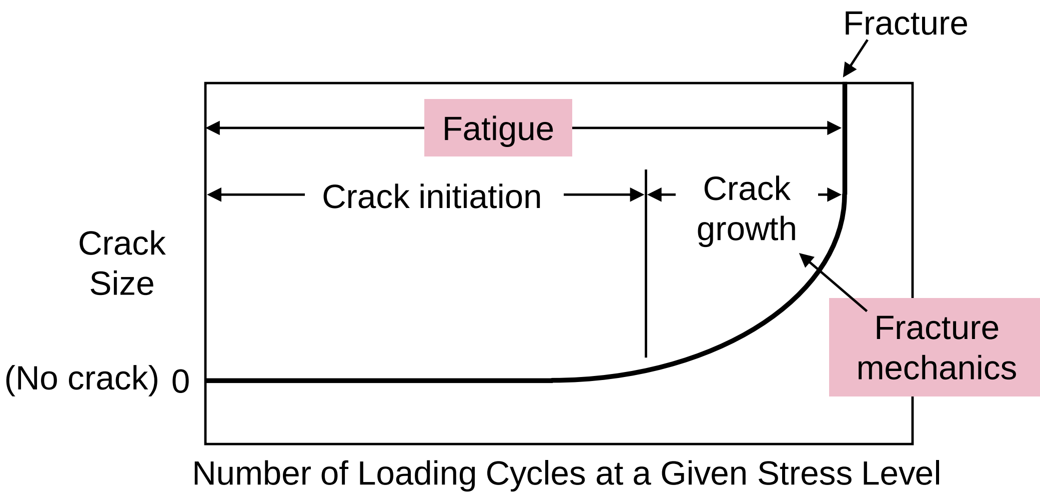
Fatigue failure of an aluminum part

Dark area—slow crack growth.
Bright area—sudden failure.

Images from Wikipedia

Crack Initiation and Growth

Three phases of fatigue: crack initiation, crack growth, fracture (full rupture)



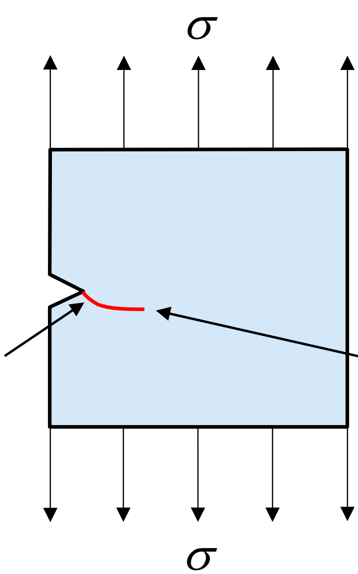
The number of cycles to failure in most fatigue data or curves is the number of cycles to fracture.

***Fracture mechanics*—the mechanics of crack growth—is a subset of the field of fatigue.**

When we're concerned that a crack is present or may be present, we do <u>fracture mechanics crack-growth analysis</u>, based on testing of specimens with initial cracks.

Otherwise, we do general <u>fatigue analysis</u> to assess life, based on data from testing of specimens without initial cracks.

Cyclic Shear Stress Causes Crack Formation; Normal Stress Causes Crack Growth



A crack usually starts in a region of stress concentration, in a plane of principal shear stress

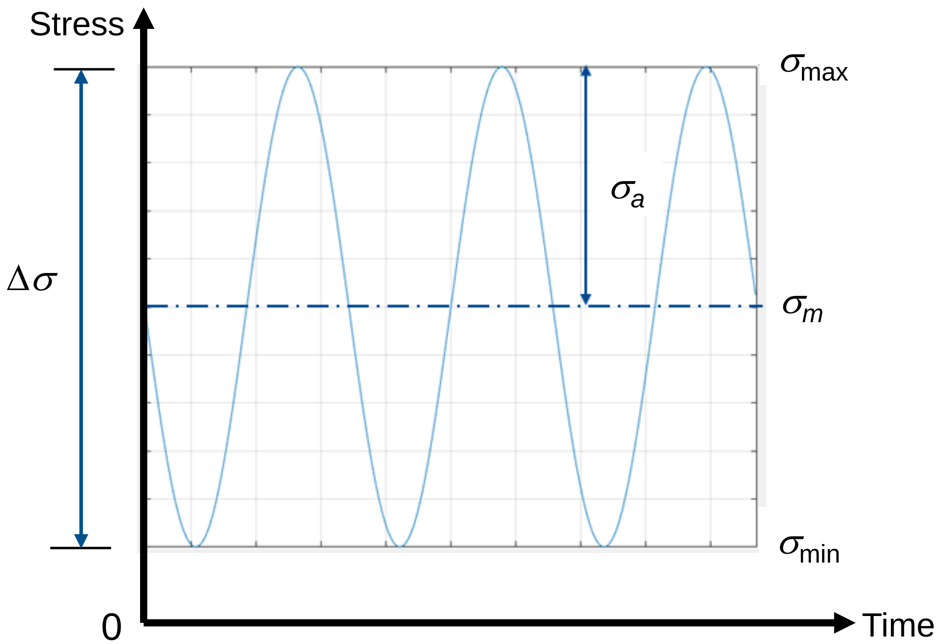
As the crack grows, it typically reorients to be normal to the principal tensile stress

Von Mises stress is most applicable for high-cycle fatigue analysis, in which most of the material's life is in crack initiation.

Principal normal stress is most applicable for crack-growth analysis.

Terms Defining a Loading Cycle

Example: three identical loading cycles



When stress is fully reversed (σ_{min} = -σ_{max}), *R* = -1.

This situation causes the most fatigue damage.

Stress range, $\Delta\sigma$

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \qquad \textbf{(Eq. 5.2)}$$

Alternating stress, σ_a

$$\sigma_a = \frac{(\sigma_{max} - \sigma_{min})}{2} \qquad \textbf{(Eq. 5.3)}$$

Mean stress, σ_m

$$\sigma_m = \frac{(\sigma_{max} + \sigma_{min})}{2} \qquad \textbf{(Eq. 5.4)}$$

Stress ratio, R

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad \textbf{(Eq. 5.5)}$$

Note: σ_{min} and σ_{max} in this sense are the min and max stresses in a loading cycle, not necessarily including the effects of stress concentration.

Fatigue Analysis Process

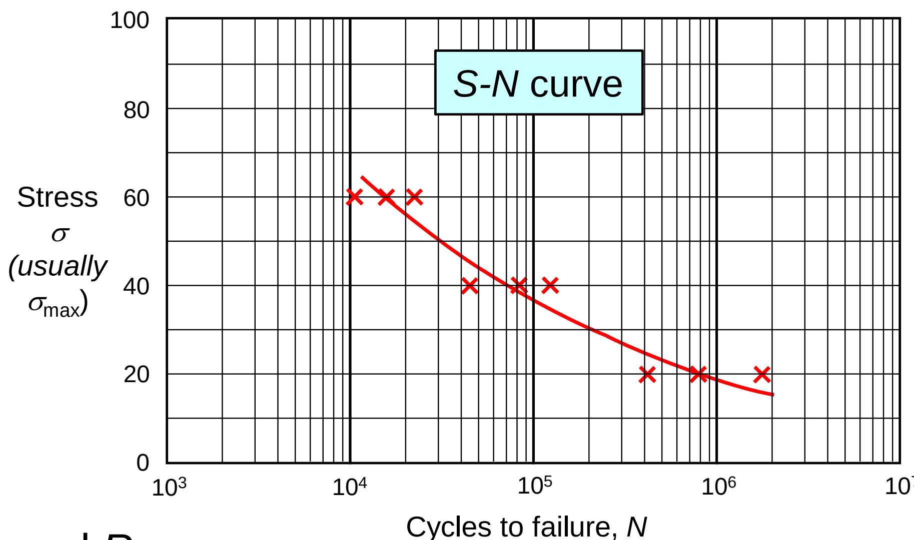
1. Define or assemble fatigue analysis criteria.
   – Life factor and fatigue analysis factor, discussed later in this section
2. Generate a ***loading spectrum*** (a.k.a. ***load spectrum***):
   – expected numbers of loading cycles at different alternating stress levels and associated mean stress
3. For a given part, identify regions of potential fatigue failure.
   – High stress, sharp filet radii, sudden changes in cross section
4. Identify the applicable stress concentration factor.
5. Identify a suitable method of fatigue analysis.
6. Assemble fatigue data that are consistent with the selected method and that (preferably) apply to the applicable stress concentration factor.
7. Assess fatigue life.

Again: improve design if warranted!

Iteration is often necessary, with the first iteration based on a simple, conservative loading spectrum. Refining the loading spectrum to reduce conservatism can be very time consuming.

Quantifying Fatigue Life by Test

- Test a coupon of a certain material and geometry with cyclic load.
- Continue the test until the coupon fails (normally defined as full fracture, but can be defined as formation of a crack of a certain size).
  - Same peak stress and stress ratio for each cycle
- Plot number of cycles to failure (N) vs. peak stress (S or σ) or strain (ε).
  - Semi-log plot
- Test multiple coupons at different stress (or strain levels), and plot results.
- Draw a best-fit curve through the scattered data.

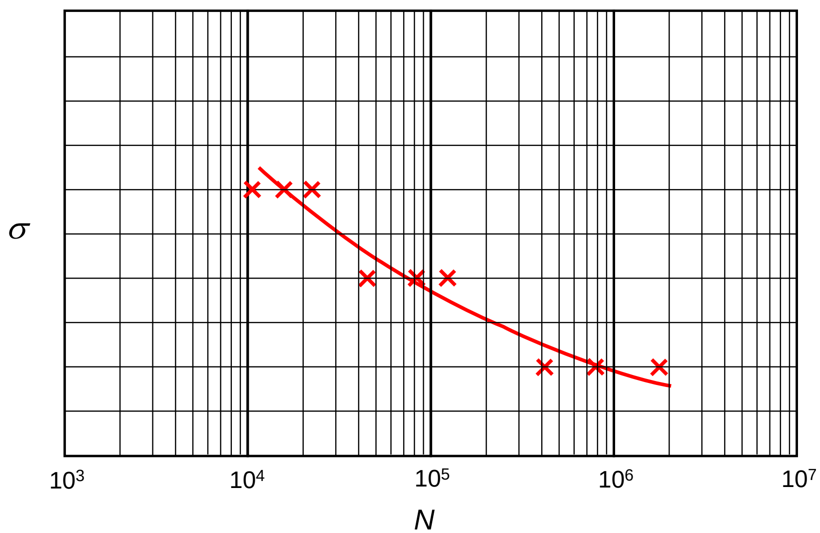


Repeat this process for different K_t and R.

Stress concentration factor

Stress ratio

The *S-N* Curve is the Most Common Format for Fatigue Data



S-N curves are used to assess fatigue life with the ***stress-life*** approach when there is little to no yielding and many cycles are needed to cause failure (***high-cycle fatigue***).

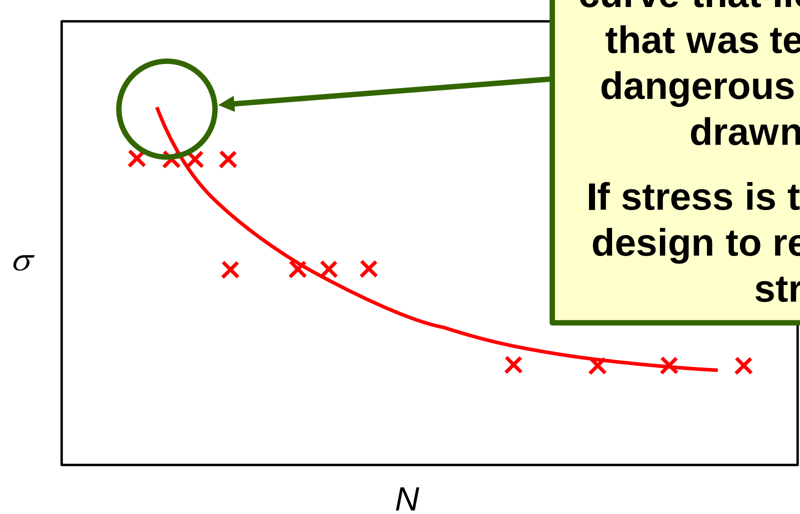
The ***strain-life*** approach, with ε-*N* curves (ε being strain), is more appropriate for ***low-cycle fatigue***, with high-strain (yielding) conditions.

Some practical advice:

When to Use the Strain-Life Approach

In most practical situations for flight structures, we use the stress-life approach.

We should use the strain-life approach if the linearly calculated peak stress exceeds the limits of the available *S-N* data.



Don't use any portion of the plotted *S-N* curve that lies above the highest stress that was tested. This is a potentially dangerous extrapolation—even if the drawn curve extends there.

If stress is that high, either change the design to reduce the stress or use the strain-life approach.

Example of Not Trusting the Presented Data

MIL-HDBK-5J
31 January 2003

Same figure appears in MMPDS-10

This point on the R = 0 curve can't be right; it's above the tensile ultimate strength (45 ksi) for the tested specimens!

The actual curve must look something like this, but the tests didn't go up high enough in stress to find it.

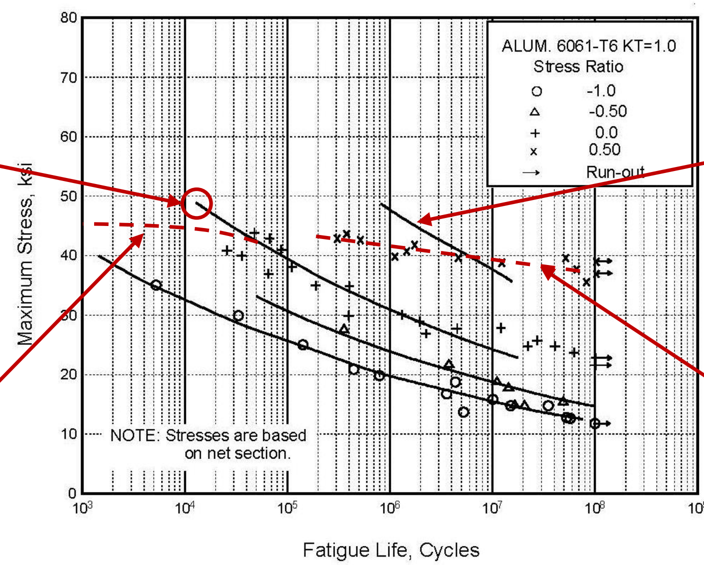


Figure 3.6.2.2.8. Best-fit S/N curves for unnotched 6061-T6 aluminum alloy, various wrought products, longitudinal direction.

Here's another problem with this data: The curve for R = 0.50 clearly doesn't match the the data!

The curve should look more like this.

Correlative Information for Figure 3.6.2.2.8

Product Form: Drawn rod, 0.75 inch diameter
Rolled bar, 1 x 7.5 inch

Properties:

TUS, ksi	TYS, ksi	Temp., °F
45	40	RT

Test Parameters:
Loading - Axial
Frequency - 2000 cpm
Temperature - RT
Environment - Air

Use caution and engineering judgment when using fatigue data, regardless of the source.

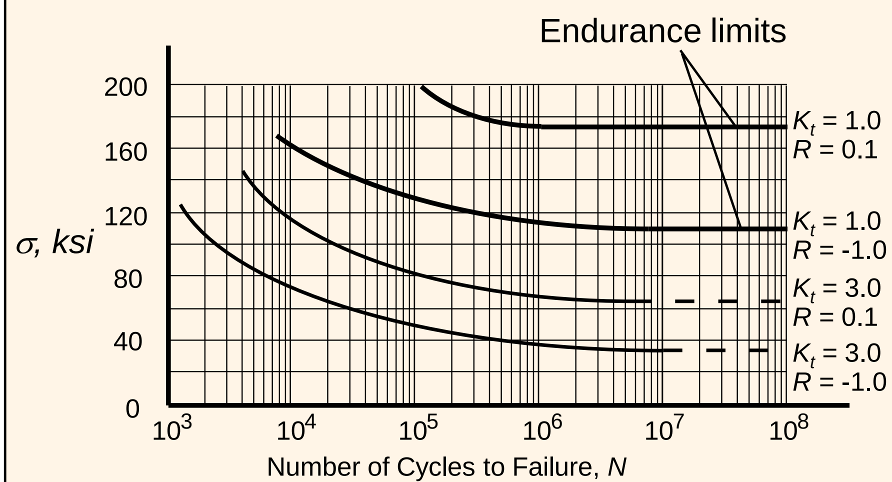
Endurance Limit and Fatigue Limit

Endurance limit—the maximum stress that can be applied at an infinite number of cycles without failure

Fatigue limit (a.k.a. ***fatigue strength***)—the maximum stress that can be applied at an extremely high number of cycles without failure

- Used in place of endurance limit for materials that don't have a true endurance limit, such as aluminum alloys.
- Usually defined for the number of cycles at which testing is stopped (often 10^7)

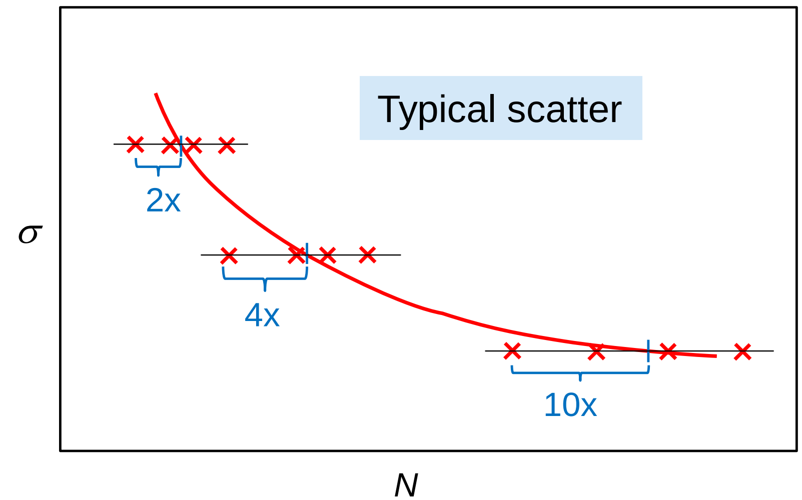
Four curves for the same material, with different combinations of K_t and R



K_t is the stress concentration factor
R is the stress ratio for the cycle

Accounting for Variation (Scatter) in Fatigue Life

- As noted in Sec. 4, allowable stress for strength analysis is statistically based to ensure there's low likelihood of actual strength being less than the allowable.
- Fatigue testing is time consuming, hence expensive, and we seldom have enough data to derive statistically based *S-N* curves.
- Instead, we draw an *S-N* curve that best fits the available test data and account for scatter by multiplying the expected number of loading cycles by a ***life factor*** (a.k.a., ***scatter factor***).
  - Life factor (LF) of 4 is somewhat standard in the space industry.
  - But, for high-cycle fatigue (typically > 10^7 cycles), LF of10 is more applicable and is used by some programs.
  - And some programs use LF of 2 for crack-growth (fracture mechanics) analysis.

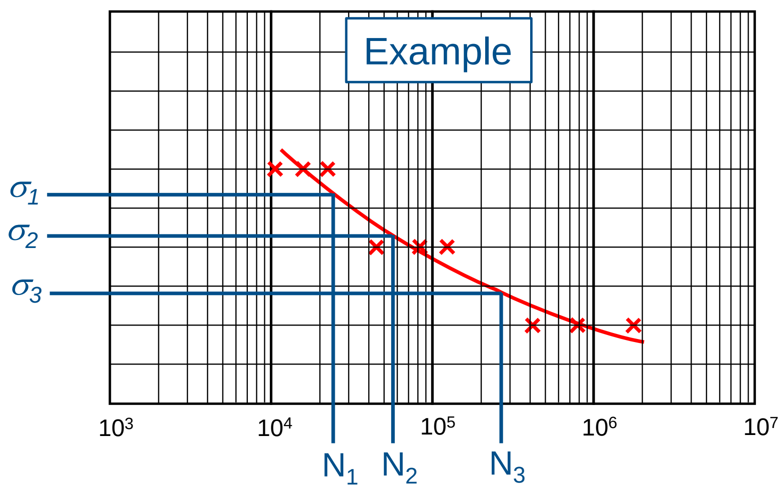


Palmgren-Miner Method for Stress-Life Analysis

The simplest and most commonly used method of assessing fatigue life is the ***Palmgren-Miner linear damage rule*** (a.k.a. ***Miner's Rule***):

Cumulative damage, $$D = \sum_{i=1}^{j} \frac{n_i}{N_i}$$ **(Eq. 5.6)** Criterion: $D \le 1$

n_i = Number of loading cycles at stress level σ_i
N_i = Number of cycles to failure at stress level σ_i
j = Number of different stress levels

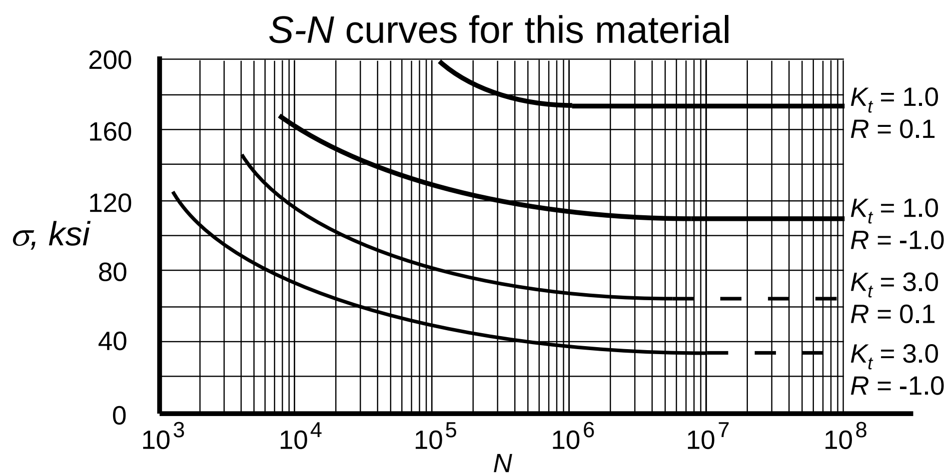


This method is used also with the strain-life approach, with N_i corresponding to strain rather than stress.

Example Problem 5-a: Assessing Fatigue Life

A part is expected to see fully reversed (R = -1) cyclic von Mises stress in the longitudinal grain direction. The part has a stress concentration factor, K_t, of 3.0.

The loading spectrum for the part's entire life cycle (component-level testing, vehicle-level testing, ground transportation, launch, on-orbit operations, etc.) is defined below.



Max Stress, σ (ksi)	Expected Cycles
70	1000
60	2500
50	8000
40	15000

Not including the stress concentration

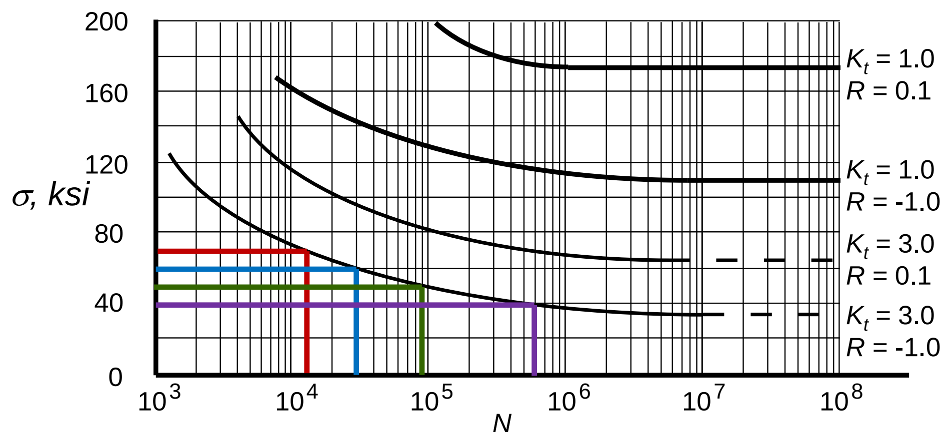
Note: The engineer assessing fatigue life often must derive the loading spectrum. (See later discussion in this section.)

Problem statement: Assess fatigue life using Miner's Rule and the applicable *S-N* curve from above, with a life factor of 4.

Example Problem 5-a: Solution

For fully reversed loading, *R* = -1.0.

With K_t = 3, we use the lowest curve shown at right.



4 times the expected cycles (life factor = 4)

Max stress, σ_{max} (ksi)	Expected cycles	Design cycles, *n*	No. cycles to failure, *N*	Damage, *n* / *N*
70	1000	4000	13000	0.31
60	2500	10000	30000	0.33
50	8000	32000	90000	0.36
40	15000	60000	500000	0.12
Sum				1.12

The sum exceeds 1, so this part does not pass the fatigue-analysis criteria.

This part may pass fatigue criteria if we can reduce its stress concentration.

See upcoming discussion of the fatigue notch factor.

Fatigue Analysis Factor

For human-rated spaceflight hardware, NASA requires that fatigue analysis use limit stress or strain multiplied by a 1.15 ***fatigue analysis factor (FAF)*** when using typical (best fit) fatigue properties. (Refs. 14 and 15)

- Although this factor is similar to a factor of safety, note that it's applied to stress rather than to load.
- Rationale provided: "intended to provide margin to account for material fatigue curve data scatter"
  - In addition to use of a life factor (LF) of 4 for low-cycle and medium-cycle fatigue and LF of 10 for high-cycle fatigue

Based on this rationale, it has taken me a while to accept use of a 1.15 FAF.

- Two factors intended for the same purpose!

But, after additional research, I believe the FAF makes sense for ensuring high reliability, given the uncertainty associated with fatigue analysis, unless the analysis is otherwise sufficiently conservative.

The next chart shows one example of justification. Other reasons for a FAF will become apparent later in this section.

Example of When a FAF Makes Sense

From Ref. 5b, MIL-HDBK-5J:

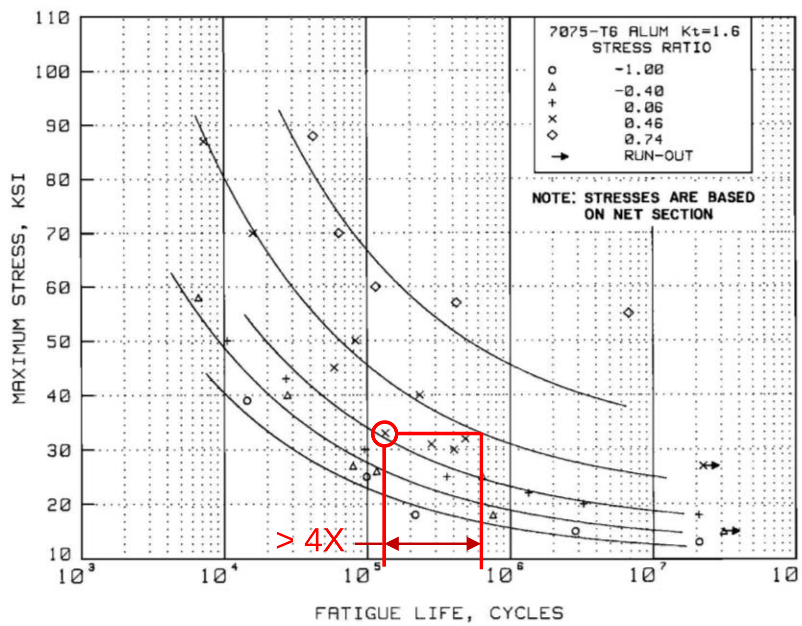


Figure 3.7.6.1.8(b). Best-fit S/N curve for notched, K_t = 1.6, 7075-T6 aluminum alloy rolled bar, longitudinal direction.

Test specimens:

Tensile ultimate strength

Product Form: 1.125-inch-diam. rolled bar

Properties:	TUS, ksi	TYS, ksi	Temp., °F
	99.2	—	RT

Specimen Details: Notched, K_t = 1.6
Notch-root-radius = 0.100
0.400-inch Test section diameter (Net)
0.450-inch gross diameter
60° groove

All of this data is from test specimens made of an unusually strong lot of material. The A-basis ultimate strength, F_{tu}, for this material is 77 ksi.

As ultimate strength drops, so do the expected *S-N* curves.

A life factor (LF) of 4 doesn't quite cover the circled test result. LF of 4 certainly wouldn't be high enough if the *TUS* of our material is only 77 ksi.

Without other compensating conservatism, it would be unwise to use data such as this without a FAF (or without scaling the *S-N* data down to F_{tu}, an optional approach).

Other Factors Affecting Fatigue Life

Most available *S-N* curves are from testing of smooth specimens.

The following characteristics of actual structural parts may significantly reduce fatigue life:

- Surface roughness
  - Typically no impact to fatigue life for aluminum alloys if finish < 138 RMS
- Surface scratches
- Anodizing or plating
- Residual stresses from fabrication and processing
- Environment

For guidance, see Refs. 16 – 19.

Accounting for Stress Concentration Factor, K_t , in Fatigue Analysis

Options:

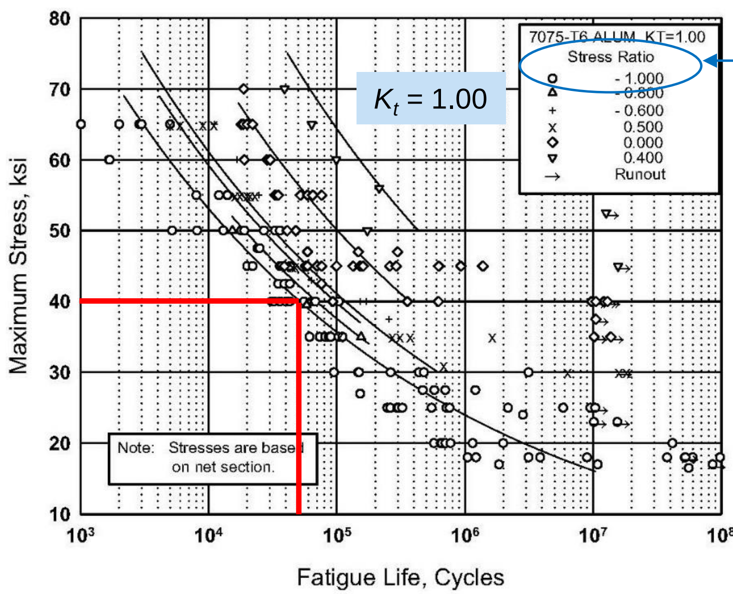
1. Multiply max. nominal stress by K_t and use an *S*-*N* curve that applies for $K_t = 1$
2. Use max. nominal stress without K_t and an *S*-*N* curve that applies for the appropriate K_t

Example, using the *S*-*N* plots from MIL-HDBK-5J for 7075-T6 sheet:

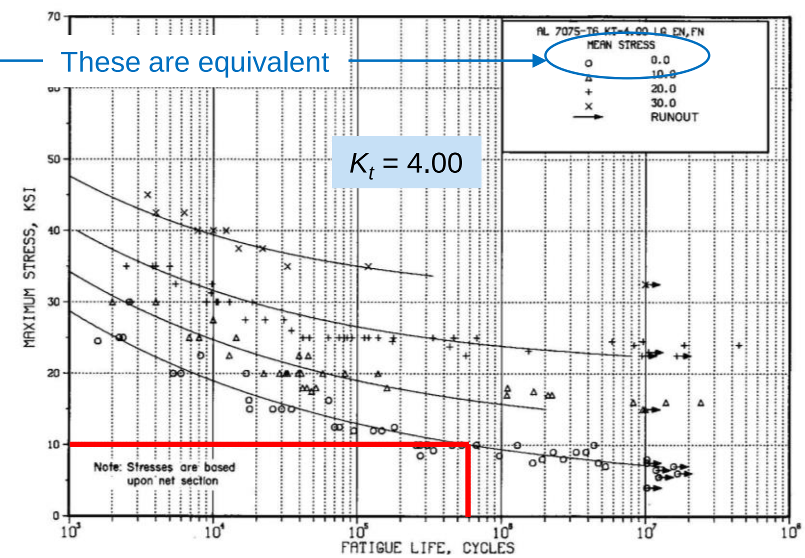
Nominal stress = 10 ksi, $K_t = 4.0$, and $R = -1$

1. At stress = 4(10) = 40 ksi, ***N* = 50,000 cycles**

2. At stress = 10 ksi, ***N* = 600,000 cycles**



These are equivalent



Accounting for K_t, continued

As shown in the example on the preceding page, multiplying stress by K_t typically overpredicts the reduction in fatigue life caused by concentrated stress.

It's better to use fatigue data from testing of specimens with the appropriate K_t.

But what do we do when we can't find data for the K_t that applies to our design—and we can't tolerate the conservatism associated with multiplying nominal stress by K_t?

Let's define two other material properties that may help with this problem …

Fatigue Notch Factor, K_f

The ***fatigue notch factor, K_f***, is more representative of the actual effect on fatigue life of stress concentrations than K_t because K_f is based on test data.

- ***Notch*** is a generic term for a geometric feature that results in concentrated stress.

For a given $K_t > 1$,

$$K_f = \frac{S_{e-un}}{S_{e-n}} \quad \textbf{(Eq. 5.7)}$$

S_{e-un}: Fatigue limit for un-notched specimen (K_t = 1)

S_{e-n}: Fatigue limit for notched specimen (K_t > 1)

The premise here is that the following two methods of analysis would result in the same fatigue damage:

1. Multiply max. nominal stress by K_f and use an S-N curve that applies for K_t = 1
2. Use max. nominal stress without multiplying by K_t or K_f and an S-N curve that applies for the appropriate K_t

However, these methods agree only if the number of cycles being assessed is the same as the number of cycles corresponding to the fatigue limit used to derive K_f. *

When assessing significantly fewer loading cycles, such as for single-mission spaceflight hardware, K_f should be derived from fatigue limits that apply to a lower, more applicable number of cycles.

*It's also hard to get agreement with these methods because of scatter and limited test data.

Notch Sensitivity Factor, *q*

Derived from K_f, the ***notch sensitivity factor, q***, is a measure of the degree to which the full theoretical effect of the stress concentration is realized by the reduction in a member's fatigue limit.

fatigue notch factor

$$q = \frac{K_f - 1}{K_t - 1} \quad \textbf{(Eq. 5.8)}$$

stress concentration factor

If $q = 0$, then $K_f = 1$, and the material has no sensitivity to notches.

If $q = 1$, then $K_f = K_t$, and the material has full notch sensitivity.

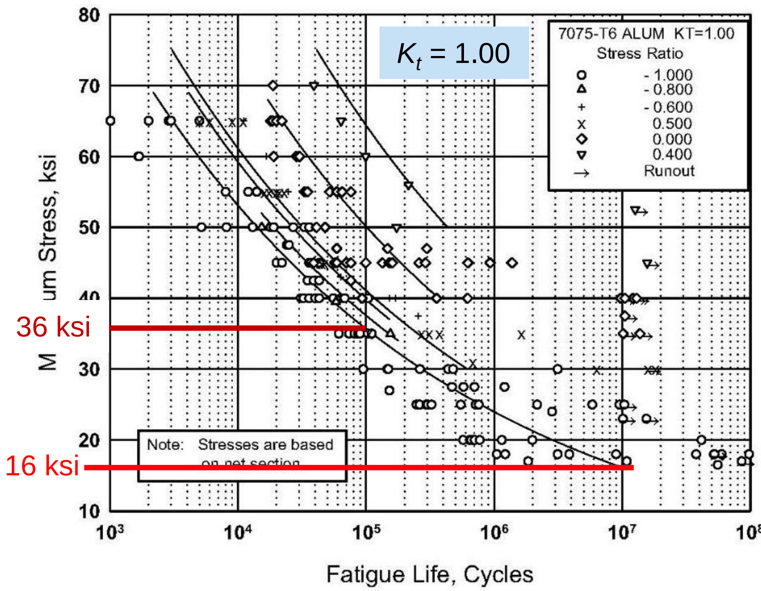
We can use q calculated for one value of K_t to estimate K_f for another K_t :

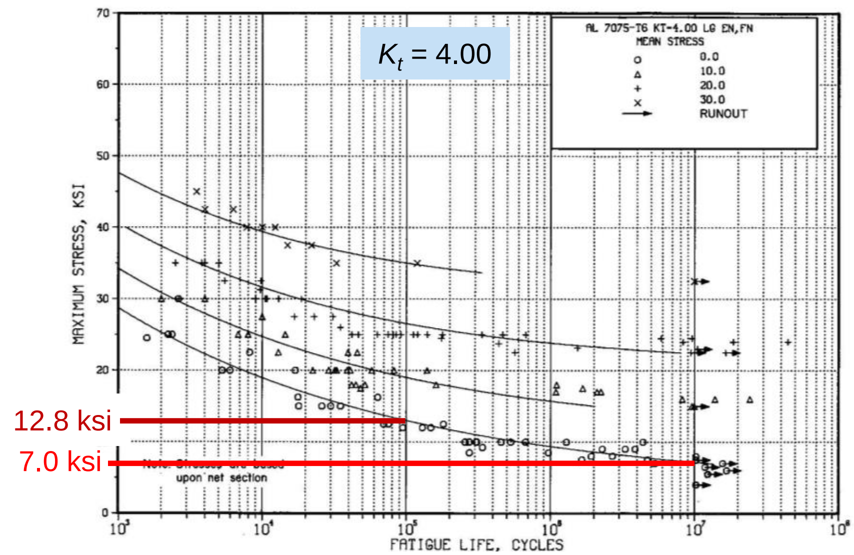
$$K_f \approx q\left(K_t - 1\right) + 1 \quad \textbf{(Eq. 5.9)}$$

Caution: This approach is approximate and can be unconservative.

It's prudent to use a fatigue analysis factor unless there is other compensating conservatism.

Example of Deriving K_f and q from Test Data





Using fatigue limit defined at 10^7 cycles, from Eqs. 5.7 and 5.8,

$$K_f = \frac{16}{7.0} = 2.3 \qquad q = \frac{2.3-1}{4.0-1} = 0.43$$

Using fatigue limit defined at 10^5 cycles,

$$K_f = \frac{36}{12.8} = 2.8 \qquad q = \frac{2.8-1}{4.0-1} = 0.60$$

Note the difference, based on number of cycles.

Derive these properties based on an appropriate number of cycles for your application.

The Goodman Method

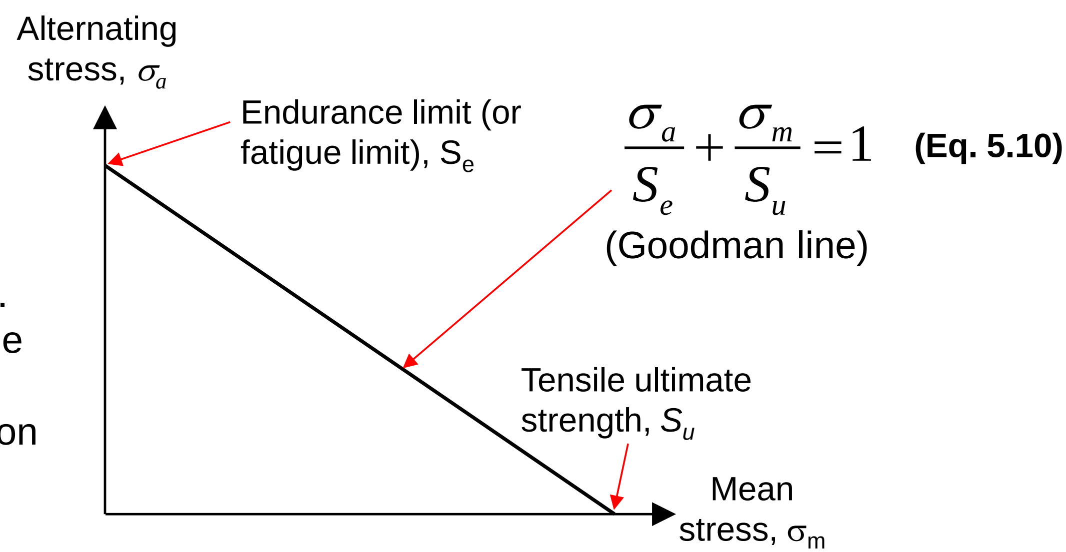
S-*N* data are often available only for fully reversed stress (*R* = -1).

The ***Goodman Method*** lets us use this *S*-*N* data for other *R* values.

Goodman criterion:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} < 1$$

This criterion is conservative. The Gerber parabola, with the same end points, is less conservative but not always on the safe side of test data.



If you have as-tested ultimate strength for the production lot of material used for the structure you are assessing, I would use the minimum value for the specimens tested for S_u. Otherwise, I'd use F_{tu}, the allowable stress used for strength analysis.

Equivalent Alternating Stress

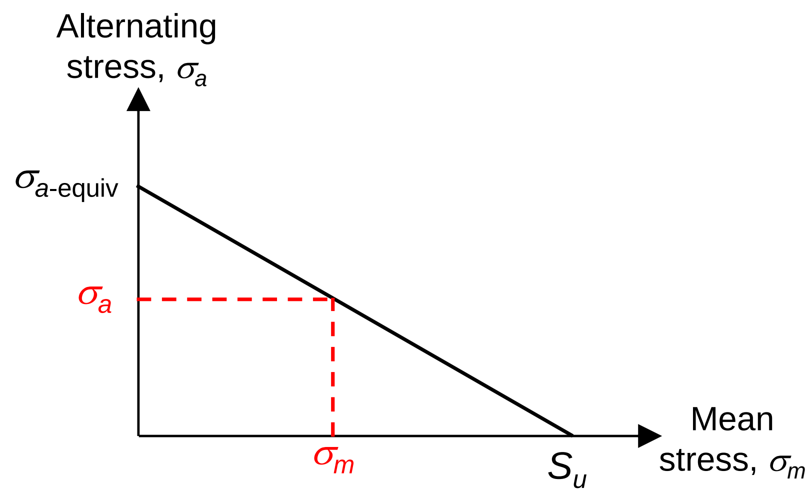
At any combination of mean and alternating stress (or any peak stress and stress ratio), we can use the Goodman method to estimate an ***equivalent alternating stress, $\sigma_{a\text{-equiv}}$***

Based on linear extrapolation from the line segment connecting S_u to the plotted σ_a-σ_m point,

$$\frac{\sigma_{a\text{-equiv}}}{S_u} = \frac{\sigma_a}{S_u - \sigma_m}$$

$$\sigma_{a-equiv} = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{S_u}\right)}$$

(Eq. 5.11)



We can use $\sigma_{a\text{-equiv}}$ with an *S-N* curve for *R* = -1 to approximate the fatigue damage expected for any other *R* value.

However, as we'll see shortly, this method can be unconservative (unsafe).

Example Problem 5-b:
Calculating Fatigue Damage with Equivalent Alternating Stress

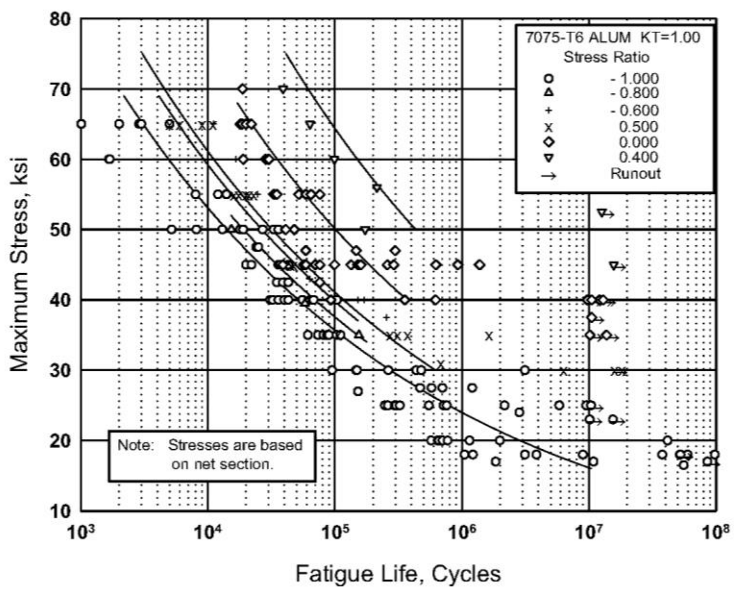


Figure 3.7.6.1.8(d). Best-fit S/N curves for unnotched 7075-T6 aluminum alloy sheet, longitudinal direction.

(Ref. 5b, MIL-HDBK-5J)

Tensile ultimate strength (TUS) of material tested: 82 ksi

A part made of 7075-T6 aluminum sheet has the following loading spectrum, with R = 0 and K_t = 1.

Max stress, ksi	Expected cycles	Design cycles (4X)
60.0	500	2000
50.0	5000	20000
40.0	20000	80000

Problem statement:

Calculate fatigue damage with Miner's rule two ways, and compare results:

1. Using the S-N curve for R = 0
2. Using equivalent alternating stress and the S-N curve for R = -1

continued

Solution to Example Problem 5-b

Damage calculated from S-N curve for R = 0

Max stress, ksi	Expected cycles	Design cycles (4X)	*N*	*D*
60.0	500	2000	35000	0.057
50.0	5000	20000	100000	0.200
40.0	20000	80000	370000	0.216
Total damage				**0.473**

$$\sigma_{\min} = R\sigma_{\max} = 0$$

$$\sigma_a = \frac{(\sigma_{\max} - \sigma_{\min})}{2}$$

$$\sigma_m = \frac{(\sigma_{\max} + \sigma_{\min})}{2}$$

$$\sigma_{a-equiv} = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{S_u}\right)}$$

S_u = 82 ksi (given)

Very close match in this case!

Damage calculated using equiv. alternating stress and S-N curve for R = -1

Max stress, ksi	Expected cycles	Design cycles (4X)	σ_a	σ_m	$\sigma_{a\text{-equiv}}$	*N*	*D*
60.0	500	2000	30.0	30	47.31	20000	0.100
50.0	5000	20000	25.0	25	35.96	90000	0.222
40.0	20000	80000	20.0	20	26.45	500000	0.160
Total damage							**0.482**

Before we get confident in this approach though, let's try again with K_t = 2 …

Class Problem 5-1:
Calculating Fatigue Damage with Equivalent Alternating Stress

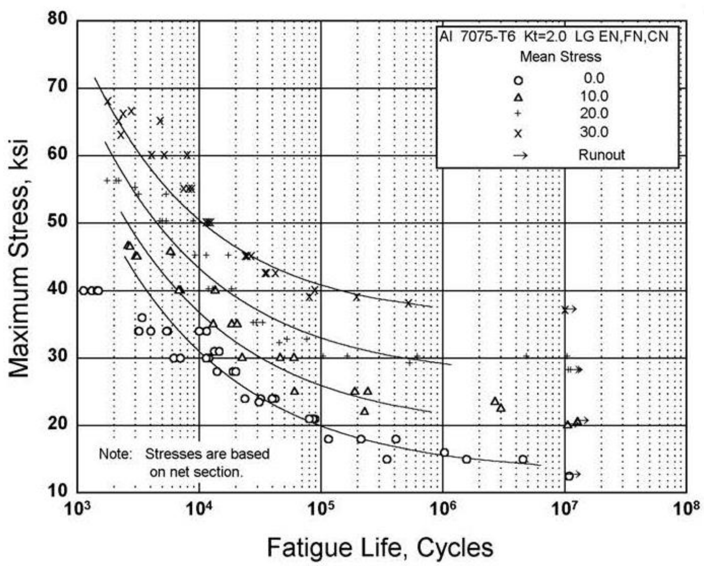


Figure 3.7.6.1.8(f). Best-fit S/N curves for notched, K_t = 2.0, 7075-T6 aluminum alloy sheet, longitudinal direction.

(Ref. 5b, MIL-HDBK-5J)

A part made of 7075-T6 aluminum sheet has the following loading spectrum, with a mean nominal stress of 20 ksi and K_t = 2.

Max stress, ksi	Expected cycles	Design cycles (4X)
45.0	500	2000
35.0	5000	20000
30.0	50000	200000

Tensile ultimate strength (TUS) of material tested: 88 ksi

Problem statement:

Calculate fatigue damage with Miner's rule two ways, and compare results:

1. Using the *S-N* curve for mean stress = 20 ksi
2. Using equivalent alternating stress and the *S-N* curve for mean stress = 0

Conclusions Regarding the Use of Equivalent Alternating Stress

Summary of cases studied:

- For 7075-T6 and assumed loading spectra …
  - Good match with test data for R = 0 and K_t = 1
  - Unconservative for mean stress of 20 ksi and K_t = 2
- For 6061-T6, R = 0, K_t = 1, and assumed loading spectra (not documented herein) …
  - Excessively conservative: damage ratio > 9 with equivalent alternating stress vs. 0.80 from the R = 0 *S-N* curve

Conclusions and a recommendation:

Using equivalent alternating stress as described, when only data for *R* = -1 can be found, is more accurate than using the *R* = -1 data, but that's not saying much! It can be very inaccurate.

Unless you are convinced this approach is conservative, use an appropriate fatigue analysis factor (FAF).

What If the Material Has an Undetected Crack-like Defect?

If a material has a significant flaw or crack, it will not provide the life that we would expect using published S-N data.

- It's highly unlikely that any of the specimens tested to get the fatigue data had significant flaws or cracks.
- In other words, the scatter found when testing a small number of specimens typically does not encompass the reduced strength or shorter life of a material that starts with an undetected crack-like defect.

Why would the material have a significant crack?

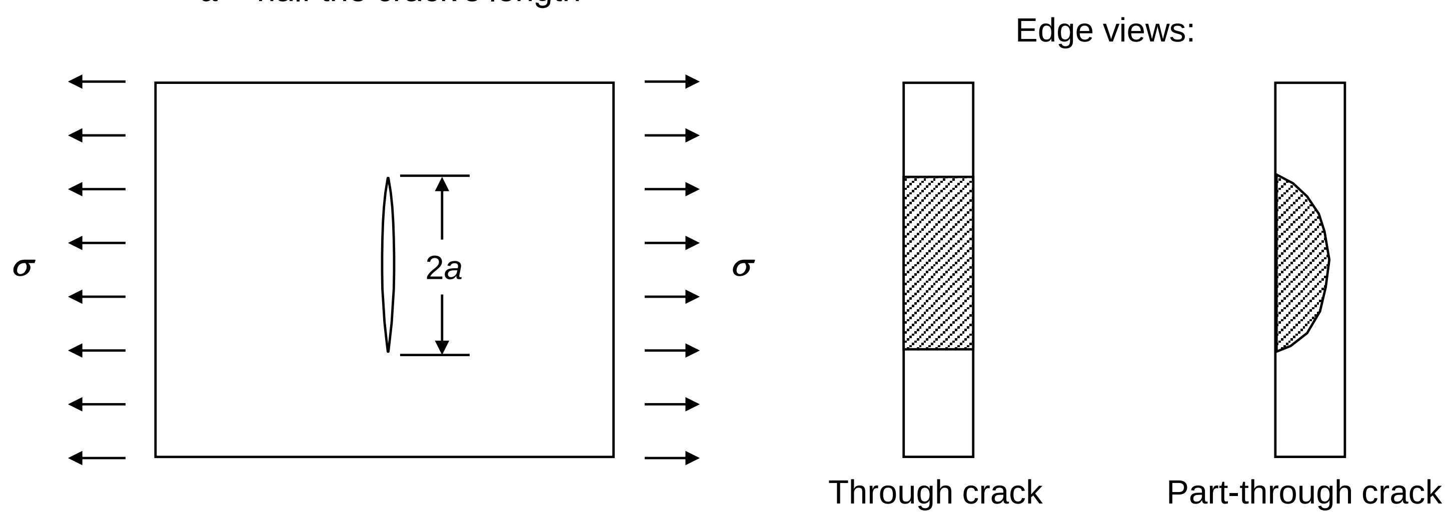
- Crack formed by fatigue from previous cycles of loading (vibration testing, transportation, previous missions)
- Crack or flaw formed by material processing, forming, or welding
- Crack caused by an incident during ground handling (e.g., dropped hardware)

Linear-Elastic Fracture Mechanics (LEFM) Theory

Crack growth depends on the material, the number of loading cycles, the range of stress intensity at the crack tip, and the state of stress relative to the crack orientation.

Stress intensity, $K_I = M\sigma\sqrt{\pi a}$ **(Eq. 5.12)** units: $ksi \cdot \sqrt{in}$ or $MPa \cdot \sqrt{m}$

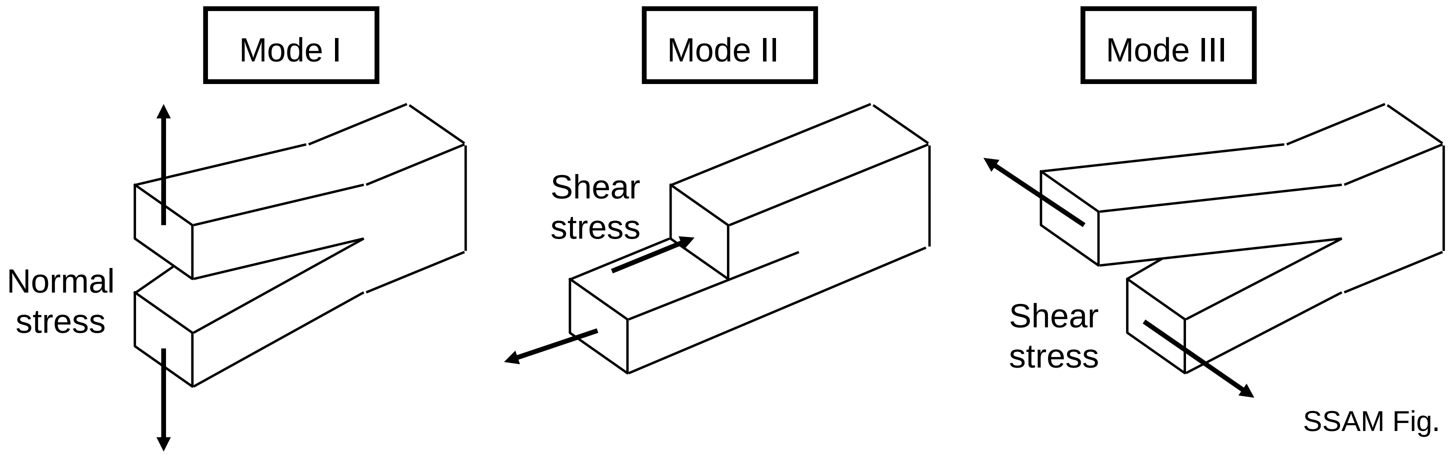
where M = coefficient to account for type of stress and geometry of crack and part
σ = stress on the full section if there were no crack
a = half the crack's length



Range of stress intensity, $\Delta K = K_{max} - K_{min}$

(peak stress intensity minus minimum stress intensity for a loading cycle)

Modes of Crack Growth



SSAM Fig. 9.2

- Most crack-growth data and fracture mechanics analyses apply to Mode I, which is most critical (fastest crack growth).
- The "*I*" subscript in "K_I" stands for "Mode I".
- Crack growth is rapid and unstable when stress intensity reaches the material's ***fracture toughness***, K_{Ic} .
- The fracture toughness increases as part thickness decreases; the thickness-adjusted fracture toughness is called the ***critical stress intensity***, K_c .

Three Stages of Crack Growth

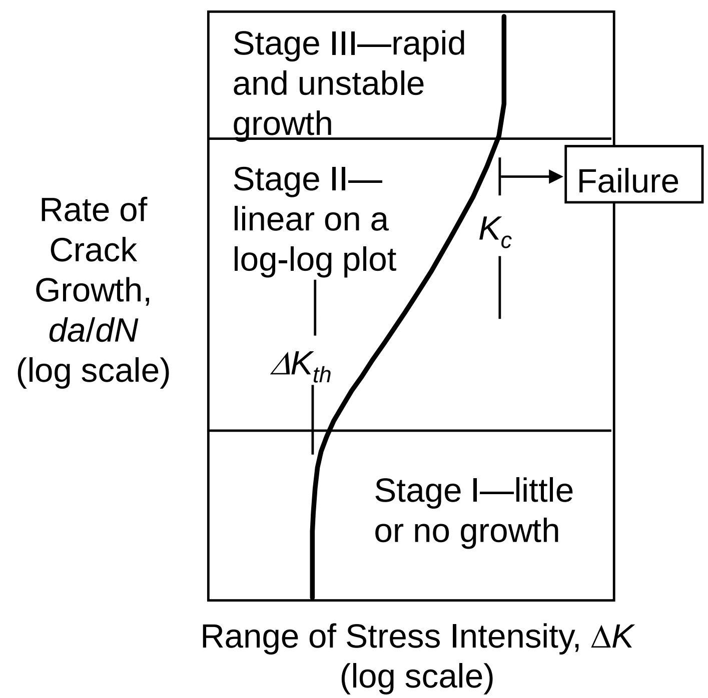
- Most* crack-growth analysis applies to Stage II crack growth.
- Rate of Stage II crack growth (***Paris equation***):

$$da/dN = C(\Delta K)^n \quad \textbf{(Eq. 5.13)}$$

(change in length per cycle)

C and *n* are material-dependent coefficients used to match test data

- As the crack grows, the stress intensity increases, driving the need for iterative analysis.



ΔK_{th} = ***threshold stress intensity range***

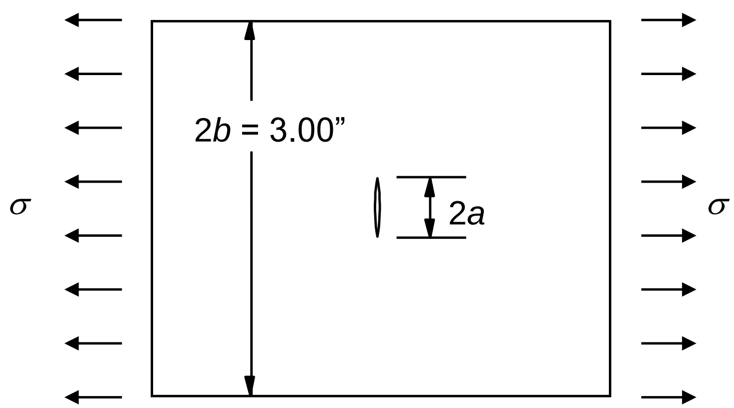
*The NASGRO software, introduced in a later page, can also do crack-growth analysis in stages I and III, with more complex equations for crack growth.

Limitations of LEFM Theory

- Linear-elastic fracture mechanics theory applies when there is no plastic deformation or when plastic deformation occurs only at the crack tip.
- Elastic-plastic fracture mechanics theory is not nearly as well developed; crack-growth rates are not available for most materials.
- For safe use of LEFM theory and data, a common approach is to keep the net stress (based on the gross area minus the crack area) below the material's allowable yield stress.
  - Exceeding the allowable yield stress is considered failure.

Example Problem 5-c: Crack-growth Analysis

Problem statement: The plate below has a through crack of initial length 2*a* = 0.150” and a cyclic limit stress of ±38.0 ksi. Determine whether the plate has enough life to withstand 100 cycles of load, using a life factor of 4 (analyze for 400 cycles).



Material: Aluminum alloy with the following properties:

Fracture toughness, $K_{Ic} = 33 \text{ ksi}\sqrt{\text{in}}$

In this example, we’ll assume no thickness adjustment is warranted for fracture toughness.

Crack-growth rate,

$$da/dN = 2.0 \times 10^{-8} (\Delta K)^{2.5} \text{ in/cycle}$$

Allowable tensile yield stress, $F_{ty} = 55 \text{ ksi}$

For a through (Mode I) crack in a plate of finite width, the stress intensity is

$$K_I = M\sigma\sqrt{\pi a} \qquad \text{where} \qquad M = \sqrt{\frac{2b}{\pi a}\tan\frac{\pi a}{2b}} \qquad \text{(Ref. 20)}$$

continued

Example Problem 5-c: Solution

Calculate crack growth iteratively:

Units: in, kip

Cycle	Crack half-length, a	Net Stress, σ_n	Stress Intensity, K_I	Stress-Intensity Range, ΔK_I	Crack Growth, Δa
1	0.075000	40.00	18.46	36.93	0.000166
2	0.075166	40.00	18.48	36.97	0.000166
3	0.075332	40.01	18.51	37.01	0.000167
4	0.075499	40.01	18.53	37.05	0.000167
...	...	...	...	...	...
398	0.202781	43.94	30.56	61.12	0.000584
399	0.203366	43.96	30.61	61.21	0.000586
400	0.203952	43.98	30.65	61.30	0.000589

$$\left(\sigma_n = \left(\frac{3.00}{3.00 - 2a}\right)\sigma\right)$$

Results (end of life):

Crack size, $2a = 2(0.204) = 0.408$ in (0.0104 m)

Net stress, $\sigma_n = 44.0$ ksi (303 MPa) $<$ 55 ksi (380 MPa) allowed

Stress intensity, $K_I = 30.7\ \text{ksi}\sqrt{\text{in}} < 33\ \text{ksi}\sqrt{\text{in}}$ allowed

$(33.8\ \text{MPa}\sqrt{\text{m}} < 36\ \text{MPa}\sqrt{\text{m}})$

The plate is safe for use.

NASGRO—Automated Crack-Growth Analysis

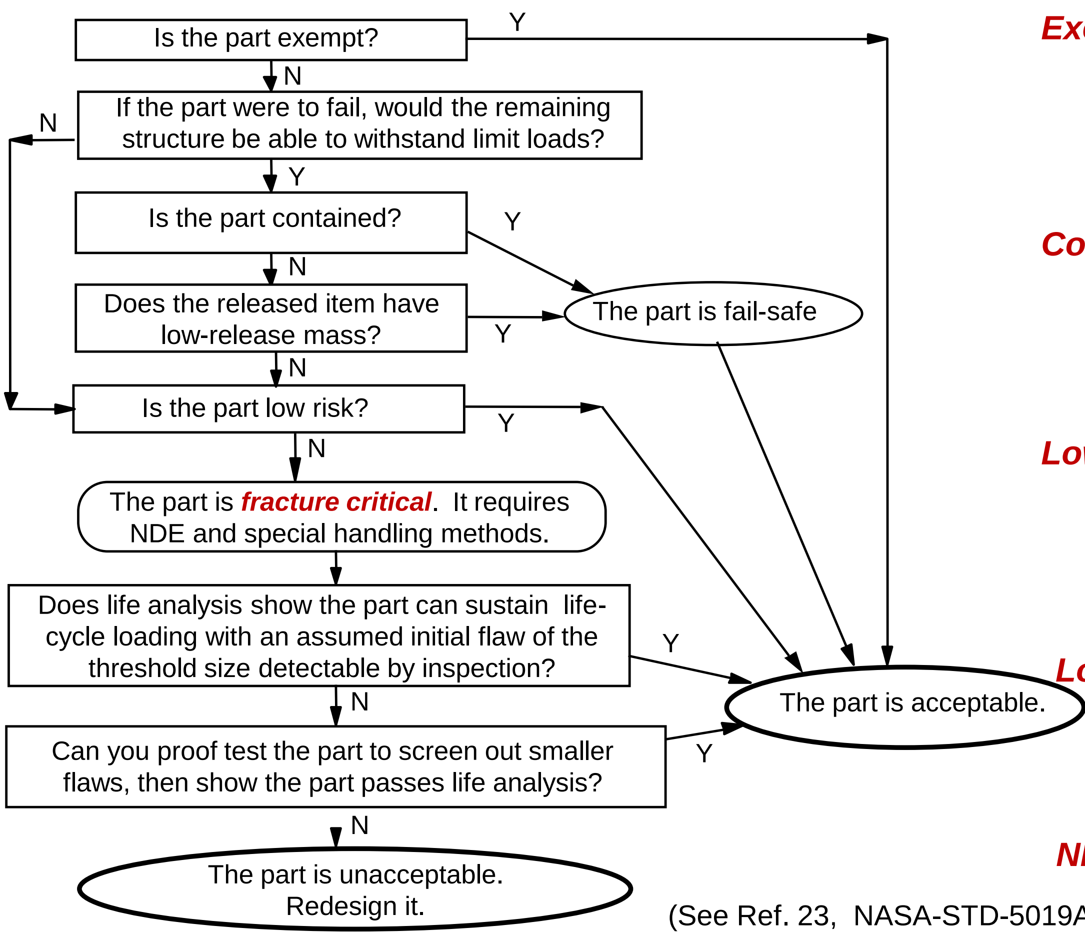
- Contains a large, regularly updated database of material fracture properties
- Can analyze many different crack types and sizes and part geometries
- Steps through complex loading spectrums
- Iteratively calculates crack growth
- Calculates stress intensity at each loading cycle and compares it to the critical stress intensity, K_c, and the stress-corrosion cracking threshold, K_{scc}
- Can compute critical crack size for given geometry, material, and stress
- Can define *da/dN* curves for new materials from the input of test data
- Available from Southwest Research Institute (free for NASA programs only)

Damage Tolerance and Fracture Control

When human safety is at stake, most aerospace programs establish a fracture control program to ensure damage tolerance.

- ***Damage tolerance***: "the attribute of a structure that permits it to retain its required residual strength for a period of unrepaired usage after the structure has sustained specific levels of fatigue, corrosion, accidental, and/or discrete source damage" (Ref. 22, MIL-STD-1530C)
- ***Fracture control***: "rigorous application of those branches of engineering, assurance management, manufacturing, and operations technology dealing with the analysis and prevention of crack propagation leading to catastrophic failure" (definition from the no-longer-active NASA-STD-5003, Fracture Control Requirements for Payloads Using the Space Shuttle; the current standard, NASA-STD-5019A (Ref. 23), does not provide a definition.)
  - Or, more simply, the process of protecting against a detected or undetected crack leading to catastrophic failure
- ***Damage tolerance analysis***: analysis performed to ensure damage tolerance
  - Crack-growth analysis is a form of damage tolerance analysis.

Flow Diagram Capturing the Gist of Fracture Control for Human-rated Spaceflight Hardware



(See Ref. 23, NASA-STD-5019A for detailed requirements)

Exempt parts: Thermal blankets
Electrical wires
Washers
Shims
Others

Contained: Upon failure, would not be released or cause release of something else from the structural assembly

Low release mass: Release would not be a catastrophic hazard (see Ref. 24, NASA-STD-5019 Sec. 4.1.1.1)

Low risk: Low stress, well-controlled material and process (see NASA-STD-5019 Sec. 4.1.1.12)

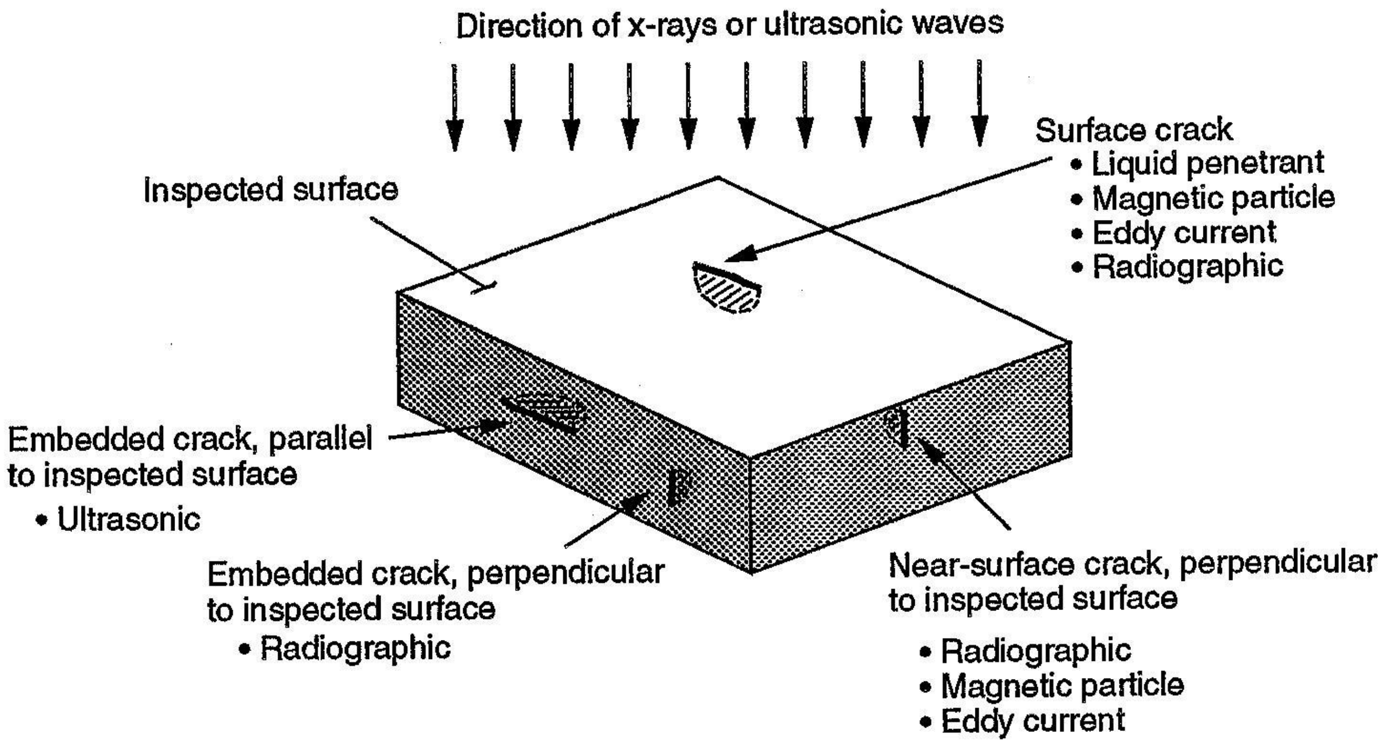
NDE: Non-destructive evaluation (inspection for cracks and other defects)

Standard Assumptions in Fracture Analysis

- A crack exists in the worst possible location and orientation.
  - In a region of high stress concentration
    - At the threads in a bolt
    - At a bolt hole
    - At a fillet radius or a notch
  - Plane of crack normal to direction of stress (Mode I)
- The crack size used in crack-growth analysis is the largest that has up to a 10% chance of escaping NDE (90% probability at 95% statistical confidence). (Ref. 24, NASA-STD-5009B)
  - As determined in certification tests for the inspection process
    - "Standard NDE" refers to processes that conform to common industrial standards
    - "Special NDE", which is more expensive, refers to processes that exceed common standards and thus have been demonstrated to reliably find smaller cracks
- For each evaluation, only one crack is assumed to be present.

Detectable Cracks for Various Methods of NDE

(Ref. 1, SSAM, Fig. 13.4)



See SSAM Table 13.3 for descriptions of these methods.

Generating a Loading Spectrum

To do fatigue or crack-growth analysis, we need to assemble a loading spectrum that envelops the damage potential from all life-cycle loading events at maximum expected statistical levels.

A loading spectrum might take the form of the simple examples shown previously in this section, with a given stress ratio *R* or mean stress σ_m:

Max stress	Cycles
10	53000
20	29000
30	4800

Each number of cycles at a given stress level is referred to as a ***bin***

Or it might be presented in the form of a matrix. Hypothetical example showing number of cycles for each combination of mean stress and alternating stress:

16 bins for this example

Mean stress	Alternating stress			
	10	20	30	40
0	47000	93000	25000	4300
10	54000	107000	35000	5100
20	32000	76000	19000	2500
30	18000	39000	7500	1200

This format is often used when steady-state, low-frequency, and high-frequency dynamic loads are combined.

Generating a Loading Spectrum for Random Vibration

Random vibration is the most common source of fatigue damage for flight hardware.

- During flight
- During ground testing
- And during ground or air transportation

Random vibration combines with any co-existing steady-state and low-frequency dynamic loads to cause fatigue damage.

As mass of an item decreases, random vibration during the above events typically causes a higher percentage of total stress in the materials, hence more of the fatigue damage.

When random vibration contributes significantly to fatigue damage, the stress analyst must derive a loading spectrum that envelops the maximum expected loading but that is not overly conservative.

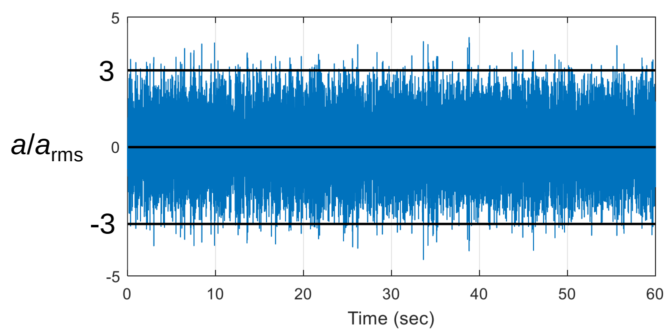
Doing so requires an understanding of some important statistics related to random vibration.

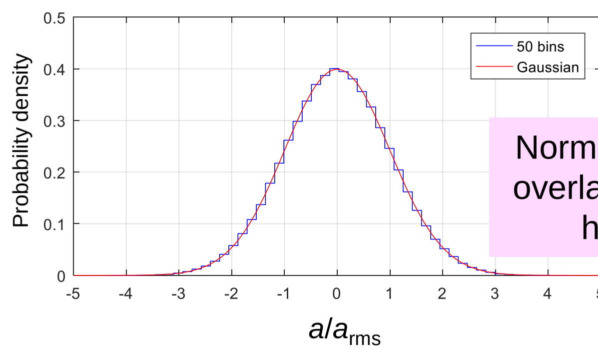
Probability Distributions for Random Vibration:

The Normal Distribution for Instantaneous Signal Level

For random vibration, if we consider acceleration, *a*, at any point in time as a random variable, that variable tends to follow a normal (Gaussian) distribution.

When we calculate a root-mean-square (RMS) acceleration, a_{rms}, that value is the standard deviation ("sigma") of the random acceleration at any point in time.



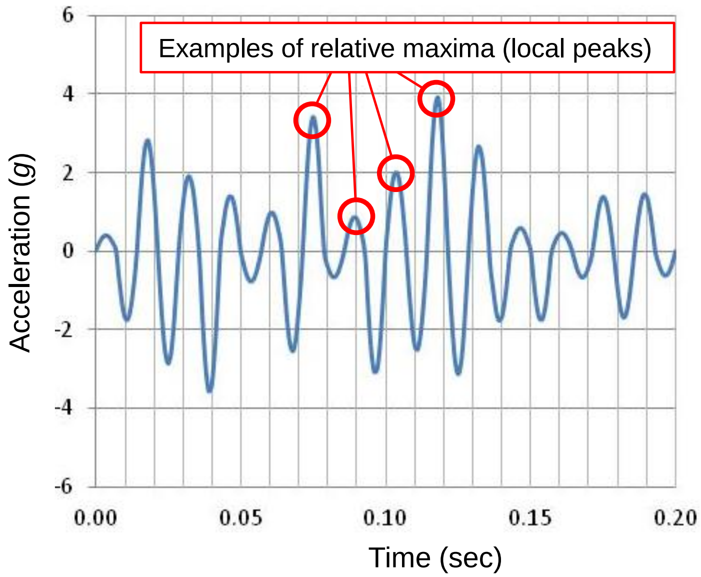


Normal distribution overlaid on top of a histogram

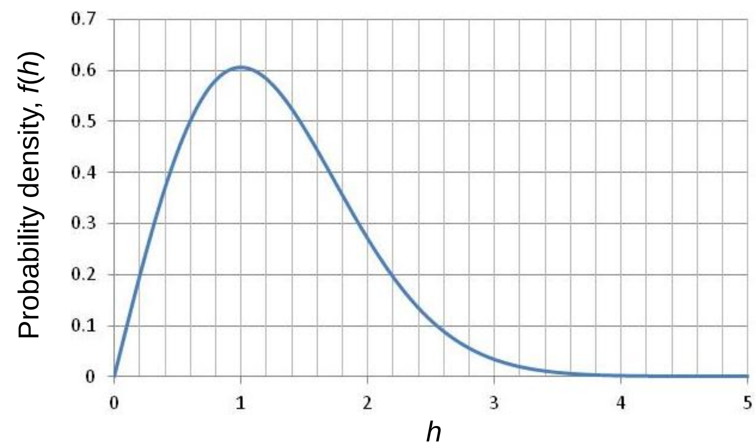
At any particular *instant*, there is a 99.74% (3 sigma) probability that the acceleration is contained within +/-3a_{rms}, but over a long duration, such as 60 seconds, there will be many absolute peaks of acceleration that exceed +/-3a_{rms}.

Probability Distributions for Random Vibration:
The Rayleigh Distribution for Relative Maxima

The ***Rayleigh distribution*** represents the distribution of relative maxima for ***narrow-band*** random vibration (single frequency, e.g., mass on a spring).



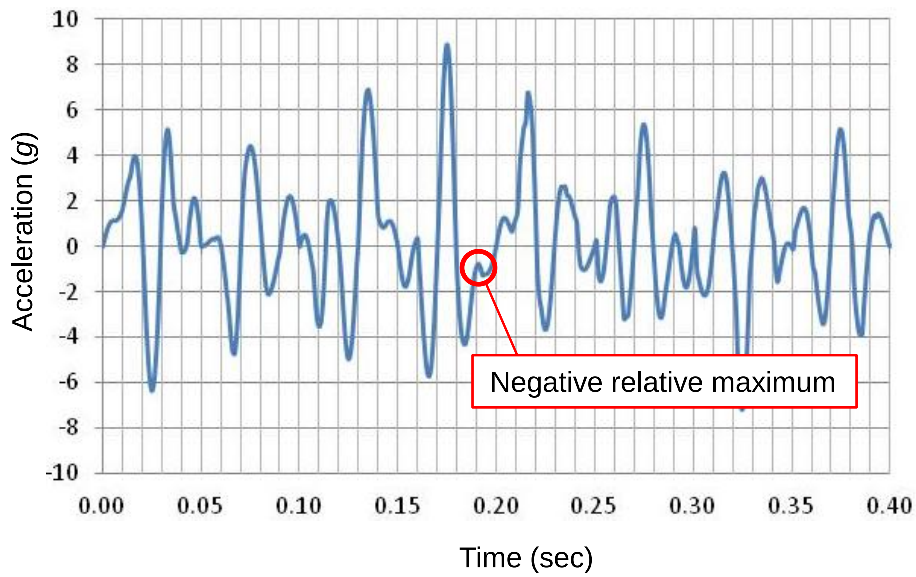
Let $h = \dfrac{a}{a_{\text{rms}}}$ = multiple of RMS level for acceleration a
= number of standard deviations



Rayleigh probability density function (PDF): $f\left(h\right) = h \cdot e^{\left(\frac{-h^2}{2}\right)}$ **(Eq. 5.14)**

Broad-Band Vibration

For random vibration that includes multiple frequencies, some of the relative maxima are negative.

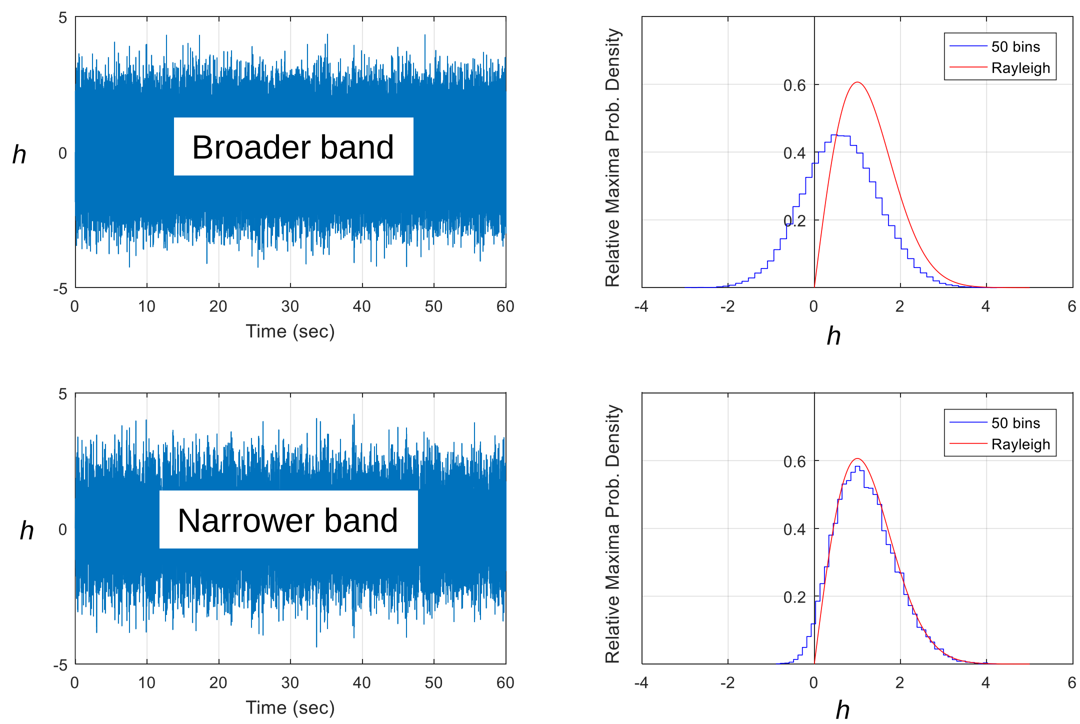


This time history includes only 3 frequencies of vibration.

The term ***broad band*** means a wide frequency range of content, which is common for random vibration.

Rayleigh Distribution for Broad-Band Random Vibration

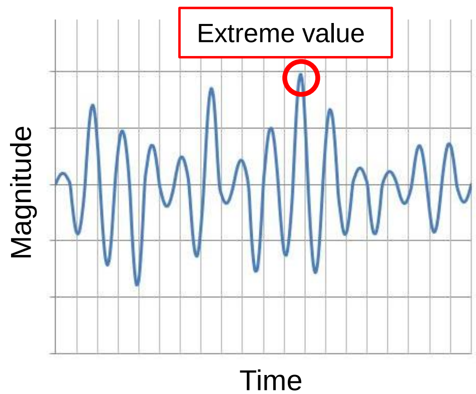
The plots below illustrate the distribution of relative maxima for two different signals, showing that the broader-band process has more negative maxima.



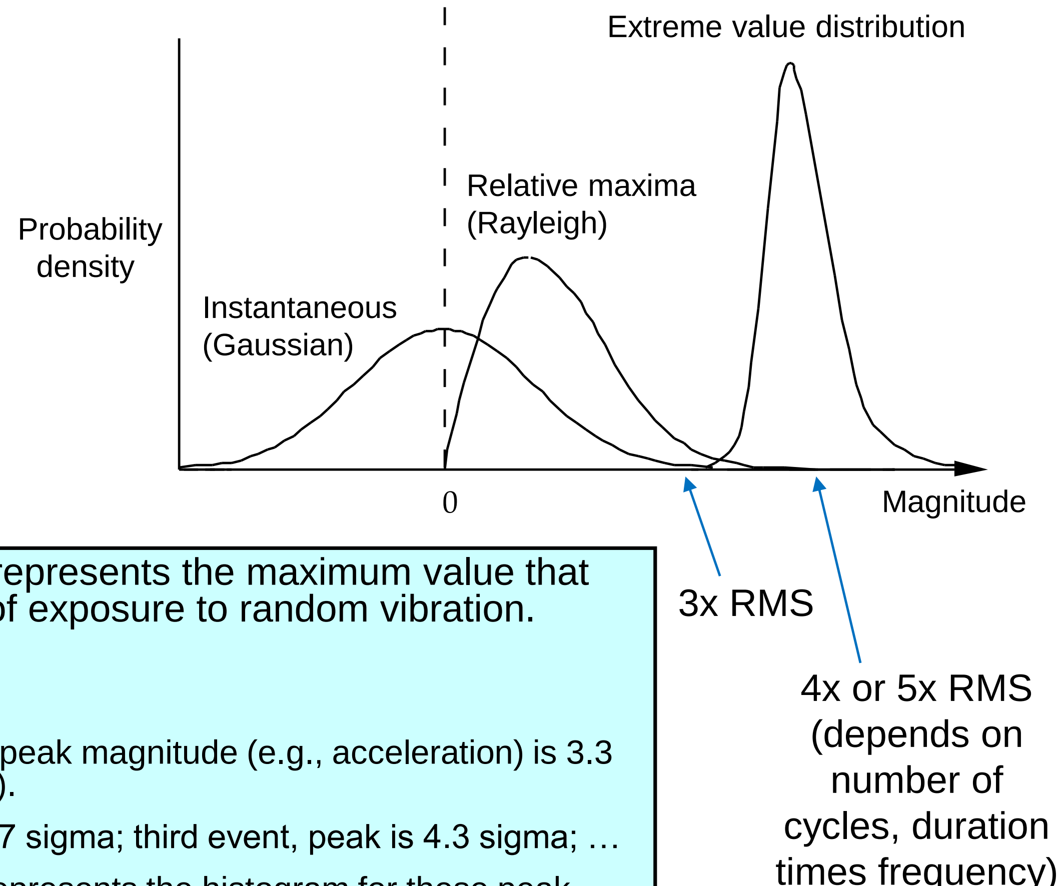
The Rayleigh distribution becomes less accurate for relative maxima as the process becomes broader band.

But the use of Rayleigh to generate a loading spectrum is conservative.

The Extreme Value Distribution for Narrow-band Random Vibration



Adapted from SSAM Fig. 12.13



The ***extreme value distribution*** represents the maximum value that occurs during the entire duration of exposure to random vibration. Example:

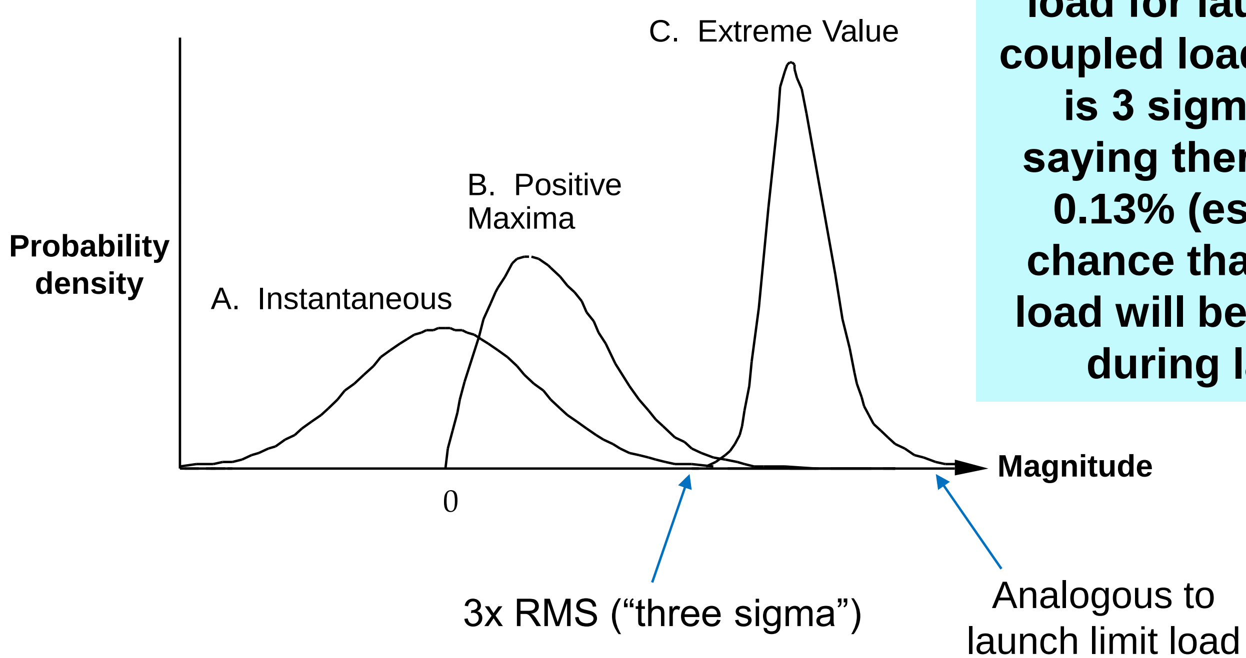
- Duration = 10 seconds
- In the first 10-second event, the peak magnitude (e.g., acceleration) is 3.3 sigma (3.3 times the RMS value).
- Second 10-sec event, peak is 4.7 sigma; third event, peak is 4.3 sigma; …
- The extreme value distribution represents the histogram for these peak (extreme) values.

The Difference between 3 Sigma for Random Vibration and 3 Sigma for Launch Load

For stress analysis, many engineers use the 3-sigma load as the highest load (limit load) for random vibration—an assumption that is often accepted in the aerospace industry.

This is most likely because of not understanding probability associated with random vibration.

When we say a limit load for launch from coupled loads analysis is 3 sigma, we're saying there's only a 0.13% (estimated) chance that the limit load will be exceeded during launch.



Is 3 Sigma for Random Vibration (3 times the RMS value) an Appropriate Limit Load for Stress Analysis?

Not only does theory tell us to expect peaks in excess of 3 sigma, we often see 4- and 5-sigma peaks in test.

- Even when using "3-sigma clipping" (Ref. 32).

The assumption of 3 sigma as the peak load for random vibration has been successful for the most part because of other compensating conservatism:

- Factor of safety and A-basis allowable stresses for strength analysis
- Damping usually higher than assumed, especially at full test levels
- Using the calculated RMS response acceleration as a quasi-static load in analysis (can be very conservative)
- Linear-elastic analysis for ductile failure modes
  - Yielding absorbs energy and can reduce the peak response acceleration.
- Single-mission hardware often not seeing enough loading cycles to cause fatigue failure

But we must be cautious in counting on past success with the 3-sigma assumption, especially if we remove some of the above conservatism.

Suggested Sigma Level in Stress Analysis for Random Vibration

For strength analysis, with a factor of safety …

- Brittle failure (composites, ceramics, glass): Use the maximum level predicted by the Rayleigh distribution; reduce excess conservatism elsewhere if needed.
  - I would treat failures of ductile materials for which there is little plastic displacement prior to rupture as brittle failure (example: stripping of bolt threads).
- Ductile failure: Use of 3 sigma as the limit load is probably adequate when making other assumptions that are conservative.
  - But fatigue failure can occur even with positive strength margins, and is more likely if the strength analysis is based on 3-sigma loads and ignores stress concentrations.

For fatigue or damage-tolerance (crack-growth) analysis …

- My preference is to use the Rayleigh distribution to derive a loading spectrum, including cycles higher than 3 sigma, and reduce excess conservatism elsewhere if needed.
- But truncating the loading spectrum to 3 sigma (lumping all cycles above 3 sigma in the Rayleigh distribution into the 3-sigma bin) can be successful when other conservatism compensates.

Apparent Frequency

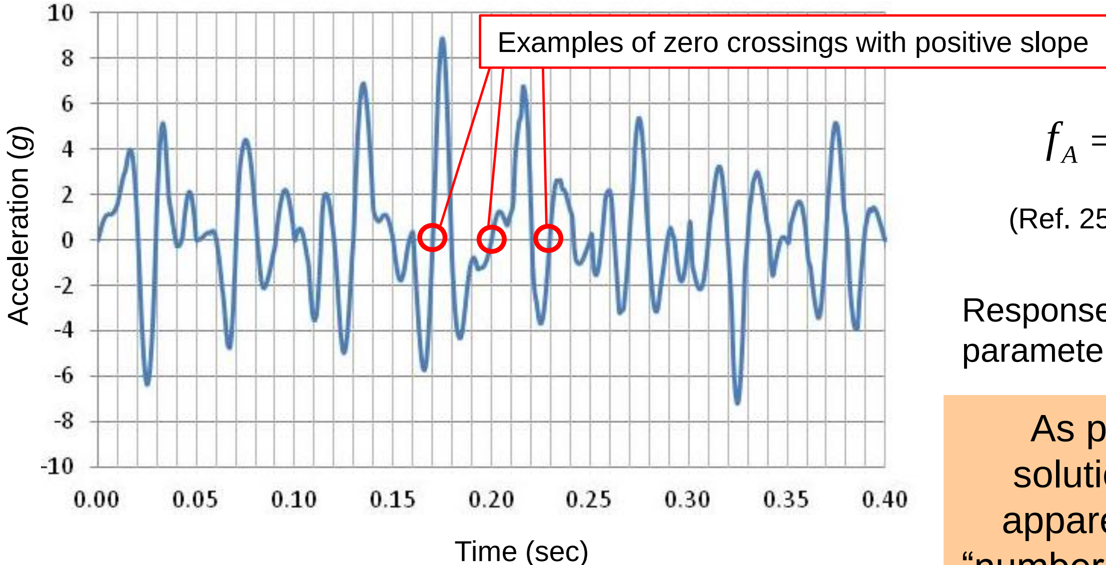
- To derive a loading spectrum with the Rayleigh distribution, we need an estimate of the total number of loading cycles, *n*.

$$n = \underbrace{f_A}_{\text{Apparent frequency}} \underbrace{T}_{\text{Duration of event}} \qquad \textbf{(Eq. 5.15)}$$

- The ***apparent frequency*, f_A**, is the effective average frequency associated with the parameter of interest, such as acceleration or stress at a particular location.
- For assessing stress near a component's mounting interface, f_A is approximately equal to the component's fundamental frequency if the component has a dominant mode with nearly all of the modal mass participation.
- When multiple frequencies of acceleration cause significant stress, we can calculate the apparent frequency from the response PSD. (See next page.)

Apparent Frequency (continued)

An accepted way to estimate the apparent frequency is to assume it's equal to the ***frequency of zero crossings with positive slope, f_{cross}*** (number of times per second the random variable goes from negative to positive).



$$f_A = f_{\text{cross}} \approx \sqrt{\frac{\int f^2 \cdot \text{PSD}(f) \cdot df}{\int \underline{\text{PSD}(f)} \cdot df}}$$

(Ref. 25)

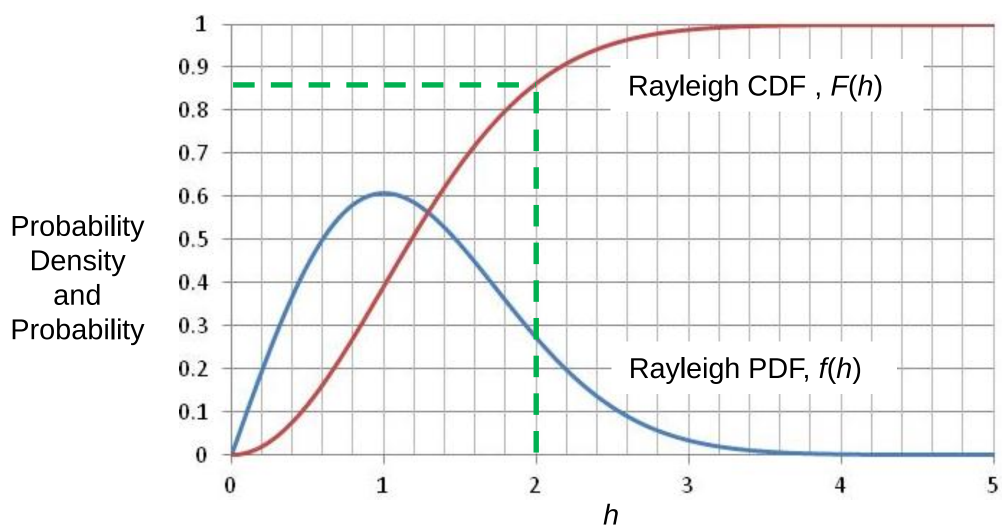
(Eq. 5.16)

Response PSD at frequency *f* for the load parameter of interest, such as force or stress

As part of the random vibration solution, Nastran can provide the apparent frequency (referred to as "number of zero crossings with positive slope per unit time" for Nx Nastran) for each response parameter identified.

Rayleigh Cumulative Distribution Function (CDF)

The Rayleigh CDF, *F*(*h*), represents the probability that any particular relative maximum will be below *h*, which is the RMS multiple (number of standard deviations away from the mean).



Note: This is not frequency; it's the probability density function per Eq. 5.14

$$F(h) = \int_0^{\infty} \underbrace{f(h)}\, dh$$

$$= 1 - e^{\left(\frac{-h^2}{2}\right)}$$

(Eq. 5.17)

Example: The probability of a local peak being less than or equal to 2 sigma is …

$$F(2) = 1 - e^{\left(\frac{-2^2}{2}\right)} = 0.86466$$

… which means about 13.5% of all local peaks will be > 2 sigma.

Example Problem 5-4:

Generating a Loading Spectrum for Random Vibration

Problem statement: Generate a 10-bin loading spectrum for one minute of exposure to random vibration, with an assumed apparent frequency of 100 Hz.

Solution: First calculate the number of loading cycles, *n*, then use the Rayleigh distribution to generate a loading spectrum.

Given: Duration of exposure, T = 60 sec
Apparent frequency, f_A = 100 Hz

$$\text{No. cycles } = f_A T$$

$$= 100(60) = 6{,}000$$

With a life factor of 4,

$$\text{Design cycles, } n = 4(6000) = 24{,}000$$

(continued)

Example Problem 5-4, continued:

Calculate the Rayleigh Distribution of Cycles

Design cycles, n = 24,000

$$h = \frac{\sigma}{\sigma_{rms}}$$

(stress level as a multiple of RMS stress)

$$n \cdot F(h)$$

h	Rayleigh PDF, $f(h)$	Rayleigh CDF, $F(h)$	Cumulative Cycles
5.0	0.000019	0.999996	24000
4.5	0.000180	0.999960	23999
4.0	0.001342	0.999665	23992
3.5	0.007656	0.997813	23948
3.0	0.033327	0.988891	23733
2.5	0.109842	0.956063	22946
2.0	0.270671	0.864665	20752
1.5	0.486979	0.675348	16208
1.0	0.606531	0.393469	9443
0.5	0.441248	0.117503	2820
0.0	0.000000	0.000000	0

$$f(h) = h \cdot e^{\left(\frac{-h^2}{2}\right)}$$

(Eq. 5.14)

$$F(h) = 1 - e^{\left(\frac{-h^2}{2}\right)}$$

(Eq. 5.17)

Example Problem 5-4, conclusion:

Now Generate the Loading Spectrum

If the RMS stress, σ_{rms}, is 8 ksi:

h	Rayleigh PDF, $f(h)$	Rayleigh CDF, $F(h)$	Cumulative Cycles	**Stress, σ**	**Design Cycles**
5.0	0.000019	0.999996	24000	**40**	**1**
4.5	0.000180	0.999960	23999	**36**	**7**
4.0	0.001342	0.999665	23992	**32**	**44**
3.5	0.007656	0.997813	23948	**28**	**214**
3.0	0.033327	0.988891	23733	**24**	**788**
2.5	0.109842	0.956063	22946	**20**	**2194**
2.0	0.270671	0.864665	20752	**16**	**4544**
1.5	0.486979	0.675348	16208	**12**	**6765**
1.0	0.606531	0.393469	9443	**8**	**6623**
0.5	0.441248	0.117503	2820	**4**	**2820**
0.0	0.000000	0.000000	0	**Total cycles:**	**24000**

23992 – 23948 = 44

Note that we've conservatively lumped all cycles within a bin at the highest load level in that bin (e.g., the third bin is from 3.5 to 4.0 times the RMS).

We can significantly reduce excess conservatism by increasing the resolution (more bins). When automating the life analysis, it doesn't take much more time to derive and use a 50-bin spectrum, with increments of 2% of max stress.

Generating a Loading Spectrum for Random Vibration Testing

Account for all three test axes, dwells at lower test levels, potential retests, and tests at different levels of assembly.

- Test levels are typically ramped up in 3dB increments, with dwell time at each increment dependent on how long it takes for the control system to bring the levels within tolerance over the full frequency spectrum.
  - In the loading spectrum, account for at least 10 seconds dwell time at -12 dB, -9 dB, -6 dB, and -3 dB.
- If the test plan calls for going to, say, -3 dB, and then powering down while the responses are reviewed, account for that test run as well.
- Each test axis may stress the same region in the structure
- If a part in an assembly fails in test, the part will need to be replaced, and the test will be repeated—everything other than the item that failed will be tested twice.
  - The loading spectrum should account for at least two potential tests.
  - In deriving qualification test durations, military programs typically account for up to six tests of flight hardware at a particular level of assembly (the most common option from Ref. 28, SMC-S-016).
- As applicable, account for planned vibration tests at different levels of assembly, including acoustic testing of the integrated spacecraft.

Account for the Full Life Cycle when Generating a Loading Spectrum

- Structural and vibration testing
- Transportation, ground and air
  - Commonly accounted for with random vibration over the transportation duration
- Launch
  - Transient and low-frequency loads (based on time histories from coupled loads analysis)
  - Combined with coexisting random vibration and steady-state loads
- On-orbit operations and thermal cycles
- Descent and landing (when applicable)

Sequence of loading is not addressed with Miner's Rule, but it actually does affect fatigue life—especially after a crack has formed. Fracture mechanics crack-growth analysis accounts for sequence.

In my opinion, this noted deficiency with Miner's Rule is of minor importance, given all the uncertainty associated with available fatigue data and loading spectra.

Still, to enable use of methods other than Miner's Rule, I suggest you construct a stacked loading spectrum in the sequence in which the events will occur.

A Common Problem—and How To Avoid Costly Impacts

Design schedules usually don’t allow time for thorough fatigue analysis.

Which means many programs commit to structural designs and build hardware before the fatigue analysis is complete.

To avoid big cost and schedule impacts associated with late fatigue analysis, we need to find simple fatigue screening criteria that we can implement during the design process.

We can derive such criteria based on selective fatigue analysis with clearly conservative loading spectra to derive allowable stresses for fatigue, for different …

– Materials

– Stress concentration factors

– Apparent frequencies

We can then use these criteria together with traditional criteria for strength analysis to release designs for manufacturing.

Example Form of a Fatigue Screening Criterion

$$\underbrace{\sigma_{\text{nom}}}_{\text{Nominal stress (limit or 3-sigma value)}} \cdot \underbrace{K_t}_{\text{Stress concentration factor}} < \underbrace{a}_{\text{Material-dependent factor}} \cdot \underbrace{F_{ty}}_{\text{Allowable yield stress}}$$

For a given material, the factor *a* can be derived as a function of apparent frequency from fatigue analysis using a worst-case assumed loading spectrum.

Other forms of screening criteria may be more practical for your program.

If a part passes the criterion, there is no need for dedicated fatigue analysis.	If a part fails the criterion, dedicated fatigue analysis is warranted.

Key Points from This Section

- Traditional strength analysis does not adequately screen out potential fatigue and fracture failures.
  - Especially for materials seeing a high number of stress cycles
- Not everything we design warrants fatigue analysis or fracture control, but we all must be on the watch for potential fatigue issues.
  - Doing selective fatigue analysis builds understanding of the design boundaries we should stay within to minimize risk of fatigue failures.
- For single-mission spaceflight hardware, we can avoid most fatigue concerns by …
  - designing gradual transitions between cross-sectional changes (e.g., large fillet radius),
  - ensuring high preload for bolted joints (which we'll discuss in Sec. 6),
  - and avoiding materials that tend to have cracks and manufacturing processes that tend to cause them.

Continued

Key Points from This Section (continued)

- When prudent, use a fatigue analysis factor (FAF) to account for uncertainty.
- When human safety is at stake, implement a rigorous fracture control program to protect against the possibility of undetected crack-like defects.
- When the design release schedule doesn't allow time for thorough fatigue analysis, derive simple, conservative fatigue screening criteria for use in the design process.
- Design release schedules usually don't allow for thorough fatigue analysis, so find simple ways to screen out potential fatigue issues during the design process.

Don't wait for your customer to specify a requirement for fatigue analysis or fracture control.

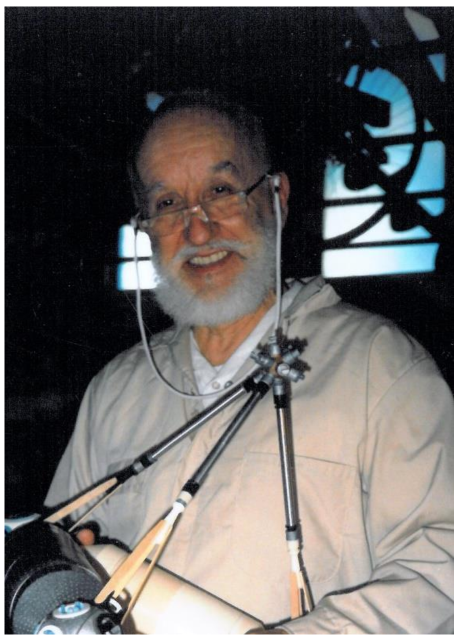
Ensuring structural integrity is part of your job!

6. Structural Design

- Opening Thoughts on Structural Design
- Material Selection
- Types of Structures and Important Things to Understand when Designing Them
  - Beams
  - Trusses and Frames
  - Forms of Lightweight Panels and Shells
  - Monocoque and Semi-Monocoque Cylinders
  - Skin-Stringer and Panel-Frame Structures
- Methods of Attachment
- Design of Bolted Joints
- Reducing Cost by Reducing the Number of Parts
- Designing an Adaptable Structure
- Summary: Structural Design Guidelines from a Master

Some Opening Thoughts on Structural Design

"Structural design is seldom seen as a disciplinary foundation for the practice of 'systems' engineering. In fact, hardly anyone employed in that 'discipline' is more than passingly acquainted with the subject. This is strange because if there is one common bond between all the subsystems in a vehicle it is the structure; everything attaches to it."



Oliver P. ("Ollie") Harwood (1922 – 2003)
Structural design engineer with over 40 years aerospace experience and one of the "fathers" of isogrid in the space industry; author of the manuscript "Right for Flight: The Structural and Architectural Design of Machines that Fly" (written in the late 1980s; yet to be published)

Thought-provoking words such as these make us wonder if we really know what "systems engineering" is all about!

A Structural Design Engineer Is a Systems Engineer

To do the job well, he or she must …

- understand the needs of the items being supported.
- understand system requirements and ensure the structure enables a successful mission.
- understand and accommodate life-cycle environments and loads.
- anticipate and accommodate uncertainty and growth.
- anticipate and avoid unnecessary cost and time associated with manufacturing, integration, test, handling, and transportation.

"Stress analysis is concerned with how strong (or stiff …) a piece of hardware is, analyzing its suitability for service … Structural design, like all design, must consider many other functions such as the architectural subdivision of spaces, leak tightness in pressurized or fluid containment volumes, provision for stowage of and access to the components of other systems, fabrication and assembly methods, and all the compromises which inevitably go into making a balanced flying machine—or, for that matter, any other kind of machine. All of this should be done with a minimum of fuss, feathers, and dollars … in a manner consistent with production quantity."

Ollie Harwood, "Right for Flight"

The Flow of This Course Section

In this section, we'll start with some basic design decisions:

- Material selection
- Types of structures
  - Beams
  - Trusses and frames
  - Panels used to support components
  - Shells; monocoque cylinders
  - Skin-stringer structures and panel-frame structures
- Methods of attachment

We'll explore how to make the designs of these structures efficient, weight-wise.

Then we'll circle back to explore some of the system considerations of structural design.

Properties to Consider when Selecting Materials

Property	Symbol
Allowable (minimum) tensile ultimate stress for design	F_{tu}
Allowable tensile yield stress	F_{ty}
Allowable compressive yield stress	F_{cy}
Young's modulus (modulus of elasticity)	E
Fatigue life	*S-N* curve
Fracture toughness	K_{Ic}
Density	ω
Coefficient of thermal expansion (CTE)	α
Thermal conductivity	K

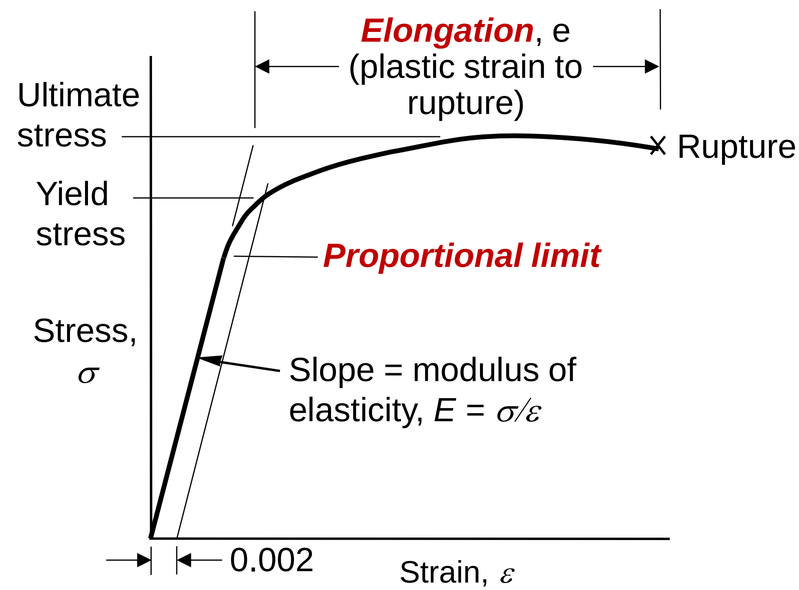
Other key considerations:

- Compatibility with environments (e.g., high/low temperature)
- Corrosion resistance
- Compatibility with other materials (e.g., to avoid galvanic corrosion)
- Raw material cost and processing cost
- Availability; lead time
- Machinability
- Weldability

Important Properties that Are Not as Often Considered

Minimum elongation—a measure of ductility. You can count on at least this much strain before rupture under uni-axial stress.

> Important for parts that have stress concentrations, such as bolts and parts that attached with fasteners. Ductility allows loads to redistribute before the highest-stress region ruptures.
>
> Also important for impact loading, as ductility allows parts to absorb more energy prior to rupture.



Proportional limit—the highest uni-axial stress that is proportional to strain

> A high proportional limit in compression is important for structures subject to potential buckling. The minimum proportional limit, F_{pl}, should be considered the allowable ultimate compressive stress for elastic buckling analysis.

Key Performance Parameters for Selecting Materials

Parameter	Calculation	Relevance	Applicable for …
Specific tensile strength	$\dfrac{F_{tu}}{\omega}$	Inversely proportional to weight for a structural member sized for tensile ultimate strength	Uniaxial tensile stress
Specific modulus (a.k.a. ***specific stiffness***)	$\dfrac{E}{\omega}$	Inversely proportional to weight for a structural member sized for stiffness under tensile load	Uniaxial tensile stress
Specific buckling strength	$\dfrac{E^{1/3}}{\omega}$	Inversely proportional to weight for a structural member sized for stability when the buckling load is proportional to thickness cubed	Flat plates, flanges, or webs under compression when the buckling stress does not exceed the proportional limit

F_{tu} = allowable tensile ultimate stress
E = Young's modulus
ω = density

Reminder: When the topic is buckling in this course, "E" is the compressive modulus of elasticity, referred to as "E_c" in the MMPDS.

Introduction to Composite Materials

- A ***composite*** material is one made of two or more base materials, each of which adds to the composite characteristics.
  – The materials bond together to act as one.
- Common examples:
  – Wood
  – Concrete and reinforced concrete
  – Fiberglass and carbon-fiber composites
- A composite material has a ***reinforcement*** (usually fibers of some sort), which provides strength and stiffness, and a ***matrix*** material, which bonds the reinforcements together.
- Most commonly,
  – Fibers are continuous.
  – A composite part is made from a lay-up (***laminate***) of plies.
    - ➢ Each ply or layer (***lamina***) has fibers running in a single direction.
    - ➢ Multiple plies at angles relative to each other to get the desired properties
    - ➢ Or multiple layers of woven plies

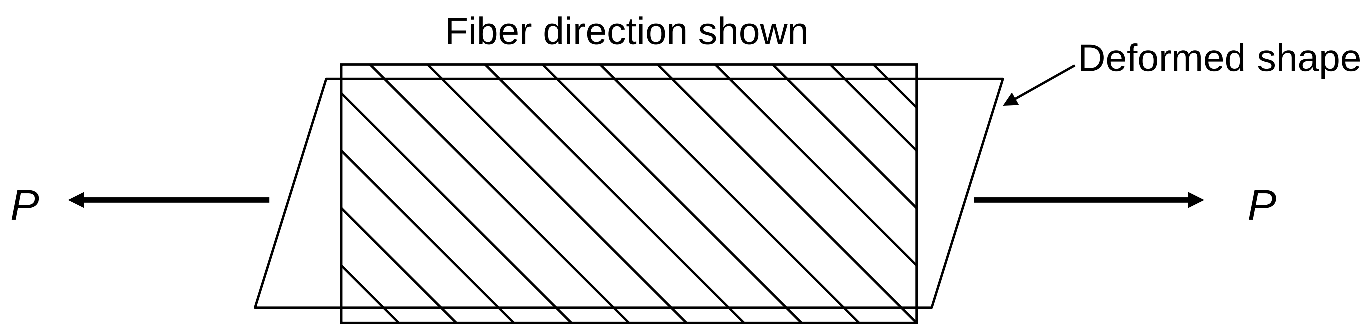
Important Differences between Metals and Composites

- Most metals are (or nearly are) …
  - ***Homogeneous*** (uniform; having the same composition and properties throughout)
  - ***Isotropic*** (having the same properties in all directions)
  - ***Ductile*** (undergoing plastic deformation before rupture)
- As a result,
  - Direction of stress usually is not important.
  - Can use methods of analysis that are based on isotropic theory.
  - Strength is not affected much by stress concentrations at microscopic defects and is thus relatively repeatable from build to build.

- Most composites are …
  - ***Heterogeneous*** (not uniform; high strength in the directions in which fibers run)
  - ***Anisotropic*** (not isotropic)
  - ***Brittle*** (no plastic deformation before rupture)
- As a result,
  - In design, we need to avoid interlaminar tensile stress (through-thickness stress).

    Key point
  - Analysis needs to account for anisotropic behavior.
  - Strength is sensitive to process variation and random defects. When production volume is low, we normally have to proof test every member and every joint in each flight article.

Example of Anisotropic Behavior for an Unbalanced Composite



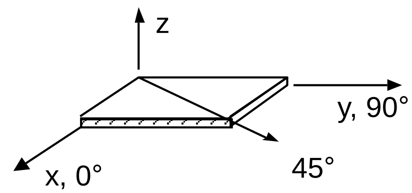
We usually want composite laminates to be balanced and symmetric:

Balanced = Each layer at angle $+\theta$ is balanced by a layer at $-\theta$.

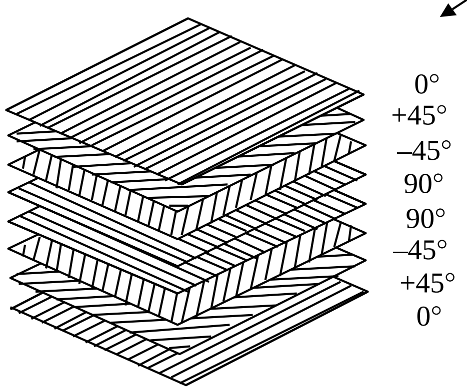
Symmetric = The layup above the laminate's center plane is a mirror image of the layup below.

A Quasi-Isotropic Laminate is Often Best

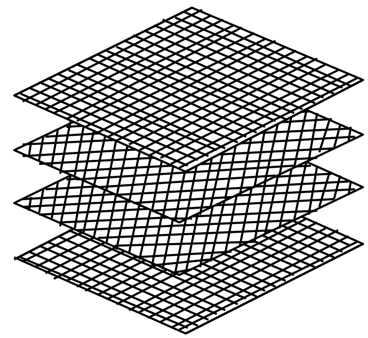
A ***quasi-isotropic laminate*** has the same properties in all planar directions.



Note that both laminates shown are balanced and symmetric.



Quasi-isotropic laminate constructed from 8 unidirectional plies



Quasi-isotropic laminate constructed from 4 woven 0/90 plies

Ref. 1, SSAM Fig. 15.7

<u>Advantages</u>

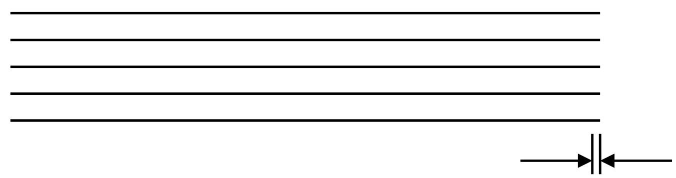
- No weak in-plane direction
- Can predict certain behavior with isotropic-shell theory
- Most efficient when strength is needed in all directions (e.g., bolted shear joints)

<u>Disadvantage</u>

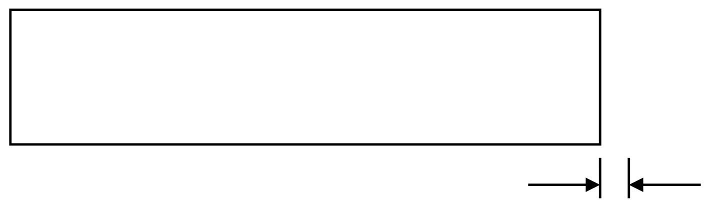
- Less efficient when higher strength or stiffness is needed in one direction

Thermal Expansion of a Carbon-Fiber Composite

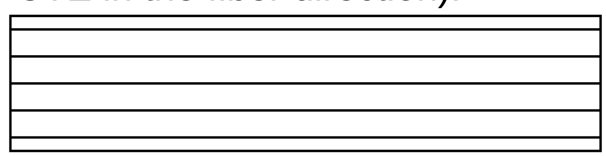
When temperature increases, carbon fibers contract (small negative CTE):



Whereas the matrix material wants to expand (positive CTE):



If you design the right volume fraction of carbon fibers, the composite will neither contract nor expand (zero CTE in the fiber direction):



But the materials will be stressed

Because the fibers and the matrix are fighting each other, temperature change generates stresses that can cause *micro-cracking* (cracking of matrix material) and reduce strength under external loads

Advantages: Why Composites are Attractive for Flight Vehicles

- High specific tensile strength
- High specific modulus
- Can be designed to have near-zero coefficient of thermal expansion (CTE)
- High-temperature applications
- Long fatigue life
  - Nonmetallic materials don't form cracks under cyclic loading like metals, and fibers tend to stop growth of any crack-like defects caused by manufacturing processes.
- Design flexibility, tailored properties
- Good wear resistance (carbon fibers resist abrasion)
- Corrosion resistance
  - But we have to be careful to avoid sustained contact between carbon fibers and aluminum, magnesium, or low-alloy steel, any of which leads to galvanic corrosion of the metal.
- Reduced cost when you can amortize development cost over large production volume

Disadvantages and Limitations of Composites

- A composite laminate is strong in the two in-plane directions only.
  - Design options limited by need to avoid through-thickness (interlaminar tensile) stress
  - Metal end fittings often required; may not achieve expected weight savings
- Most composite materials are brittle (no elongation).
  - Not tolerant of impact loads and sensitive to stress concentrations
  - Strength varies more widely than for ductile materials.
  - Proof testing (test each build) is usually necessary when production volume is low.
- Composite laminates are not as strong in compression as in tension.
  - Interlaminar buckling limits compressive strength
- Usually much more costly than using metals when production volume is low.
- Polymer-matrix composites (PMCs) outgas and desorb water in space.
  - Source of contamination for sensitive surfaces and causes dimensional change
  - Less of a problem with recent polymers, such as cyanate ester
- There are many potential pitfalls and risks:
  - Hard to obtain dependable design data
  - Unexpected failure modes
  - Key ingredient may become unavailable
  - Schedule slips and cost overruns

Using composites requires an investment. There is more to learn, and more things can go wrong.

The keys are knowledge, understanding, and a sound development approach.

My Philosophy: Use Aluminum Alloys Where You Can!

- Aluminum is low in density, low in cost, easy to machine, ductile, easy to attach (many options), and readily available, with little or no development cost.
- A ductile material such as most aluminum alloys yields before it ruptures, which makes strength less sensitive to random defects, allows the material to absorb more impact, and allows internal loads to redistribute such that a structure can carry more applied load.
- Use composites ...
  - when the design needs greater specific tensile strength, specific modulus, or thermoelastic stability than aluminum can provide.
    - antenna booms and large reflectors
  - when stresses are tensile and interlaminar tensile stress can be avoided
    - pressurized tanks, truss members
  - when processes are already established and well controlled.
    - nothing new about the design
  - or when production volume is high enough to justify an extensive development program.

Comparison of Representative Material Properties

(room temperature, U.S. customary units—not intended for design use)

Material	F_{tu} ksi	F_{ty} ksi	F_{cy} ksi	F_{pl} ksi	E msi	e %	ω lb/in^3	α (CTE) 10^{-6}/°F
Aluminum 6061-T651 plate, 2"t	42	36	35	26	9.9	8	0.098	12.7
Aluminum 7075-T651 plate, 2"t	76	69	66	48	10.3	7	0.101	12.4
Magnesium AZ31B forging, 1"t	35	22	12	10	6.5	7	0.064	14.0
Beryllium hot pressed shapes	47	35	#	#	42.0	2	0.067	6.2
Ti-6Al-4V, annealed bar, 1"t	135	125	129	110	16.9	10	0.16	4.9
A36 structural steel	58	36	36	36	29.0	20	0.284	6.5
301 SS, annealed sheet <0.187"t	73	26	23	<10	29.0	40	0.286	8.3
301 SS, half hard sheet <0.187"t	141	93	61	10	26.0	18	0.286	8.3
17-4PH H1150 SS, bar	125	100	90	65	28.5	16	0.284	6.2
Inconel 718 bar, 2"t	185	150	156	87	29.4	12	0.297	6.9
Invar® 36, annealed	64	35	#	7	20.5	30	0.291	1.0
P75/1962 Gr/Epoxy [±30,0$_4$]sym	85	N/A	N/A	54.3	32.7	0	0.062	-1.0
P75/1962 Gr/Epoxy [0,±45,90]sym	45	N/A	N/A	26.5	15.2	0	0.062	-0.4

The above properties apply in the L (longitudinal) direction, where applicable.

A-basis allowables for metals (minimum strength values):

- F_{tu} = tensile ultimate stress
- F_{ty} = tensile yield stress
- F_{cy} = compressive yield stress
- F_{pl} = approximate minimum compressive proportional limit

E = Young's modulus, tensile
e = minimum elongation
ω = density
CTE = coefficient of thermal expansion
SS = stainless steel
# = value not found

Comparison of Representative Material Properties

(room temperature, SI units—not intended for design use)

Material	F_{tu} MPa	F_{ty} MPa	F_{cy} MPa	F_{pl} MPa	E GPa	e %	ω g/cm^3	α (CTE) 10^{-6}/°C
Aluminum 6061-T651 plate, 2"t	290	248	241	179	68	8	2.71	22.9
Aluminum 7075-T651 plate, 2"t	524	476	455	331	71	7	2.80	22.3
Magnesium AZ31B forging, 1"t	241	152	83	69	45	7	1.77	25.2
Beryllium hot pressed shapes	324	241	#	#	#	2	1.85	11.2
Ti-6Al-4V, annealed bar, 1"t	931	862	889	758	117	10	4.43	8.8
A36 structural steel	400	248	248	248	200	20	7.86	11.7
301 SS, annealed sheet <0.187"t	503	179	159	#	200	40	7.92	14.9
301 SS, half hard sheet <0.187"t	972	641	421	69	179	18	7.92	14.9
17-4PH H1150 SS, bar	862	689	621	448	197	16	7.86	11.2
Inconel 718 bar, 2"t	1276	1034	1076	600	203	12	8.22	12.4
Invar® 36, annealed	441	241	#	48	141	30	8.05	1.8
P75/1962 Gr/Epoxy [±30,0_4]sym	586	N/A	N/A	374	225	0	1.72	-1.8
P75/1962 Gr/Epoxy [0,±45,90]sym	310	N/A	N/A	183	105	0	1.72	-0.7

The above properties apply in the L (longitudinal) direction, where applicable.

A-basis allowables for metals (minimum strength values):

- F_{tu} = tensile ultimate stress
- F_{ty} = tensile yield stress
- F_{cy} = compressive yield stress

F_{pl} = approximate minimum compressive proportional limit

E = Young's modulus, tensile
e = minimum elongation
ω = density
CTE = coefficient of thermal expansion
SS = stainless steel
\# = value not found

Why is Aluminum Used So Often for Flight Structures?

- Yes, raw material cost and machining cost are lower for aluminum than for stainless steel or titanium.
- But specific tensile strength and specific modulus are no higher for aluminum than what is attainable with stainless steel or titanium.
- Yet an aluminum flight structure, such as an airframe, will be lighter than one made out of steel. Why?

Answer: Low density leads to compressive efficiency (and bending efficiency as well).

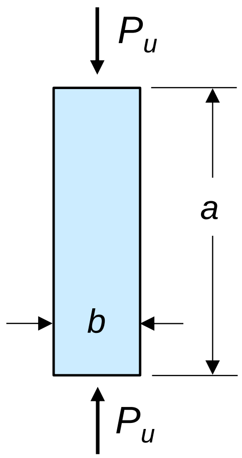
The load that causes a plate, shell, or I-beam flange to buckle elastically is proportional to thickness cubed.

For the same weight, an aluminum plate or I-beam flange is thicker than one made of steel and thus can carry more compressive load.

This is where specific buckling strength helps us select materials:

$$\frac{E^{1/3}}{\omega}$$

Derivation of the Expression for Specific Buckling Strength



Let's say we're designing a flat plate of length *a* and width *b* for a uniformly distributed design ultimate compressive load, P_u. The ends of the plate are pinned, and the long edges are unsupported.

Knowing that the plate's thickness, *t*, will be driven by the need to prevent buckling, we want to determine which of two materials would result in the lowest weight.

The relevant properties for this study are Young's modulus, *E*, and density, ω.

We can use Eq. 4.14 to calculate the elastic buckling load, P_{cr}:

$$P_{cr} = \frac{\pi^2 EI}{\left(1-\nu^2\right)a^2}$$

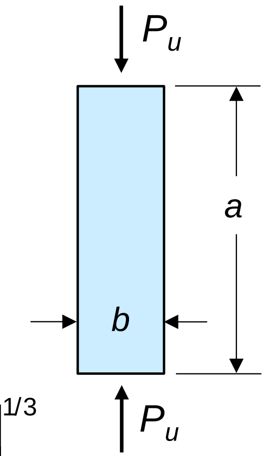
where *I* is the cross-section's lowest moment of inertia and ν is Poisson's ratio

For a rectangular cross section, $I = \dfrac{bt^3}{12}$ so $P_{cr} = \dfrac{\pi^2 Ebt^3}{12\left(1-\nu^2\right)a^2}$

continued

Derivation of the Expression for Specific Buckling Strength
(continued)

From the previous page, $P_{cr} = \dfrac{\pi^2 E b t^3}{12\left(1-\nu^2\right)a^2}$



Setting $P_{cr} = P_u$, the design ultimate compressive load, we can solve for the required thickness, t_{req}:

$$t_{\text{req}}^3 = \frac{12\left(1-\nu^2\right)a^2 P_u}{\pi^2 E b} \qquad \text{so} \quad t_{\text{req}} = \left[\frac{12\left(1-\nu^2\right)a^2 P_u}{\pi^2 E b}\right]^{1/3}$$

The weight, w, of this plate is $w = \omega b t_{\text{req}} a = \omega b a \left[\dfrac{12\left(1-\nu^2\right)a^2 P_u}{\pi^2 E b}\right]^{1/3}$

$$\text{or} \quad w = \underbrace{ba\left[\frac{12\left(1-\nu^2\right)a^2 P_u}{\pi^2 b}\right]^{1/3}}_{\text{This quantity is a constant for given dimensions } a \text{ and } b}\left(\frac{\omega}{E^{1/3}}\right)$$

This quantity is a constant for given dimensions a and b

Thus, weight is proportional to $\dfrac{\omega}{E^{1/3}}$

This expression for specific buckling strength applies to all shells for which the buckling load is proportional to t^3 (which means buckling stress is proportional to t^2)—but only if the compressive stress does not exceed the material's proportional limit.

Comparison of Performance Parameters (Ratios)

(room temperature)

Material	Specific tensile ult. strength 10^3 in	Specific tensile yld strength 10^3 in	Specific buckling strength, $in^{7/3}/lb^{2/3}$	Specific modulus 10^6 in
Aluminum 6061-T651 plate, 2"t	429	367	2191	101
Aluminum 7075-T651 plate, 2"t	752	683	2154	102
Magnesium AZ31B forging, 1"t	548	344	2921	102
Beryllium hot pressed shapes	701	522	5188	627
Ti-6Al-4V, annealed bar, 1"t	844	781	1604	106
A36 structural steel	204	127	1082	102
301 SS, annealed sheet <0.187"t	255	91	1074	101
301 SS, half hard sheet <0.187"t	493	325	1074	101
17-4PH H1150 SS, bar	440	352	1076	100
Inconel 718 bar, 2"t	623	505	1039	99
Invar® 36, annealed	222	120	940	70
P75/1962 Gr/Epoxy [±30,0_4]sym	1371	N/A	5158	527
P75/1962 Gr/Epoxy [0,±45,90]sym	726	N/A	3995	245

usually less desirable			usually more desirable	

Units for these ratios don't matter as long as they are consistent between materials.

The values for specific buckling strength apply only if stress does not exceed the material's proportional limit. So, even though magnesium has a higher specific buckling strength than aluminum, the low proportional limit for magnesium (about 10 ksi) makes it severely limited for use when buckling is of concern.

Qualitative Comparison for Material Selection

Alloy	Specific tensile ultimate strength	Specific buckling strength	Specific modulus	Propor-tional limit	CTE	High temp. usage	SCC resis-tance	Raw cost	Machin-ability	Weld-ability	Typical Applications
Alum. 6061-T651	M	H	M	L	VH	P	G	VL	VG	F	Flight structures
Alum. 7075-T6	H	H	M	M	VH	P	P	L	G	P	Flight structures
Magnesium AZ31B	M	H	M	VL	VH	P	F	M	VG	G	Test fixtures (shakers)
Beryllium	H	VH	VH	L	M	F	G	VH	P	P	Mirrors, hinges
Ti-6Al-4V	H	M	M	VH	L	F	G	H	P	F	Fittings bonded to composites
A36 steel	L	L	M	L	M	G	#	VL	F	VG	Ground support structures
301 SS, annealed	L	L	M	VL	H	F	G	M	F	G	Fasteners (light duty)
301 SS, half hard	M	L	M	VL	H	F	G	M	F	G	Welded tubes and fittings
17-4PH H1150	M	L	M	H	M	G	F	M	F	F	Turbine blades
A-286	H	L	M	H	H	VG	G	H	F	F	Bolts, turbine blades
Inconel 718	H	L	M	H	M	VG	G	H	P	G	Rocket engines
Invar® 36	L	L	L	VL	VL	F	G	H	F	G	Metering structures for optical instruments
CFRP composite	VH	VH	VH	H/M	VL	G	G	H	P	N/A	Flight structures, metering structures

SCC - stress corrosion cracking	VH	very high	VG	very good
SS - stainless steel	H	high	G	good
CFRP - carbon fiber reinforced polymer	M	medium	F	fair
	L	low	P	poor
	VL	very low	#	can't find data

usually less desirable ... usually more desirable

Types of Structures

In the following pages, we'll look individually at the following types of structures:

- Beams
- Trusses and frames
- Forms of lightweight panels and shells
- Monocoque and semi-monocoque cylinders
- Skin-stringer and panel-frame structures

The common theme in design to make the above structures efficient is to provide direct load paths.

An efficient structure weighs less relative to strength and stiffness than an inefficient structure.

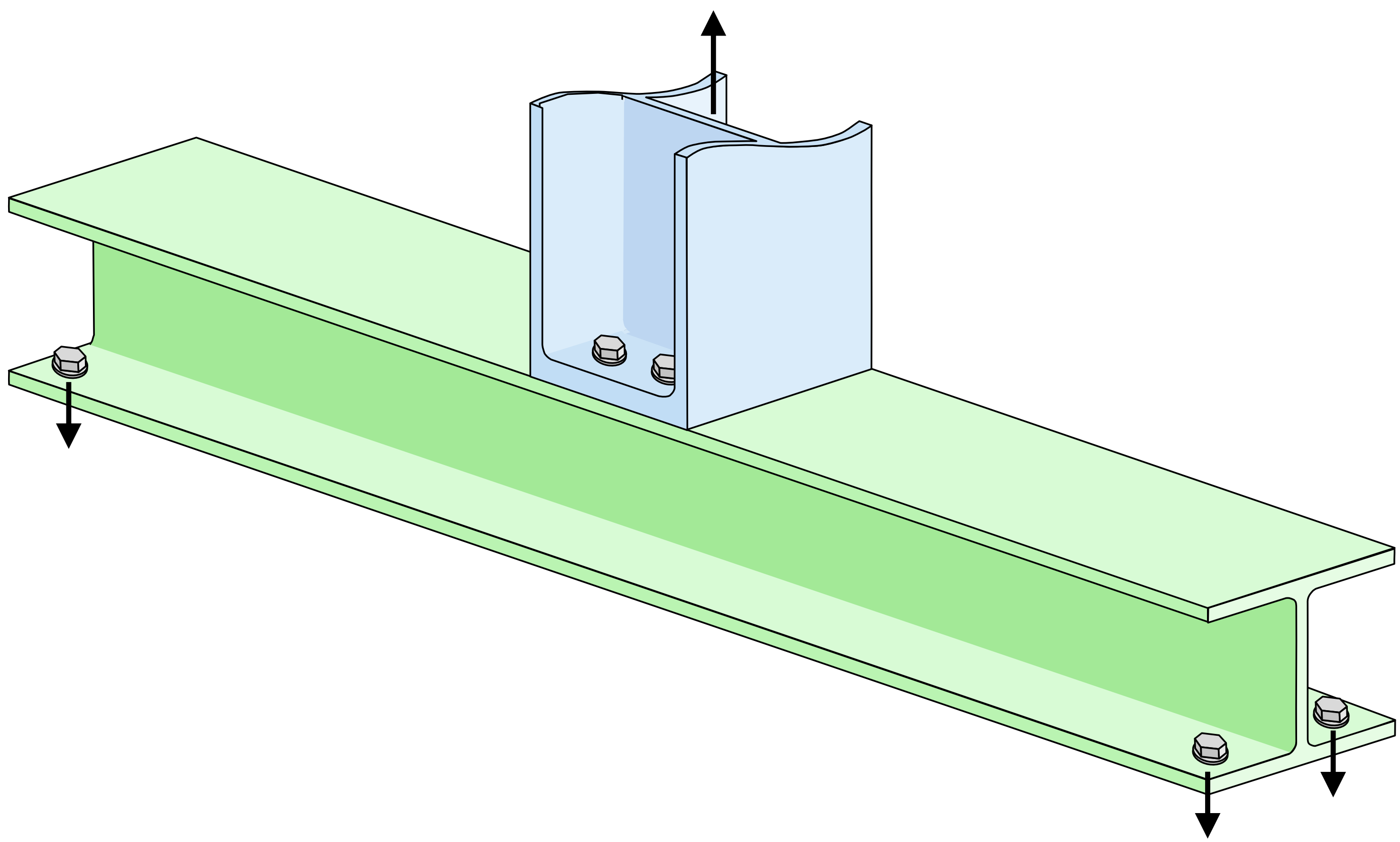
Beams

- A ***beam*** is a structural member that either is cantilevered or spans two or more supports and is intended to carry lateral loads applied in locations other than the supports.
- These lateral applied loads cause bending moments and, potentially, torsion.
  - As discussed in Sec. 3 herein
- To make a beam efficient, design it to behave according to beam theory.
  - Requires that load be uniformly distributed over the beam's cross section rather than concentrated on certain regions of the cross section.
  - Such concentrated loads make the beam less strong and less stiff than beam theory predicts.

When we learn beam theory in school, we're dealing with one-dimensional objects.

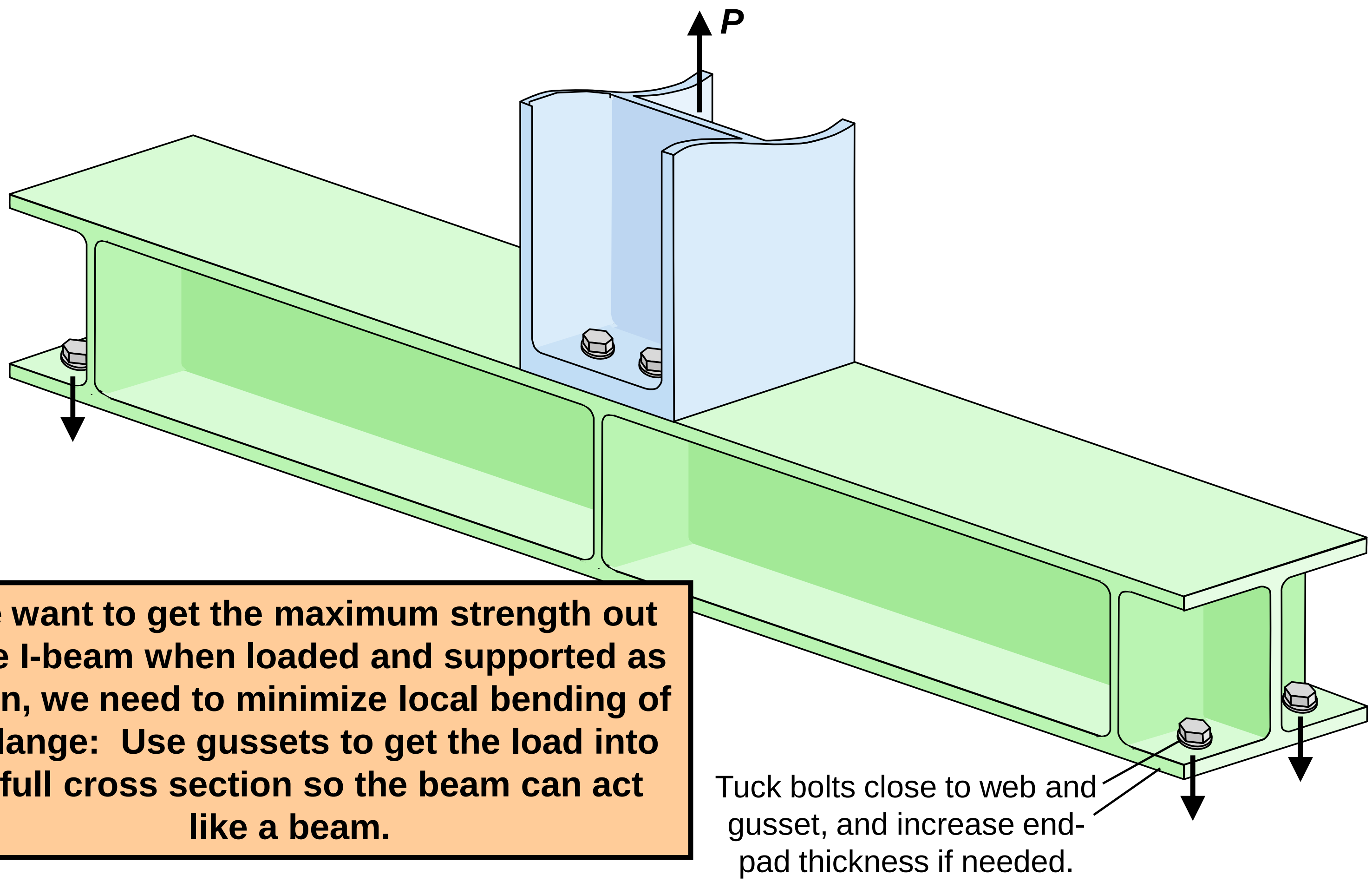
Then we enter industry and have to deal with three dimensions …

What's Wrong with This Design?



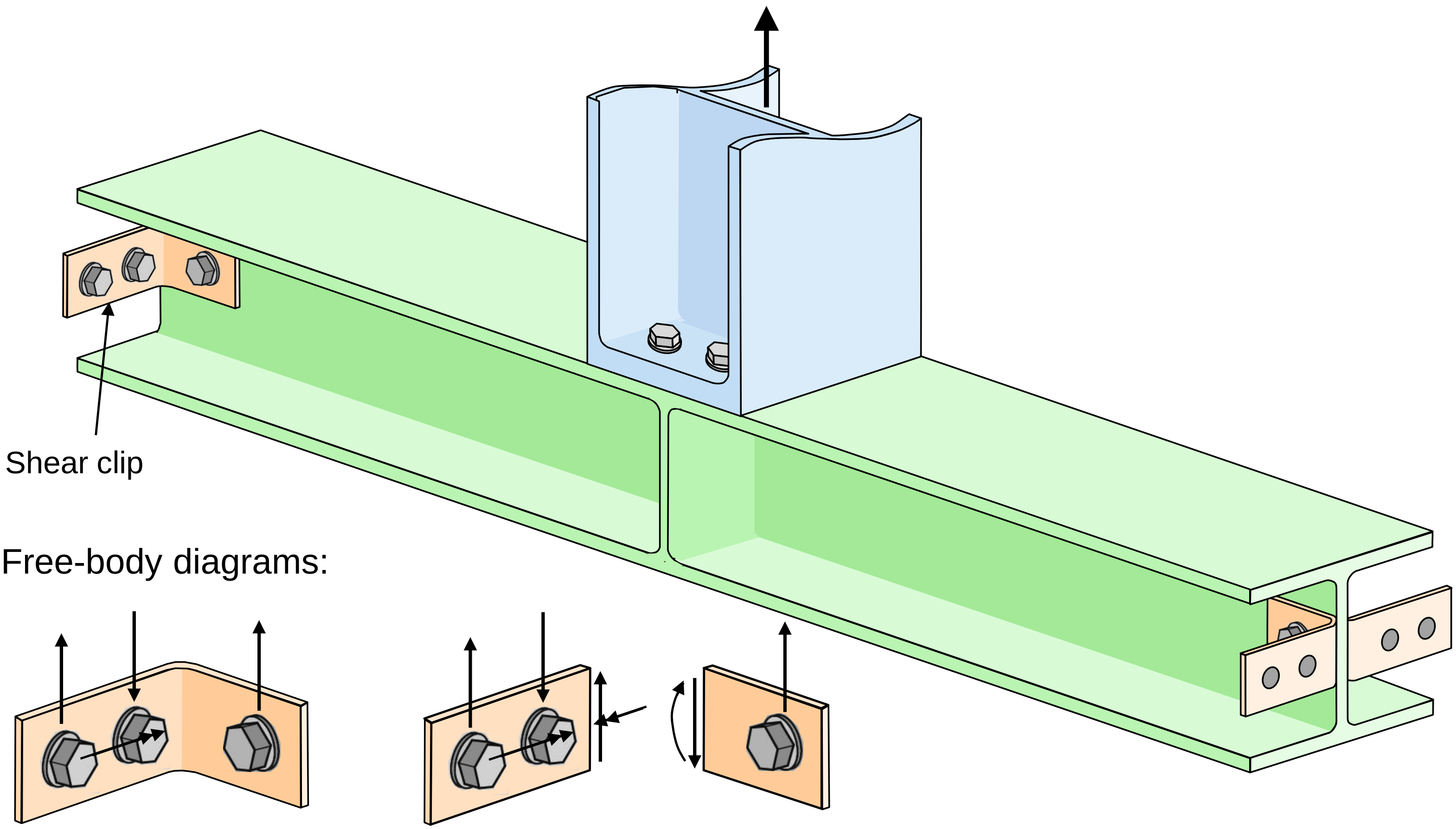
Intentionally Left Blank

Add Gussets to Reduce Flange Bending



If we want to get the maximum strength out of the I-beam when loaded and supported as shown, we need to minimize local bending of the flange: Use gussets to get the load into the full cross section so the beam can act like a beam.

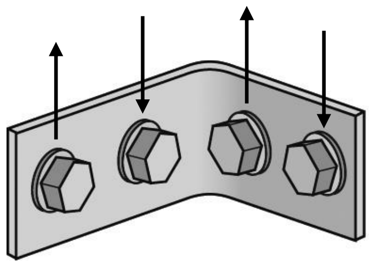
What's Wrong with This Design?



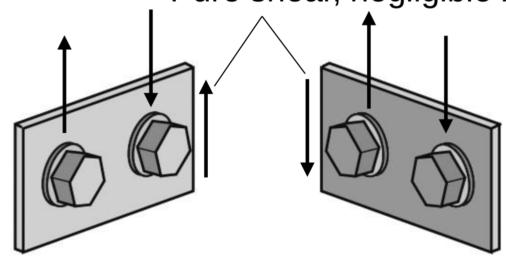
Result: torsion in a thin sheet

Designing Shear Clips

Put two fasteners on each leg:

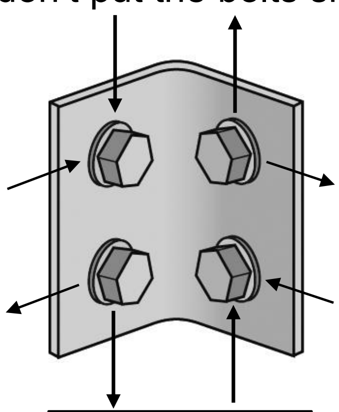


Pure shear, negligible moment at corner

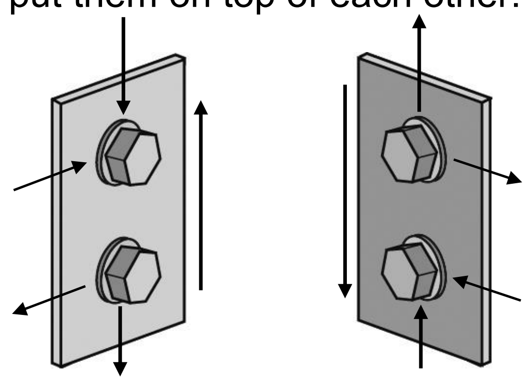


Result: The clip is able to transfer shear without torsion on either leg.

But don't put the bolts side by side; put them on top of each other:



Better design



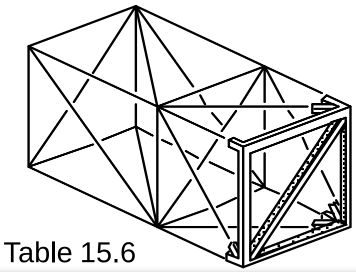
Result: Much lower peak resultant shear load on a fastener

We now have an efficient simply supported beam.

Remember to Use a Second Bolt!

But I guess one bolt is better than none!

Trusses and Frames


Adapted from SSAM Table 15.6

A ***truss*** can withstand joint-applied loads with its members loaded only axially. A ***frame*** must carry shear and bending moments through its joints.

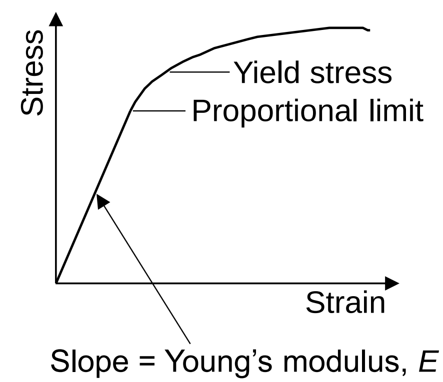
Unless the joints of a truss are intentionally pinned with spherical bearings, its members will carry shear and bending moments, which increase the member stress.

Forms of construction:	• Members extruded, machined, or formed from sheet metal • Truss sides integrally machined from plate-stock material • Composite tubes with metallic end fittings and lug-and-clevis joints
Materials:	• Aluminum, titanium, carbon-fiber composite
Attachment methods:	• Mechanical fastening (bolts or rivets), welding, or bonding, depending on the material
Packaging and access when used as a spacecraft body structure:	• Spacecraft equipment often combined into subsystem-specific modules, which can be separately tested and then integrated with the bus • Easy access for routing electrical harness
Other design considerations:	• Hard to transfer loads from a truss to a cylinder (e.g., launch vehicle) • Trusses are efficient for structures of square, rectangular, or triangular cross section; they become less efficient for polygons with more sides

Preliminary Design of a Truss

1. Assemble requirements.
2. Determine the most efficient arrangement of truss members.
3. Select material and form of construction.
4. For each member, determine the cross-sectional area needed to
   - keep the design ultimate compressive stress at or below the material's proportional limit (or any other limiting compressive stress, such as for composites).
   - keep any other design stresses at or below the corresponding material allowables.
   - meet the stiffness requirement, when applicable.
5. Find the geometry needed to keep the structural member stable at the design ultimate compressive stress.
   - Cross-sectional shape (limited by form of construction) and dimensions

Size structures to ensure the design ultimate compressive stress does not exceed the minimum proportional limit to avoid inelastic buckling, which is hard to predict.



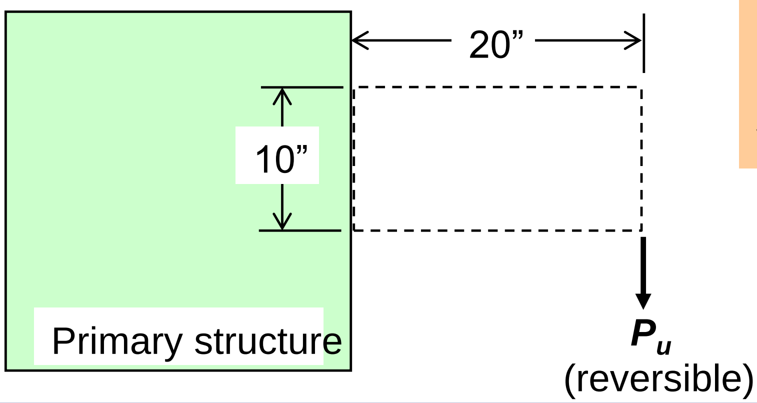
For most aluminum alloys, the compressive proportional limit is between 60 - 80% of the compressive yield stress.

Example Problem 6-a
Preliminary Structural Design of Truss

Problem statement: Design a truss that will accept a concentrated load and transmit it to the primary structure

Driving requirements (Step 1):

- The design ultimate applied load, P_u, is +/-10,000 lb in the direction shown.
- The load is applied a distance of 20” from the primary structure.
- The mounting footprint (for bolts) is 10” and is located relative to the load as shown below.
- Key constraints are weight and cost.
- Keep stiffness as high as practical, but at lower priority than weight and cost.



The problem is to be worked in two dimensions only, based on the assumption that out-of-plane support will be designed at a later date.

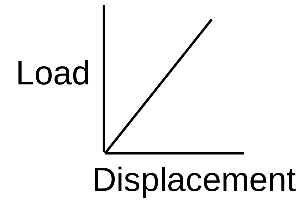
Now that we’ve identified the requirements, our next step is to configure the truss

First Let's Review Strain Energy

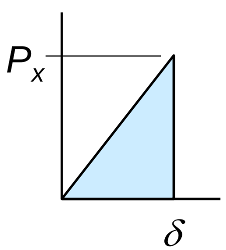
Strain energy is the energy stored in an elastic body under load.

– It's the *internal work* within the body.

Consider a tensile-test coupon under uniform axial stress:



The strain energy in the coupon is equal to the work done by the applied load (*external work*).



- Recall: work = force times distance
- If the material is linear-elastic, the force builds proportionally with displacement.
- The work is the area under the load-displacement curve.

For axial loading of a specimen of length L,

External work = $\dfrac{P_x \delta}{2} = \dfrac{P_x}{2}\left(\dfrac{P_x L}{AE}\right) = \dfrac{P_x^{\,2} L}{2AE}$ = strain energy

Designing a Minimum-Weight Truss

The work done by an applied force (external work) equals the structure's total strain energy (internal work).

For a truss made of *n* members with pinned ends, the total strain energy is

$$U = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2A_i E_i}$$

where P_i = axial load in member *i*
L_i = length
A_i = cross-sectional area
E_i = Young's modulus

The work done by a single applied force, *P*, is

$$W = \frac{P\delta}{2}$$

δ = displacement at the point of load application in the direction of applied force

Setting the external work equal to the internal work,

$$\frac{P\delta}{2} = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2A_i E_i} \quad \text{or} \quad \delta = \frac{2}{P}\sum_{i=1}^{n} \frac{P_i^2 L_i}{2A_i E_i}$$

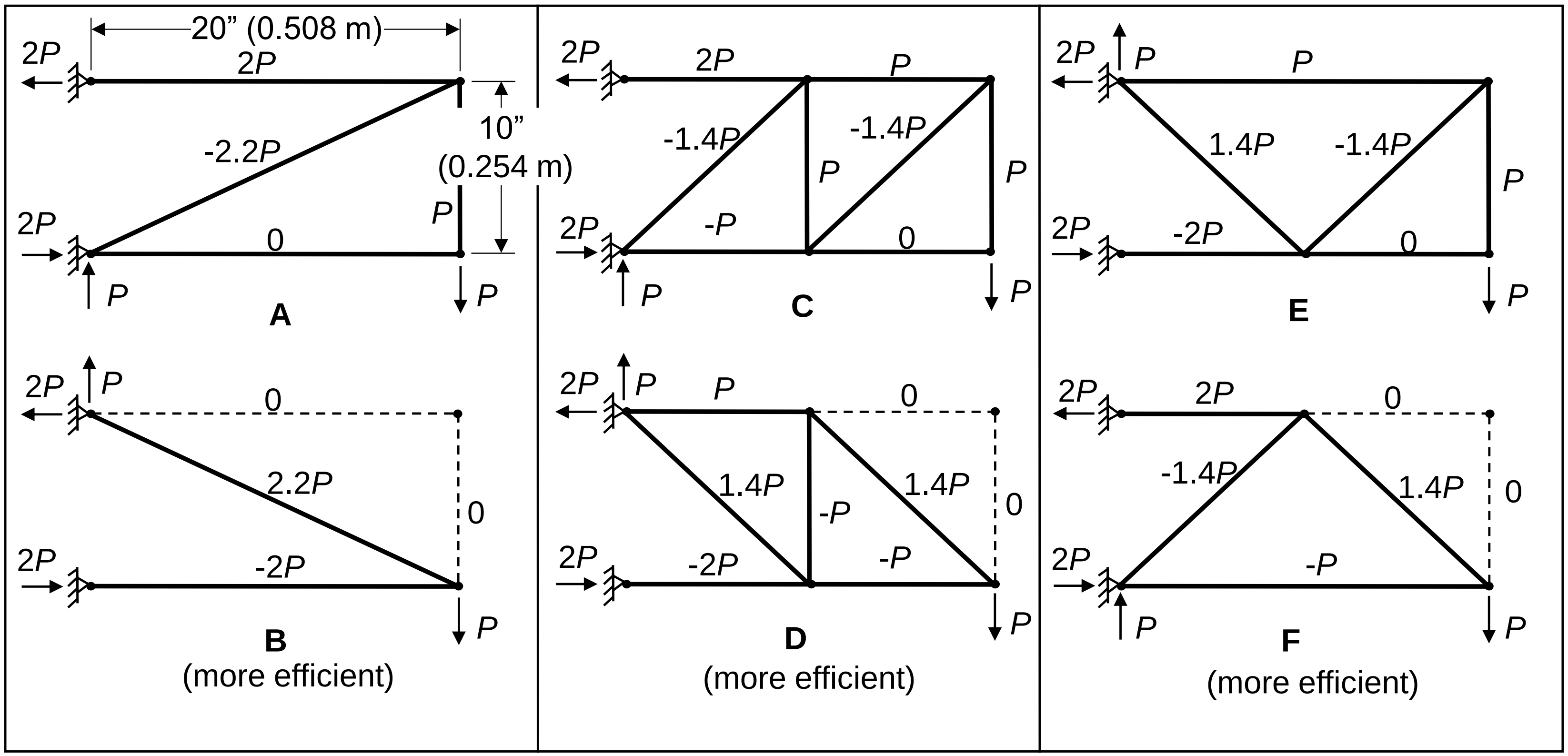
SSAM Eq. 15.4

Thus, the way to stiffen a truss is to reduce its strain energy, which we can do by increasing the members' cross-sectional areas (which adds weight) or by making load paths more direct (decreasing $\Sigma P_i^2 L_i$, which reduces weight).

Given optional truss arrangements with each member sized to provide just the required strength for a given material, the lightest truss will be the stiffest!

Back to the Truss Example 6-a

Step 2: Which Is the Most Efficient Arrangement?



Adapted from SSAM Fig. 15.14

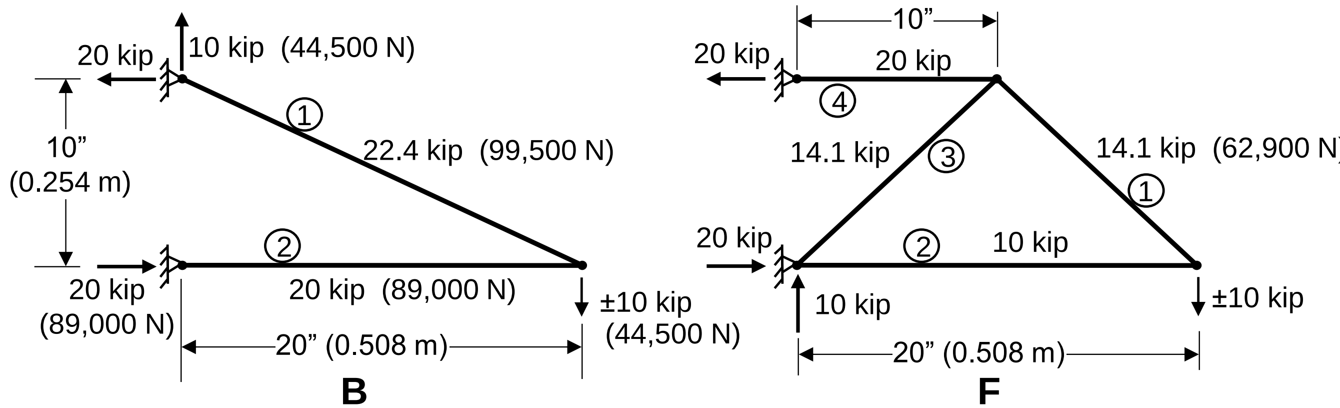
Example by Robert J. Heymans (deceased)

continued

Class Problem 6-1, as Part of the Truss Example 6-a

Steps 2 & 4 in Our Process

Select between options B and F for a reversible design ultimate applied load, P_U of 10 kip, using an aluminum alloy with Young's modulus, E, equal to 10 x 10^6 psi (69 x 10^3 MPa).



Process:

1. Calculate the design ultimate member loads using statics (already done; absolute values shown above).
2. Determine the cross-sectional area for each member that would make its design ultimate axial stress equal to its material's proportional limit, F_{pl}, the limit at which linear-elastic buckling methods still apply. Assume F_{pl} = 50 ksi (345 Mpa) (reasonable for 7075-T73 alloy).
   - This approach is a good idea for conceptual design. With it, we assume we'll later be able to find a cross section that will keep the member stable at that stress level. Selecting cross sections should be left for after arranging members.
3. Compare weight and total strain energy for the two options.

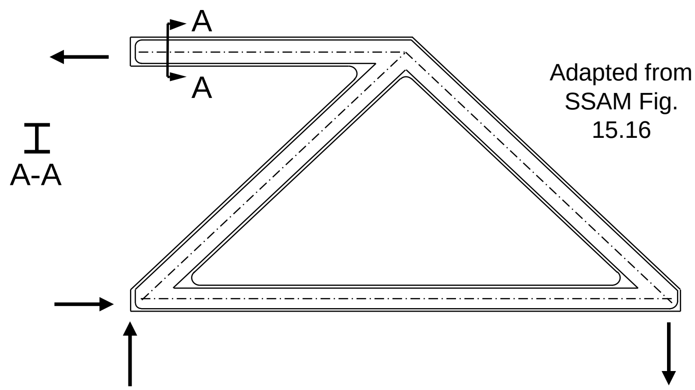
Suggestion: Build a spreadsheet.

This page intentionally left blank

Continuing the Truss Example Problem 6-a

Step 3: Select Material and Form of Construction

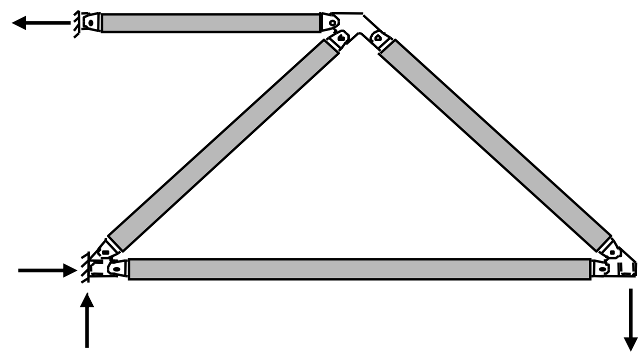
Aluminum alloy



Adapted from SSAM Fig. 15.16

- Integrally machined (usually less costly than assembling individual members)
- We can design such a truss to be machined in just two setups: front and back.

Carbon-fiber composite



Adapted from SSAM Fig. 15.17

- Bond cylindrical composite tubes to metal end fittings.
- Attach the end fittings to joint fittings with lug-and-clevis joints and spherical bearings (in the lugs) to avoid through-thickness stresses.

Which do you think would be lighter?

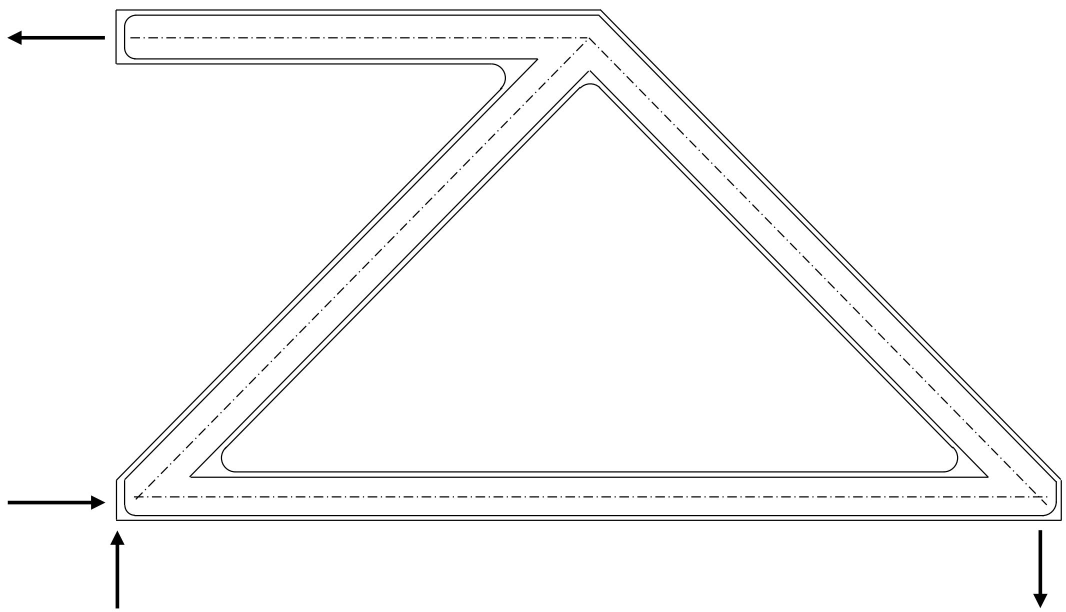
Which do you think would cost less to make?

continued

Truss Example 6-a, continued

But What's Wrong with This Design?

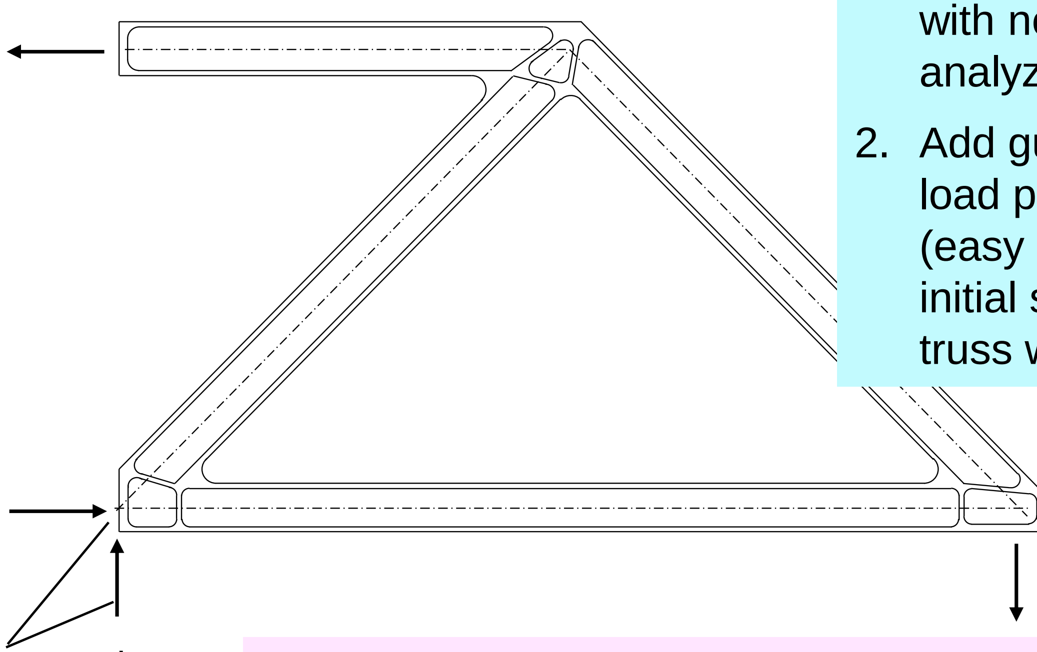
(Hint: follow the load paths.)



continued

Truss Example 6-a, continued

Add Gussets To Provide Direct Load Paths for Flanges



Options:

1. Analyze a bad design, with no gussets (hard to analyze).
2. Add gussets to make load paths more direct (easy to analyze for initial sizing—and the truss will weigh less).

And align truss members so that the shear reaction passes through the centerline intersection

Yes, this design doesn't provide pinned joints, so there will be additional stress as the result of end moments. But, for a configuration that is stable as a truss, allowing local stresses such as these to exceed the proportional limit at the design ultimate load would have little effect on a member's buckling load. So we can ignore this effect for preliminary sizing in this example.

continued

Truss Example 6-a, continued

Step 5: Preliminary Sizing of Truss Members

Now that we've found the best arrangement of members and decided on the form of construction, we're ready to size the members.

- We've already determined the required cross-sectional areas needed to keep the design ultimate stress equal to the proportional limit.
- But we need to find dimensions for the **I** sections that are producible and that will keep the members stable at the design ultimate compressive stress.
- Until we do this, we won't know whether we can actually build a truss that meets requirements at the weight estimated from the required cross-sectional areas.

Process: Generate equations that we can put into a spreadsheet, which will enable rapid iteration. Equations are needed for calculating …

- The cross-sectional moment of inertia needed to prevent column buckling
- Thicknesses for flanges and webs needed to prevent local buckling
- Section properties (area and moments of inertia) in terms of the dimensions of the cross section

Assumption: All nodes in this two-dimensional truss have out-of-plane support.

continued

Truss Example 6-a, continued

First, find I_{req}, the minimum moment of inertia needed to prevent column buckling.

From Eq. 4.13, the elastic buckling load is

$$P_{cr} = \frac{c_f \pi^2 E I}{L^2}$$

c_f = end-fixity coefficient
E = modulus of elasticity
I = moment of inertia
L = length

For a column with pinned ends, c_f = 1; for fully fixed ends, c_f = 4. For relatively long truss members that are welded together or integrally machined, a reasonable assumption is that c_f = 2.

Solve for I_{req} by setting P_{cr} equal to the design ultimate compressive load, P_{cu}:

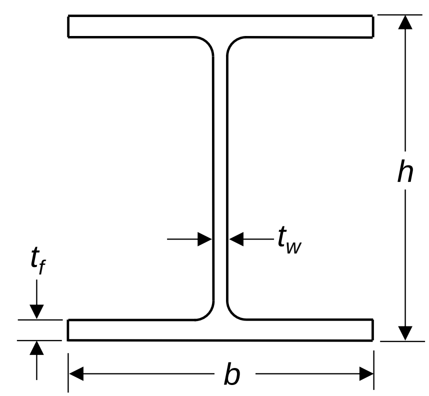
$$I_{req} = \frac{P_{cu} L^2}{c_f \pi^2 E} = \frac{P_{cu} L^2}{2\pi^2 E} \qquad \textbf{(Eq. 6.1)}$$

continued

Truss Example 6-a, continued

Next, derive equations for the flange and web thicknesses (in terms of *b* and *h*) needed to prevent local buckling at the design ultimate compressive load.

For simplicity, assume each flange is a long plate of width 0.5*b*, with pinned support along one edge and free along the other, and the web is a plate pinned along both edges. From Eq. 4.21a,



$$F_{cr-f} = 0.40E\left(\frac{t_f}{0.5b}\right)^2 \quad \text{(flange buckling stress)}$$

$$F_{cr-w} = 3.69E\left(\frac{t_w}{h-t_f}\right)^2 \quad \text{(web buckling stress)}$$

Substitute the proportional limit, F_{pl}, for $F_{cr\text{-}f}$ and $F_{cr\text{-}w}$, and compute the required flange and web thicknesses, $t_{f\text{-req}}$ and $t_{w\text{-req}}$:

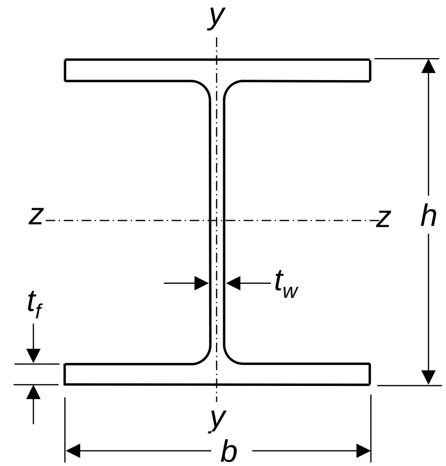
$$t_{f-\text{req}} = 0.5b\sqrt{\frac{F_{pl}}{0.40E}} \quad \textbf{(Eq. 6.2)} \qquad t_{w-\text{req}} = \left(h-t_f\right)\sqrt{\frac{F_{pl}}{3.69E}} \quad \textbf{(Eq. 6.3)}$$

In preliminary design, keep the analysis simple by allowing no local buckling at ultimate load. You can allow it during detail design, if necessary, as long as you account for interaction of local and column buckling (SSAM Chap. 8).

continued

Truss Example 6-a, continued

We also need equations for the section properties of an I-beam:



Area: $$A = 2bt_f + (h - 2t_f)t_w \quad \textbf{(Eq. 6.4)}$$

Moments of inertia: $$I_z = \frac{1}{12}\left[bh^3 - (b - t_w)(h - 2t_f)^3\right] \quad \textbf{(Eq. 6.5)}$$

$$I_y = \frac{1}{12}\left[2t_f b^3 + (h - 2t_f)t_w^3\right] \quad \textbf{(Eq. 6.6)}$$

continued

Truss Example 6-a, continued

Step 5 in Our Process, Sizing the Member Cross Sections

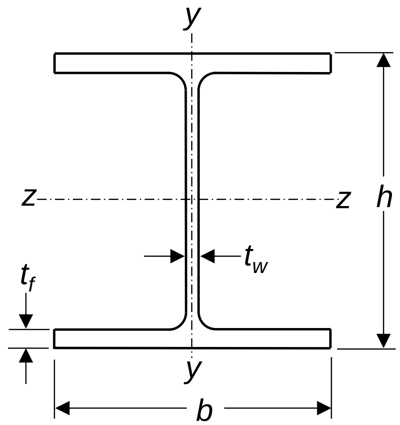
Finish the truss design by sizing the cross sections of the truss members (determine dimensions shown below). E = 10,000 ksi for all members.

Use Eqs. 6.1 – 6.6 provided in the preceding charts. Iterate the dimensions until achieving (1) the required cross-sectional areas, A_{req}, determined in Class Problem 6-1 and (2) the cross-sectional moments of inertia, I_{req}, needed to prevent column buckling (Eq. 6.1). We will generate a spreadsheet to do this.

Design ultimate compressive load

Units: inch, kip

Member	Length, L	P_{cu}	A_{req}
1	14.10	14.10	0.283
2	20.00	10.00	0.200
3	14.10	14.10	0.283
4	10.00	20.00	0.400



Keep in mind the following considerations for manufacturing:

- Flange thickness no less than 10% of dimension b/2 to avoid excessive flange bending (chatter) during machining, which would make it hard to hold dimensional tolerances and may lead to cracking (Ref. Walter Habicht in SSAM Chapter 20)
- Try to make flange width, b, the same for all members, slightly less (0.050 – 0.100”) than a standard plate thickness (0.75, 0.875, 1.00, 1.125, 1.25, 1.50, 1.75”, etc.)

continued

Truss Example Problem 6-a, continued

Sizing Truss Member Cross Sections

Determine the minimum moment of inertia to prevent column buckling:

$$I_{req} = \frac{P_{cu} L^2}{c_f \pi^2 E} = \frac{P_{cu} L^2}{2\pi^2 E}$$

Modulus, E =		10000	ksi
Member	P_{cu}, kip	L, in	I_{req}, in^4
1	14.1	14.1	0.0142
2	10.0	20.0	0.0203
3	14.1	14.1	0.0142
4	20.0	10.0	0.0101

continued

Truss Example Problem 6-a, continued

Sizing Truss Member Cross Sections, continued

Here's my preliminary design:

Units: inch

Member	A_{req}	I_{req}	h	b	$t_{f\text{-req}}$	$t_{w\text{-req}}$	t_f	t_w	A	I_z	I_y
1	0.283	0.0142	1.00	1.20	0.067	0.034	0.100	0.050	0.280	0.0509	0.0288
2	0.200	0.0203	1.00	1.20	0.067	0.034	0.070	0.040	0.202	0.0385	0.0202
3	0.283	0.0142	1.00	1.20	0.067	0.034	0.100	0.050	0.280	0.0509	0.0288
4	0.400	0.0101	1.00	1.20	0.067	0.034	0.140	0.090	0.401	0.0655	0.0404

From class problem 6-1

Calculated with Eq. 6.1

Inputs (guesses)

Calculated with Eqs. 6.2 & 6.3

Inputs (guesses)

Calculated with Eqs. 6.4 - 6.6

We now have an efficient preliminary truss design with optimal configuration and cross-sectional areas no larger than they need to be—hence minimum weight— and sections that should ensure stability at their design ultimate loads.

A more thorough assessment, accounting for end moments and dimensional tolerances, should be made in detail design.

continued

Truss Example Problem 6-a, summary

Estimated Truss Weight

A factor to account for joints (gussets, corner and fillet radii, and fasteners)

Weight density of aluminum alloy

$$w = 1.1(0.100)[0.280(14.1) + 0.202(20.0) + 0.280(14.1) + 0.401(10.0)]$$

$$= 1.75 \text{ lb}$$

(0.80 kg mass)

Cross-sectional area

Member length

A weight of 1.75 lb for a structure that can carry a load of 10,000 lb

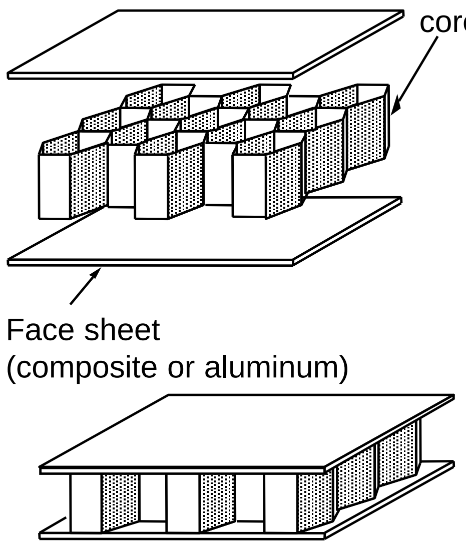
Key point of this example:

Designing efficient structures requires analysis.

Analysis is not something we do solely to assess someone else's design.

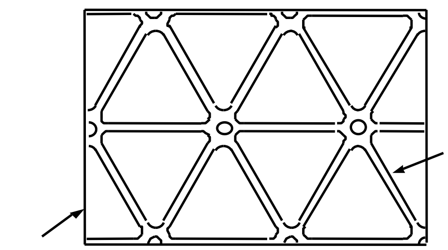
As a design engineer, either learn to do this sort of analysis or work closely with a good analyst.

Forms of Lightweight Panels and Shells

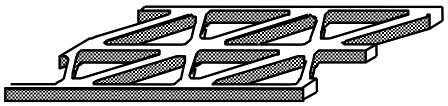


Sandwich panel **with honeycomb core**

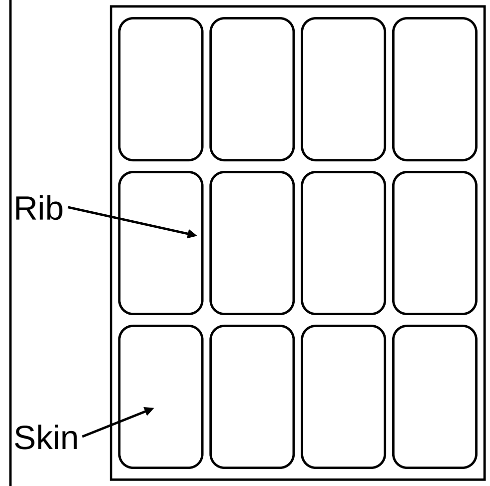
Adapted from SSAM Fig. 15.4



Isogrid **– pattern of equilateral triangles integrally machined from metal plate**



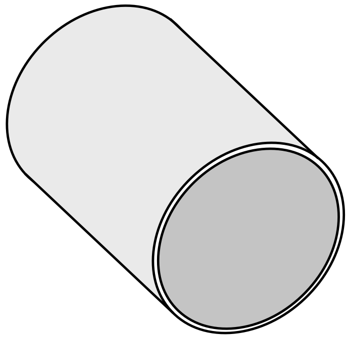
The triangle is the only stable polygon; skin not required



Orthogrid **("waffle") – rectangular pattern integrally machined from metal plate**

These forms of construction increase the ratio of bending stiffness to mass for a plate or shell, which increases buckling strength for shells under compression and improves efficiency for panels that support equipment and instruments.

Types of Primary Structures: Monocoque Cylinders



Adapted from
SSAM Table 15.8

A ***monocoque cylinder*** is an axi-symmetric shell without stiffeners or ring frames. Its strength is limited by its buckling stress. Loads must be introduced relatively uniformly over its cross section; concentrated loads can cause early local failure. To properly introduce loads, the mating structure must be either another monocoque cylinder or a stiff transition adapter.

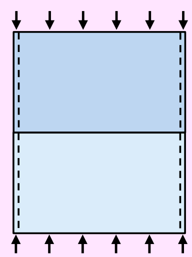
Forms of construction:	• Sheet metal, isogrid, or orthogrid with curvature formed by rolling • Sandwich construction, with segments fabricated with curvature • Typically two or three segments spliced to form a cylinder • Or one-piece forging, machined to final dimensions
Materials:	• Aluminum, magnesium, and titanium; carbon-fiber composite sandwich
Attachment methods:	• Mechanical fastening (bolts or rivets) or welding
Packaging and access when used as a flight vehicle structure:	• Hard to mount components on a curved surface; components typically mount on lateral decks instead • May require removable access panels
Other design considerations:	• Solid-skin cylinders have low buckling strength and are thus normally used only for structures loaded in tension only (e.g., pressurized tank) • Sandwich, isogrid, and orthogrid are more efficient for buckling

Avoid Concentrated Loads on Shells

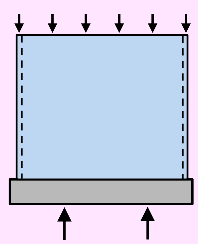
The key to using any shell or plate efficiently is to introduce loads uniformly rather than as concentrated loads.

– Regardless of whether the shell is solid skin, sandwich, isogrid, or orthogrid

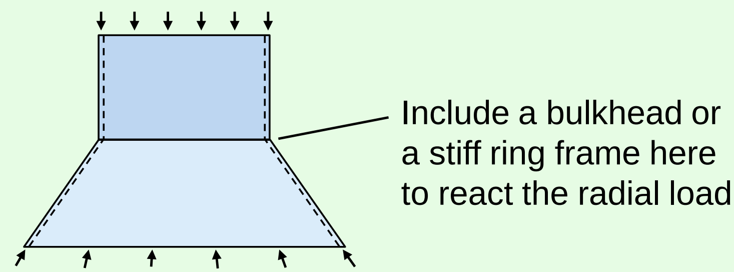
Examples:



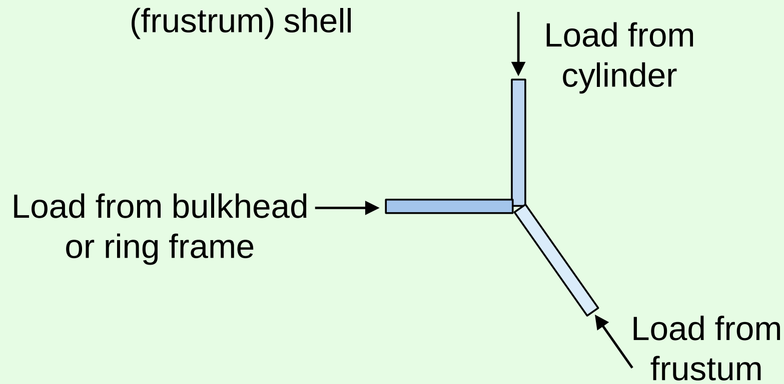
Monocoque cylinder mated to another monocoque cylinder



Or mated to a structure stiff enough to distribute loads uniformly



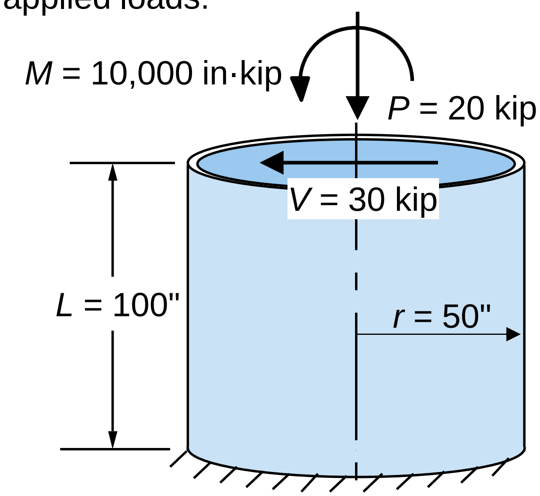
Monocoque cylinder mated to a monocoque conical (frustrum) shell



Example Problem 6-b:
Preliminary Sizing of an Isotropic Sandwich Cylinder

Example problem: Size a monocoque sandwich cylinder with 7075-T73 aluminum* face sheets for the geometry and requirements shown below.

Limit applied loads:



SSAM Example 15.3

Ultimate factor of safety, $FS_u = 1.25$

Stability factor, $S = 1.15$

Required bending stiffness, $k_{\theta req} = 1.5 \times 10^6$ in·kip/rad

Design ultimate loads at base:

$$P_u = 1.25(1.15)(20) = 28.8 \text{ kip}$$
$$V_u = 1.25(1.15)(30) = 43.1 \text{ kip}$$
$$M_u = 1.25(1.15)\left[10{,}000 + 30(100)\right] = 18{,}700 \text{ in} \cdot \text{kip}$$

Ultimate equivalent axial load for thin-wall cylinder:

$$P_{equ} = P_u + \frac{2M_u}{r} = 28.8 + \frac{2(18{,}700)}{50} = 777 \text{ kip}$$

The ***equivalent axial load*** is the axial load that causes the same normal stress as the design axial load and moment combined.

*The process demonstrated in this example is easily adapted for a sandwich with quasi-isotropic composite face sheets, such as [0, ±45, 90]$_{sym}$.

continued

Cylinder Example Problem 6-b, continued

In this example, the basic process is much like we used in the earlier truss example:

First, calculate the cross-sectional area needed to make the design ultimate compressive stress equal to the material's minimum proportional limit.

Then determine the geometry needed to make the structure stable when stressed at the proportional limit.

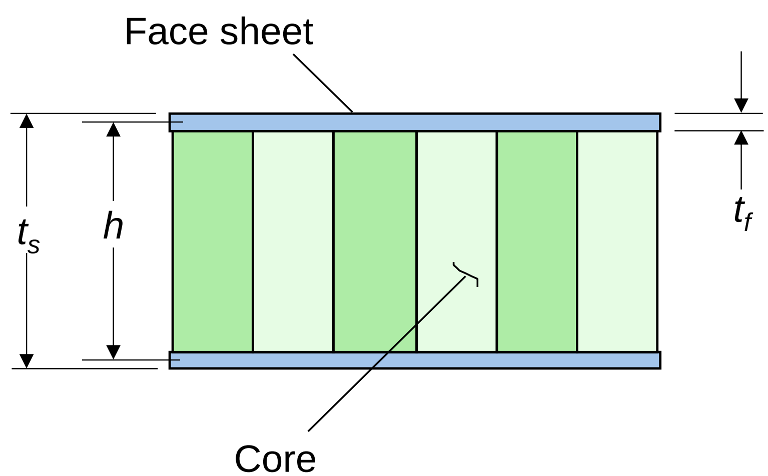
For this step, we will make use of the recognition that two thin-wall cylinders of the same diameter, Young's modulus, and radius of gyration (for unit width of shell wall) will buckle at the same stress.

(Credit goes to James R. McCandless (deceased) for this observation.)

continued

Cylinder Example Problem 6-b, continued

The important dimensions in sizing a sandwich cylinder are face-sheet thickness, t_f, and spacing, h



Adapted from SSAM Fig. 15.18

continued

Cylinder Example Problem 6-b, continued

Sizing process:

1. Determine the thickness of solid skin that will provide the necessary stiffness.
   (Each face sheet must be at least half this thick.)

Moment of inertia for thin-wall circular cross section,

$$I \approx \pi r^3 t$$

r = radius t = shell thickness L = length E = Young's modulus

Bending stiffness, $k_\theta = \dfrac{EI}{L} = \dfrac{E\pi r^3 t}{L}$

Thus, the required thickness for stiffness, $t_1 = \dfrac{k_{\theta req} L}{E\pi r^3}$

In our example:

For 7075-T73 aluminum, $E = 10.3 \times 10^3$ ksi (tension) and $E_c = 10.5 \times 10^3$ ksi (compression). For bending stiffness, use the average, 10.4×10^3 ksi.

$$t_1 = \frac{1.5 \times 10^6 (100)}{10.4 \times 10^3 \pi (50)^3} = 0.037 \text{ in.}$$

continued

Cylinder Example Problem 6-b, continued

2. Determine the solid-skin thickness that makes the design ultimate compressive stress equal to the material's minimum proportional limit.
   - Each face sheet must be at least half this thick for elastic buckling analysis to be dependable. For most aluminum alloys, the compressive proportional limit is between 70 - 80% of the compressive yield stress.

Cylinder's cross-sectional area, $A \approx 2\pi rt$

Design ultimate compressive stress, $f_{cu} = \dfrac{P_{equu}}{A} = \dfrac{P_{equ}}{2\pi rt}$

Thus, setting $f_{cu} = F_{pl}$, the thickness required for strength is …

$$t_2 = \frac{P_{equ}}{2\pi r F_{pl}}$$

r = radius
t = shell thickness
P_{equ} = design ultimate equivalent axial load
F_{pl} = minimum compressive proportional limit

For our example:

For 7075-T73 sheet, the A-basis allowable compressive yield stress, F_{cy}, is 54 ksi (Ref. 5a, MMPDS-08 Table 3.7.9.0(b_4). Based on the stress-strain curves shown in Ref. 5a Fig. 3.7.9.2.6(a), let's assume $F_{pl} = 0.75F_{cy}$ = 40.5 ksi.

$$t_2 = \frac{777}{2\pi(50)(40.5)} = 0.061 \text{ in.}$$

continued

Cylinder Example Problem 6-b, continued

3. Select a face-sheet thickness, t_f, greater than or equal to half the larger of t_1 and t_2. For metal face sheets, select a standard thickness (SSAM Table 20.3) to avoid the cost of a special order.

For our example:

From steps 1 and 2, t_1 = 0.038” t_2 = 0.061”

$$t_f \geq \frac{0.061}{2} = 0.0305 \text{ in.}$$

The next-larger standard thickness for sheet metal is 0.032 in. Thus,

$$t_f = 0.032 \text{ in.}$$

continued

Cylinder Example Problem 6-b, continued

4. Calculate the ultimate compressive stress, f_{cu}, for the selected face sheets.

$$f_{cu} = \frac{P_{equ}}{A} = \frac{P_{equ}}{2\pi r\left(2t_f\right)}$$

Example:

$$f_{cu} = \frac{777}{2\pi(50)(2)(0.032)} = 38.6 \text{ ksi}$$

continued

Cylinder Example Problem 6-b, continued

5. Calculate t_3, the thickness of solid skin that will buckle at the stress f_{cu}

Linear-elastic buckling stress for an isotropic monocoque cylinder, F_{cr}:

$$F_{cr} = 0.6\gamma \frac{E t_3}{r} \quad \text{(Eq. 4.18)}$$

E = compressive Young's modulus
t = wall thickness
r = radius of the cylinder

$$\gamma = 1 - 0.901\left(1 - e^{-\phi}\right)$$ for axial compression (Eq. 4.19)

$$\gamma = 1 - 0.731\left(1 - e^{-\phi}\right)$$ for applied moment (Eq. 4.20)

in either case, $$\phi = \frac{1}{16}\sqrt{\frac{r}{t_3}} \quad \text{(Eq. 4.21)}$$

For sizing, assume a value for t_3, calculate F_{cr}, then iterate the value for t_3 until $F_{cr} = f_{cu}$, the design ultimate compressive stress.

continued

Cylinder Example Problem 6-b, continued

5. Calculate t_3, the thickness of solid skin that will buckle at the stress f_{cu} (continued)

Example: Let's start with the assumption that t_3 = 0.500".

$$\phi = \frac{1}{16}\sqrt{\frac{r}{t_3}} = \frac{1}{16}\sqrt{\frac{50}{0.500}} = 0.625$$

Nearly all of the equivalent axial load is from applied moment, so

$$\gamma = 1 - 0.731\left(1 - e^{-\phi}\right) = 0.660$$

$$F_{cr} = 0.6\gamma\frac{E_c t_3}{r} = 0.6(0.660)\frac{10.4\times10^3(0.500)}{50} = 41.2\text{ ksi} > \overbrace{38.6\text{ ksi}}^{f_{cu}}$$

so iterate with a smaller assumed value of t_3 .

After such iteration, we determine that t_3 = 0.473", for which F_{cr} = 38.6 ksi.

continued

Cylinder Example Problem 6-b, continued

6. Calculate the face sheet spacing, *h*, that will give the sandwich shell the same radius of gyration per unit width as a solid skin of thickness t_3.

For a solid skin, for unit width:

Area, $A = t_3$ Moment of inertia, $I = \dfrac{t_3^3}{12}$

Radius of gyration (per Eq. 4.12), $\rho = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{t_3^3/12}{t_3}} = \dfrac{t_3}{\sqrt{12}}$

For a sandwich, $A = 2t_f$ $I \approx 2t_f\left(\dfrac{h}{2}\right)^2$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{t_f h^2/2}{2t_f}} = \frac{h}{2}$$

t_f = face sheet thickness
h = face sheet spacing

Equating the radii of gyration,

$$h = \frac{2t_3}{\sqrt{12}} = 0.5774t_3$$

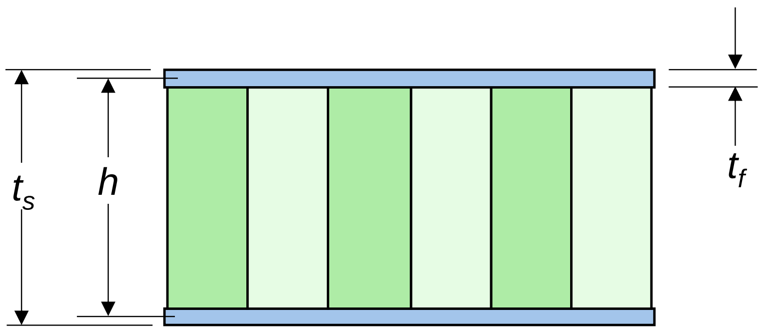
Example:

h = 0.5774(0.473) = 0.273 in.

continued

Cylinder Example Problem 6-b, conclusion

7. Calculate the sandwich shell's required thickness, t_s



$$t_s = h + t_f$$

Example:

$t_s = 0.273 + 0.032 = 0.305$ in.

This method is intended for preliminary design only. It compares well with a more complex, empirically based method in NASA's Astronautic Structures Manual (Ref. 9) when the core's shear modulus is at least 0.003 times the Young's modulus of the face sheets and when the cylinder's radius is at least 100 times the sandwich thickness.

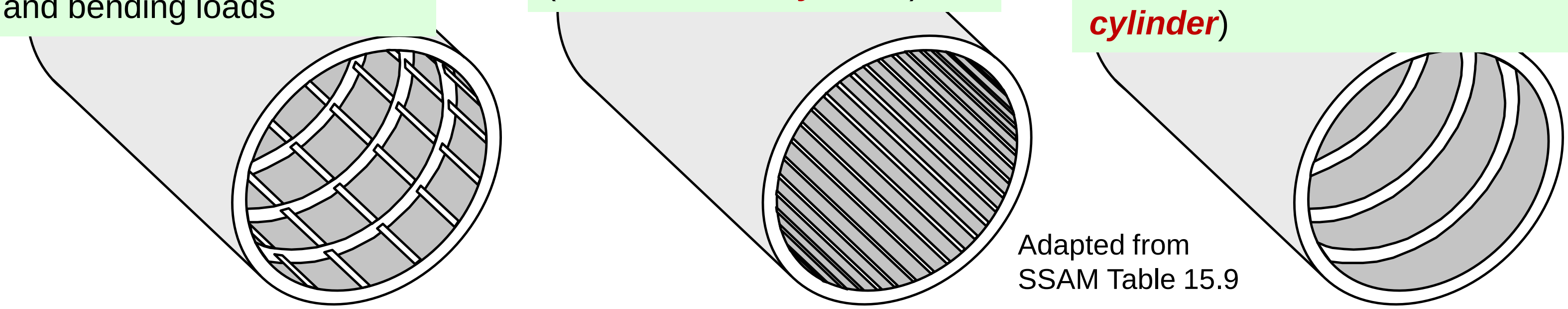
Other Types of Cylindrical Primary Structures

Two types of ***semi-monocoque cylinders***:

Skin-stringer cylinder—skin (sheet metal) typically designed to buckle; stringers carry most of axial and bending loads

Lightweight, closely spaced stiffeners, which increase the skin's buckling strength (***stiffened-skin cylinder***)

Intermediate ring frames to increase the skin's buckling strength, but no axial members (I don't know of any name other than ***semi-monocoque cylinder***)



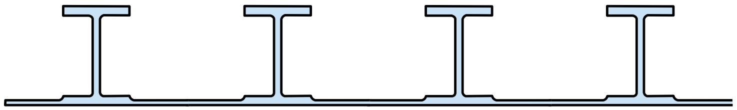
Adapted from SSAM Table 15.9

The term ***skin*** typically refers to sheet metal.

A ***stringer*** is a longitudinal member intended to accept concentrated loads.

Similar to a stringer, a ***longeron*** is typically larger and fewer in number.

A ***stiffener*** is a longitudinal member whose function is to increase a shell's buckling strength.



Stiffened skin with integrally machined I-sections (machined flat and then rolled to make cylindrical sections)

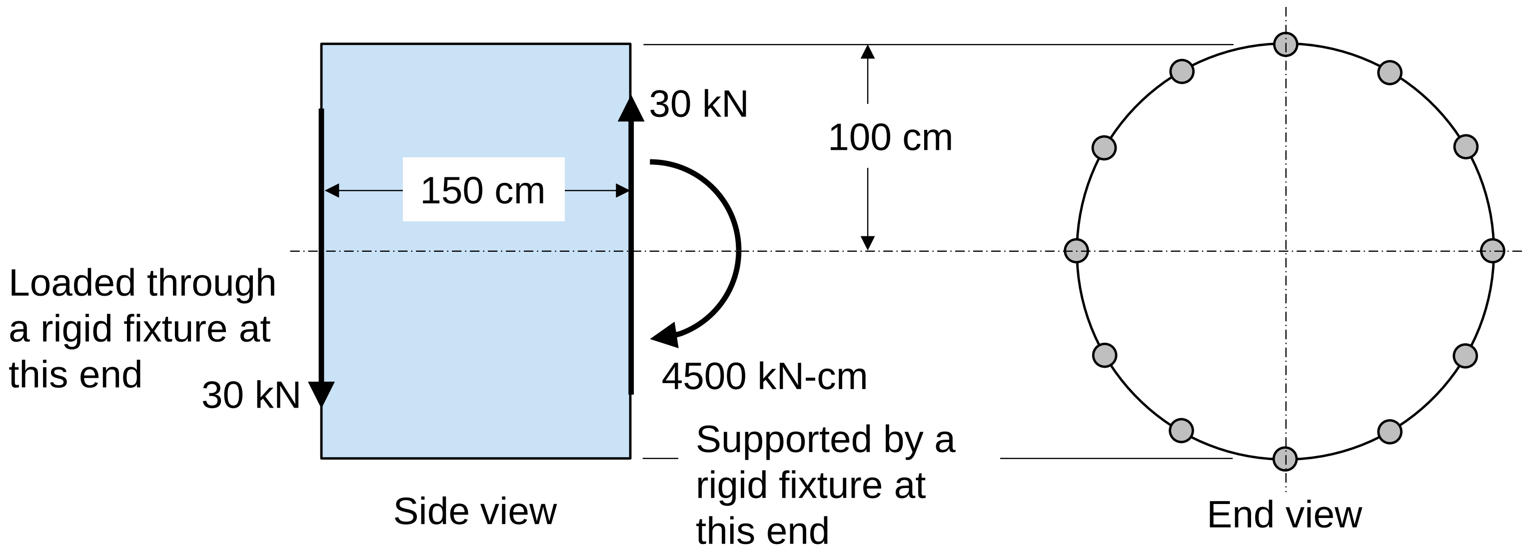
Orthogrid and closed isogrid can be used for stiffened skin as well

Example Problem 6-c: How a Skin-Stringer Cylinder Carries Loads

Problem statement: A cantilevered skin-stringer cylinder of 100 cm (39.37”) radius and 150 cm length, with 12 evenly spaced stringers, is under a design ultimate applied lateral load of 30 kN (6744 lb).

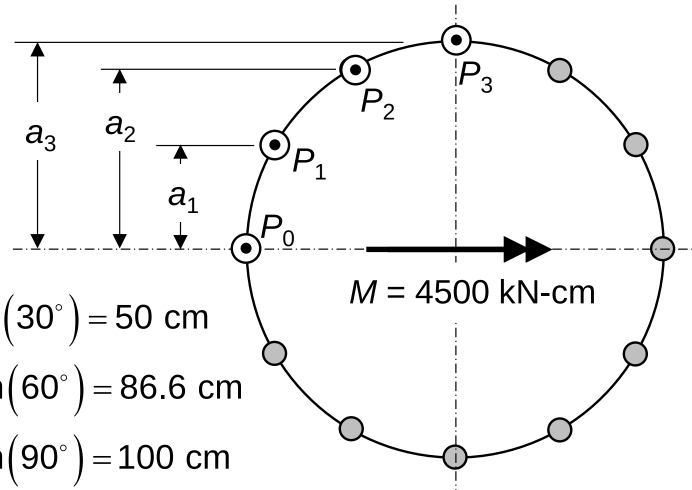
Assuming the skin carries shear only*, calculate the stringer loads at the base of the cylinder and the shear loads in the panels.

*Skin panels take normal loads as well, limited by buckling, but we can ignore those loads for preliminary design.



continued

Example Problem 6-c, continued



$a_1 = 100\sin(30^\circ) = 50$ cm

$a_2 = 100\sin(60^\circ) = 86.6$ cm

$a_3 = 100\sin(90^\circ) = 100$ cm

Step 1. Calculate stringer loads, $P_0 - P_3$, at the cylinder base

Stringer 0 is on the neutral axis, so $P_0 = 0$

4 stringers at distance a_1

2 stringers at distance a_3

$$M = 4500 = 4a_1P_1 + 4a_2P_2 + 2a_3P_3$$

If all the stringer cross-sectional areas are the same, with linear theory we can assume loads are proportional to the distance from the neutral axis:

$$4500 = 4(50)P_1 + 4(86.6)\left(\frac{86.6}{50}\right)P_1 + 2(100)\left(\frac{100}{50}\right)P_1$$

$$= 1200P_1$$

$$P_1 = \frac{4500}{1200} = 3.75 \text{ kN}$$

$$P_2 = \left(\frac{86.6}{50}\right)P_1 = 6.50 \text{ kN}$$

$$P_3 = \left(\frac{100}{50}\right)P_1 = 7.50 \text{ kN}$$

continued

Example Problem 6-c, continued

Step 2. Calculate the panel shear loads needed to generate the stringer loads at the base, recognizing that the stringer loads on the opposite end of the cylinder are zero.

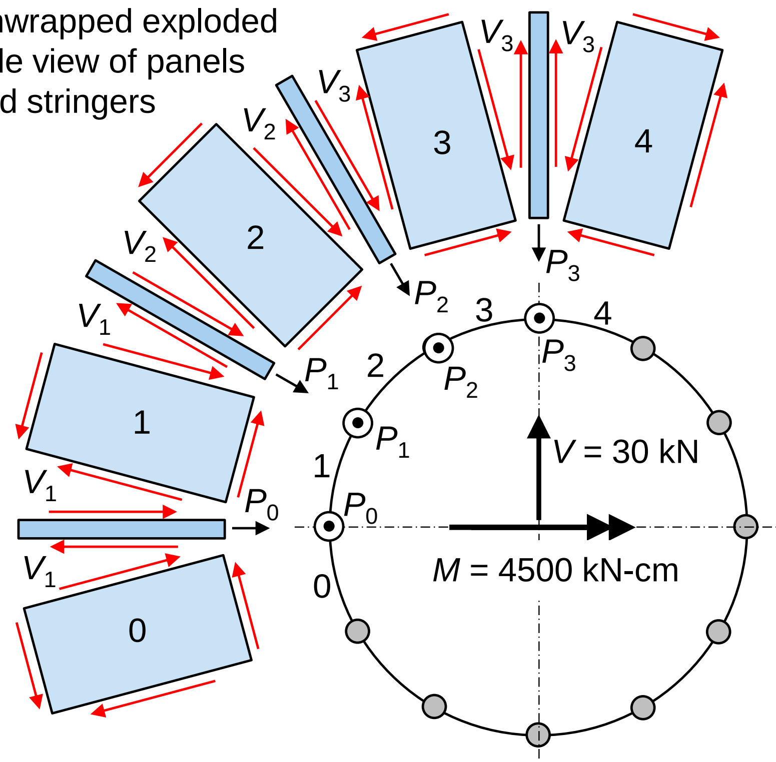
Start at stringer 3 (top): For equilibrium, …

$$P_3 = 2V_3 \quad ; \quad V_3 = \frac{7.50}{2} = 3.75 \text{ kN}$$

Now move down:

$$P_2 + V_3 = V_2 \quad ; \quad V_2 = 6.50 + 3.75 = 10.25 \text{ kN}$$

$$P_1 + V_2 = V_1 \quad ; \quad V_1 = 3.75 + 10.25 = 14.00 \text{ kN}$$

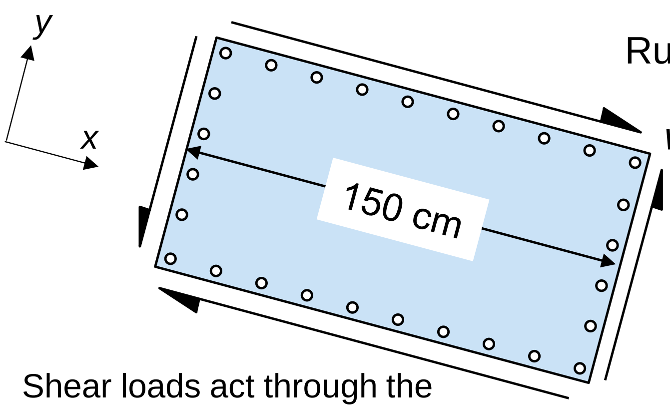


Panel 1 has the highest shear load.

continued

Example Problem 6-c, conclusion

In the quadrant we've been analyzing, panel 1 has the highest shear load.



Shear loads act through the fasteners but are shown this way for clarity.

A ***running load*** is force divided by distance, e.g., lb/in.

Running shear load,

$$w_{xy} = \frac{14\text{ kN}}{150\text{ cm}} = 93.3\text{ N/cm} \;\; (53.3\text{ lb/in})$$

If the rivets attaching this panel to the ring frames and stringers are spaced 10 cm and are bearing critical, we can estimate the rivet shear load as the running load times the spacing:

$$P_x = P_y \approx 93.3(10) = 933\text{ N} \;\; (210\text{ lb})$$

Rivets attaching skin to stringers

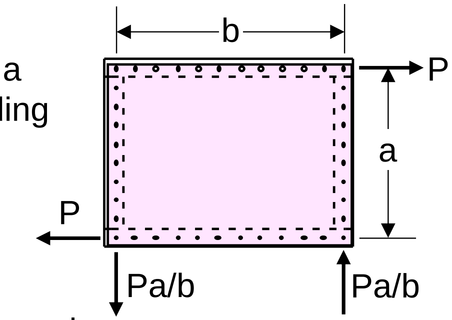
Rivets attaching skin to ring frames

The rivets at the corners see both loads, P_x and P_y, with a resultant shear load of …

$$P_s \approx \sqrt{(933)^2 + (933)^2} = 1310\text{ N} \;\; (295\text{ lb})$$

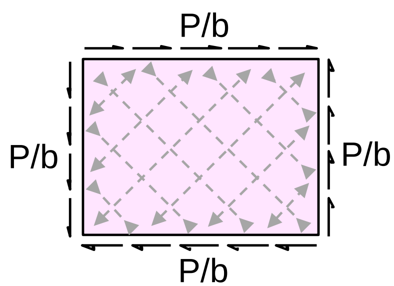
Diagonal Tension in a Buckled Shear Panel

Rectangular skin panel fastened to a frame, subjected to pure shear loading

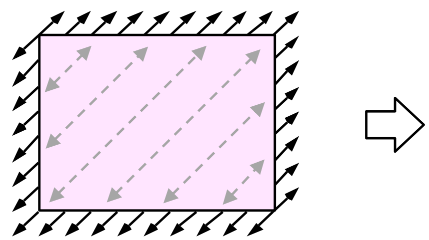


Adapted from SSAM Fig. 15.3

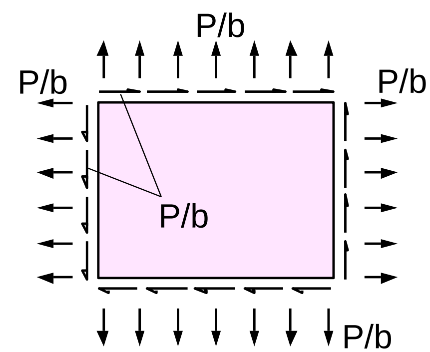
Running loads (lb/in or N/cm) acting on a skin panel:



Nonbuckled panel in pure shear: Shear consists of tension and compression at ±45°. As the load increases, the compression can cause the panel to buckle.



After the panel buckles, it transfers all additional applied shear load by tension at ±45° (***diagonal tension***).



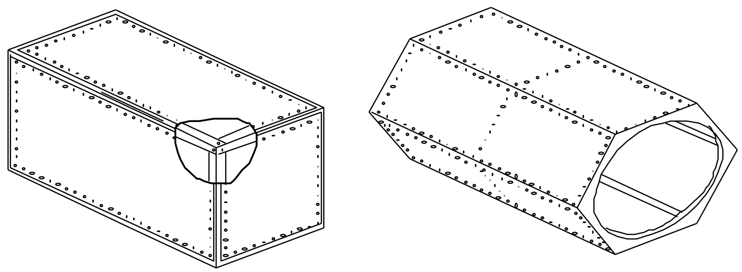
Diagonal tension resolves into equal shear and normal running loads on the panel edges. The fasteners and frame members must be designed to withstand these loads.

(See Ref. 8 (Bruhn) for methods of analysis)

Buckled Shear Panels

Image credit: Jodoin, A., et al. "Diagonal Tension in Fibre-Metal Laminates." ICAS 2002 Congress.

Non-cylindrical Skin-Frame or Panel-Frame Structures



Adapted from SSAM Table 15.7

Can be used with skin, similar in construction to cylindrical skin-stringer, or with panels that are stiff and strong in bending in order to accommodate mounted equipment. The structure's ends must be either closed or stabilized with stiff frames.

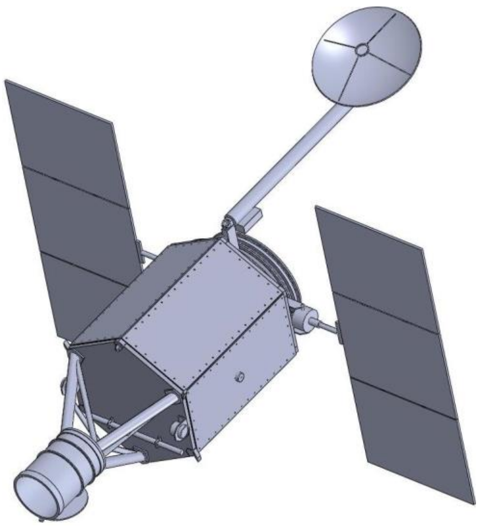
Forms of construction:	• Framework made of extruded, machined, or formed members • Panels made of sheet metal, sandwich construction, isogrid, or orthogrid. Sheet metal would be used only if there is no mounted equipment.
Materials:	• Aluminum for skin; aluminum or carbon-fiber composite for sandwich face sheets
Attachment methods:	• Mechanical fasteners (rivets often used with skin, bolts otherwise)
Packaging and access:	• Equipment mounted inside or outside • Interior access requires removable panels or open ends

Again: Avoid Concentrated Loads on Shells

As noted previously for monocoque cylindrical shells, the key to using any shell or plate efficiently is to introduce loads uniformly rather than as concentrated loads.

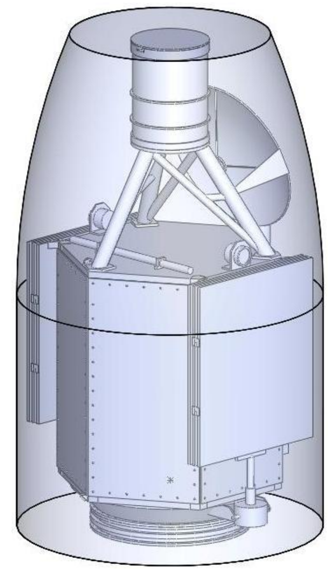
Wherever a concentrated load is introduced, add a frame member (e.g., stringer or longeron).

Example: FireSat Bus Structure



Reference 26, *Space Mission Analysis and Design*, uses the hypothetical FireSat as an example of how to design a space mission. The FireSat mission: detect forest fires in the U.S. from space.

Reference 1 (SSAM) and Instar's course "Space Mission Structures" (SMS) follow up by showing an example of how to develop a FireSat configuration as an example (written in SMAD by John Leritz).

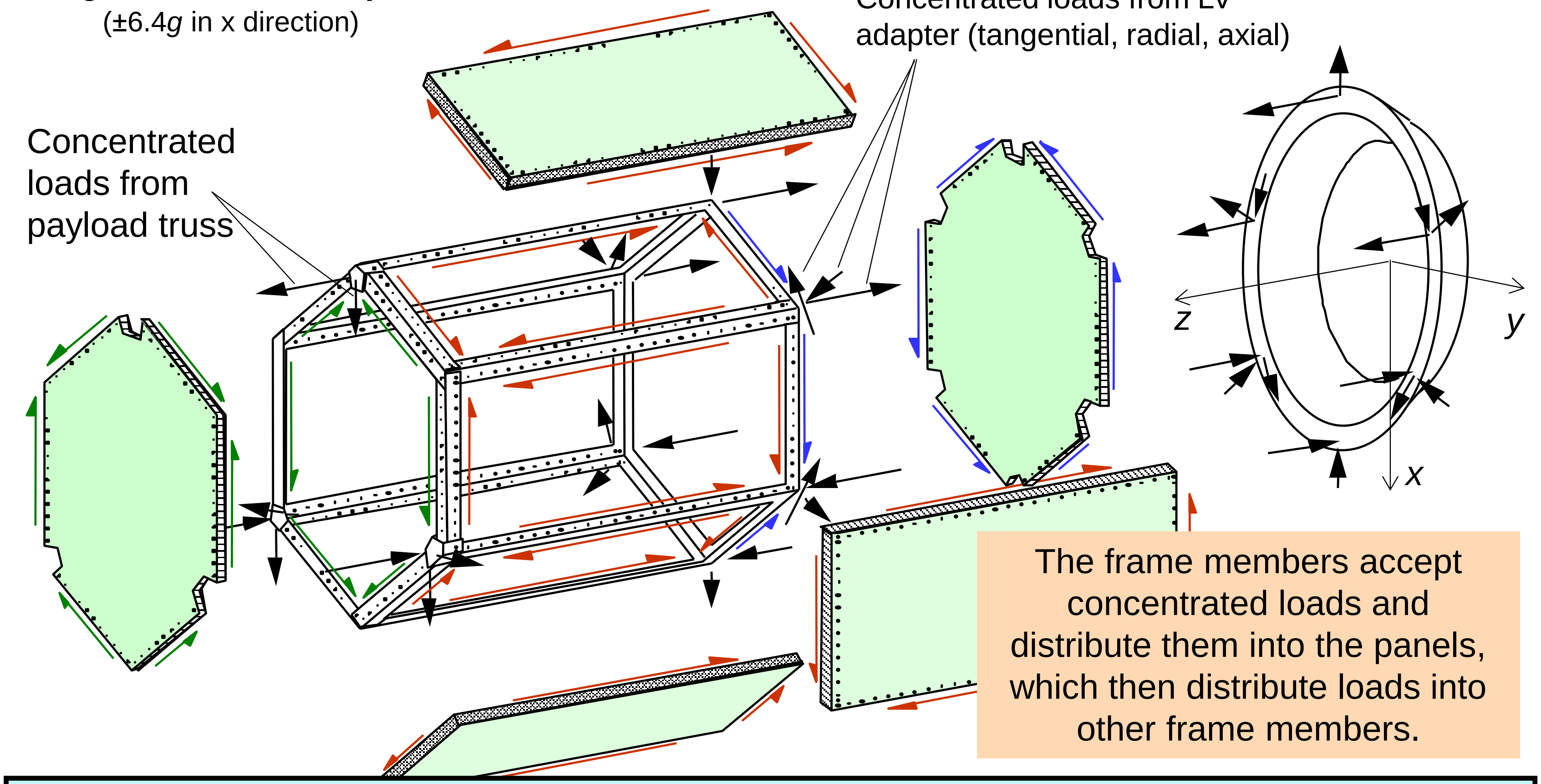


Here we'll take a closer look at FireSat's bus structure.

Driving load case: Sudden release and drop of the launch vehicle (Pegasus) from the aircraft—over 6 *g* equivalent lateral quasi-static load.

FireSat Example, continued:
Establishing Efficient Load Paths

Pegasus Launch Drop Load Case
(±6.4g in x direction)



The frame members accept concentrated loads and distribute them into the panels, which then distribute loads into other frame members.

Chase loads through the structure to build understanding and ensure efficient load paths.

Methods of Attachment

The three main methods of joining structural parts are …

- Welding
- Bonding
- Fastening

Let's compare pros and cons …

Welding is Sometimes an Option, Depending on the Materials

Advantages and Applications	Disadvantages and Limitations
• Can achieve direct load paths and high stiffness • Can be economical – Often used for ground support equipment and test fixtures made of structural steel • Dependable for sealing pressurized structures • Can be designed to ensure failure is ductile so that load spreads before rupture (depends on materials and weld geometry)	• Can't disassemble parts nondestructively • Limited to identical or compatible alloys; some alloys can't be welded • Heat from welding reduces the strength of certain alloys, such as aluminum • Can be difficult to control quality and avoid cracks – May require costly development program and proof testing • Often hard to maintain dimensions because of warping from residual stresses – May require stress relief in an oven – Critical interfaces machined after stress relief

Bonding is Often Best for Brittle Materials in Shear Joints

Advantages and Applications	Disadvantages and Limitations
• Lightweight • Spreads loads out with lower stress concentrations than for fasteners – Good for joining brittle materials, including composites • Can joint parts made of different materials • Results in smooth surfaces • Adds structural damping	• Can't disassemble parts nondestructively • Joint strength is extremely sensitive to variation in manufacturing processes – May require costly development program and proof testing • Good in shear only; weak in tension (peel) – Can be difficult to design a bonded joint that introduces low peel stress • Usually can't develop full strength of attached members • Can have narrower temperature limits than attached members

Fastening is Most Versatile but Has Drawbacks

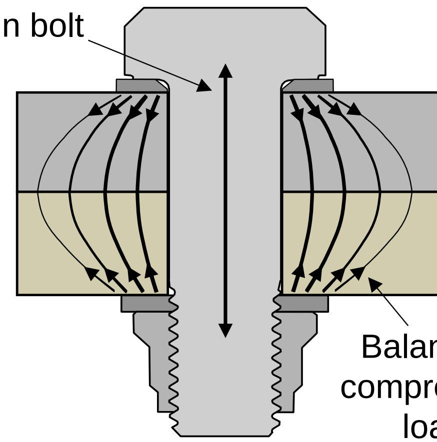
Advantages and Applications	Disadvantages and Limitations
• Use of threaded fasteners allows disassembly • Can join parts made of different materials • Joints can be designed to be at least as strong as the attached members if the members are made of ductile metal • There are many types of fasteners with quality controlled by specifications • Joints can be very dependable if designed well • Assembly can be in almost any facility without expensive tools or processes • Adds structural damping	• Adds weight • Can require costly tooling and labor • Hard to avoid loss of stiffness and nonlinear stiffness (tension vs. compression) • Introduces stress concentrations near fasteners, which can cause strength failure in brittle materials and fatigue failure in ductile materials • Assembly and disassembly can be time consuming • Easy to misuse; many potential problems if joints are not properly designed

Bolted Joints:

The Importance of Preload

Tightening the nut or bolt creates *preload*: a tensile load in the bolt and an equal clamp load between fittings. It's the clamp load that's important.

- Creates a new load path and makes joints stiff
- Keeps bolts and ***fittings*** (joint members) from failing in fatigue
  - Bolt load doesn't change much under applied tensile loads.
  - Stress in fittings doesn't change much near bolts, where stress concentrates.
- Prevents joints from slipping back and forth within clearance holes
  - See next page.



In design, we want to ensure preload is high enough to prevent *gapping* (total loss of compressive load between fittings) and slipping in shear within clearance holes under the highest expected (limit) service or mission loads.

This slide and those that follow were adapted from my course "Design and Analysis of Bolted Joints" (DABJ), Ref. 33

Preventing Cyclic Slip within Bolt Clearance Holes

Why is cyclic slip bad?

– Causes preload loss, even with most locking features. If any lateral motion causes threads to slip relative to each other, given the clearance between threads, the nut or screw tends to rotate down the helical angle of the threads.

– Can lead to ***fretting*** (corrosion resulting from breakdown of protective oxides on surfaces from rubbing).

– Can lead to detrimental permanent deformation in alignment-critical assemblies.

– Causes nonlinearity that makes structural behavior hard to predict.

– Can cause threaded fasteners to fail in fatigue, especially if threads are in or near the shear plane, which is the case for screws going into threaded inserts, and especially for flush-head fasteners as a result of prying load under the head.

How can we prevent cyclic slip or at least minimize its effects?

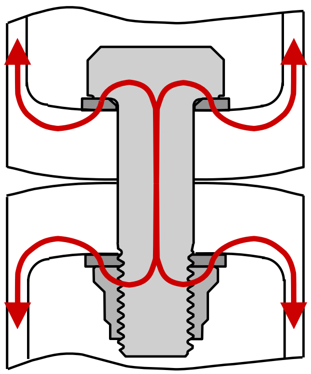
– Ensure preload is high enough for friction to carry the shear load dependably. (Ref. 2, NASA-STD-5020B provides guidance.)

– Use a dedicated feature such as a shear pin to carry the shear load with little or no slip.

– Or, as a minimum, reduce hole clearance to a few thousandths of an inch. (This strategy should be used only when threads are out of and away from the shear plane and there will be relatively few loading cycles, such as is often the case for single-mission spaceflight structures.

How Preload Increases Fatigue Life of Bolts and Fittings

Joint under applied tensile load—main load path shown with bold arrows:

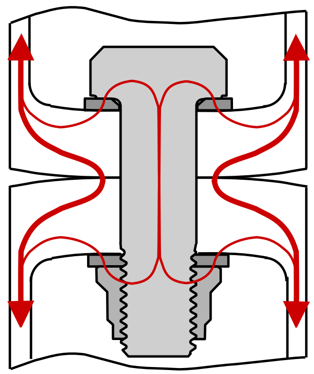


No preload, full gapping

All of the applied load must be carried by the bolt.

Fitting end pads bend like plates, and stress concentrates at bolt hole.

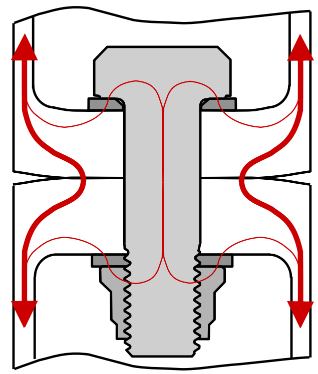
Bolt and end pads subject to fatigue failure from cyclic loading.



Just enough preload to prevent full gapping

The bolt sees a significant increase in load.

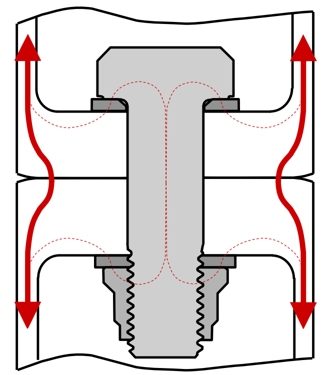
Fitting end pads still bend but have less stress than if fully gapped.



Higher preload, less partial gapping

The bolt sees a reduced increase in load.

Fitting end pads are stressed less.



Very high preload, virtually no partial gapping

The bolt load changes hardly at all.

Fitting end pads have virtually no bending or tensile stress.

Bolted Joints:
Can Preload Be Too High?

Potential outcome of high preload	Discussion
Yielding occurs, bolts break, or threads strip while torquing, during which the bolt sees combined tension and torsion.	Minor yielding is not normally a concern. Using up much of the material's elongation, breaking bolts, or stripping threads while torquing is a valid concern. Reduce installation torque if any of these is the case.
The clamped parts are crushed by preload alone or by combination of preload and applied tensile load.	Valid concern only when clamping brittle nonmetallic materials (e.g., epoxy washer).
One or more materials in the fastening system yields under the design yield tensile load combined with preload.	Such yielding is not detrimental for most joints that are designed not to gap at limit load. (Addressed in my DABJ course.)
The fastening system has lower ultimate strength under applied loads, i.e., the joint can carry less applied load as a result of preload.	Valid concern for applied tension when clamping nonmetallic materials with low modulus of elasticity (e.g., rubber gasket used to seal a joint). Rarely a concern for all-metallic joints, for which tests show the ultimate strength is the same, with and without preload, even when preload is extremely high. In a well-designed all-metallic joint, gapping occurs before rupture. (Addressed in my DABJ course.)
The bolt fails or is weakened from stress-corrosion cracking*.	Even with corrosion-resistant materials, this is a valid concern for assemblies that will be exposed to corrosive environments on Earth for many years or decades (not typically the case for space and launch vehicles).

****Stress-corrosion cracking*** is the growth of a crack starting at a corrosion pit in a material under sustained tensile stress, often leading to brittle fracture.

Torque-Preload Relationship: For Given Bolt Diameter and Materials, Preload Tends to be Nearly Proportional to Torque

$$T = KDP_{pi} \quad \text{or} \quad P_{pi} = T/KD$$

T = effective torque = total torque minus running (resisting) torque from locking feature
D = bolt nominal diameter
P_{pi} = initial preload (subject to relaxation and thermal effects)
K = nut factor, which accounts for friction, geometry, and other variables; derived from test data

With typical material combinations for fastening hardware, the nut factor, *K*, when installed without lubrication is usually between 0.20 and 0.30 but can be higher than 0.50.

With lubrication, *K* tends to be between 0.1 and 0.25 depending on lubricant and lubrication process.

- We can get higher preload with lubrication. High *K* leads to high torsional stress in the bolt, which limits the achievable preload.
- Acceptable lubricants for space use: molybdenum-disulphide dry film and Braycote 601EF and 602EF grease

Account for expected preload variation in design to ensure the joint won't gap or suffer cyclic slip under service loads.

NASA-STD-5020B (Ref. 2) specifies requirements and provides guidance on how to account for preload uncertainty.

Bolted Joints:
Fastener Selection

Use high-strength bolts for structural applications.

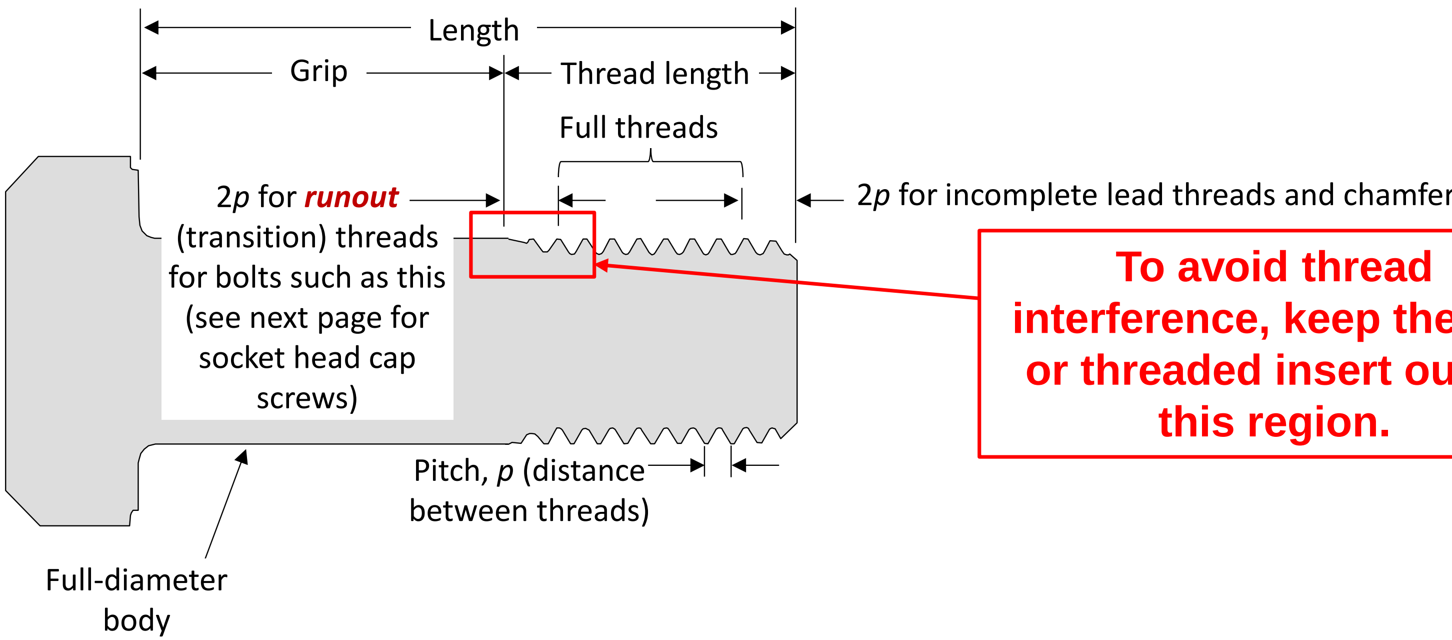
- Example: The most commonly used fastener in the space industry is the socket head cap screw, controlled by specifications such as NAS1351 (fine thread) and NAS1352 (coarse thread).
  - Use the A-286 (“heat resistant steel”) version (160 ksi ultimate tensile strength) for any fastener that can see significant applied load
  - The “corrosion resistant steel” version has very low strength, especially yield strength

Use larger-diameter bolts than you think you need if you have room for them and can tolerate the weight impact.

- Can preload them higher
- Will result in fewer problems

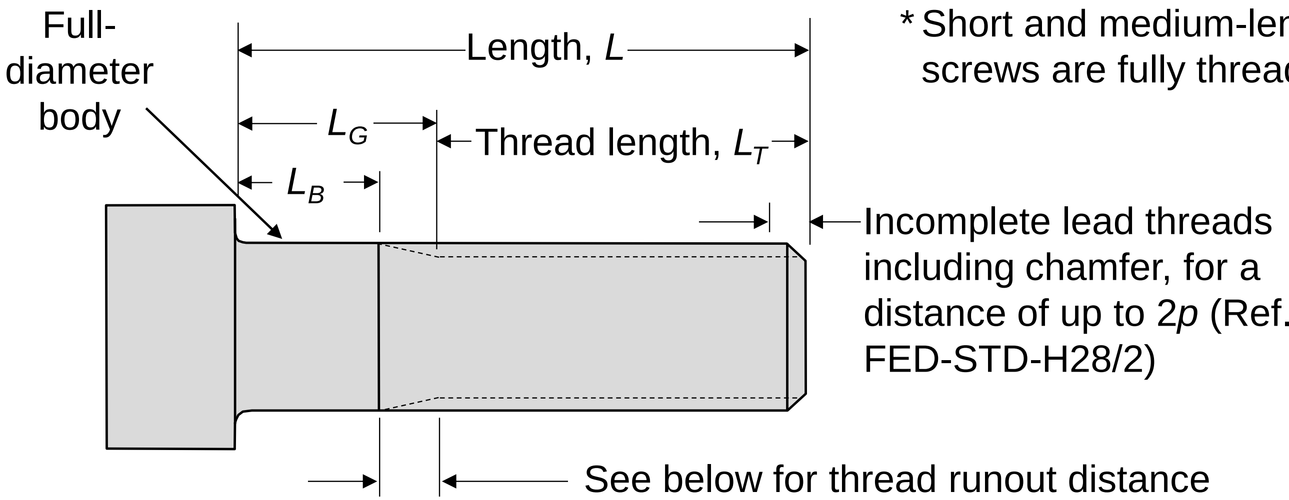
Bolted Joints:

Avoid Thread Interference that Can Prevent Preload



To avoid thread interference, keep the nut or threaded insert out of this region.

Dimensions for Relatively Long* Socket Head Cap Screws



* Short and medium-length cap screws are fully threaded.

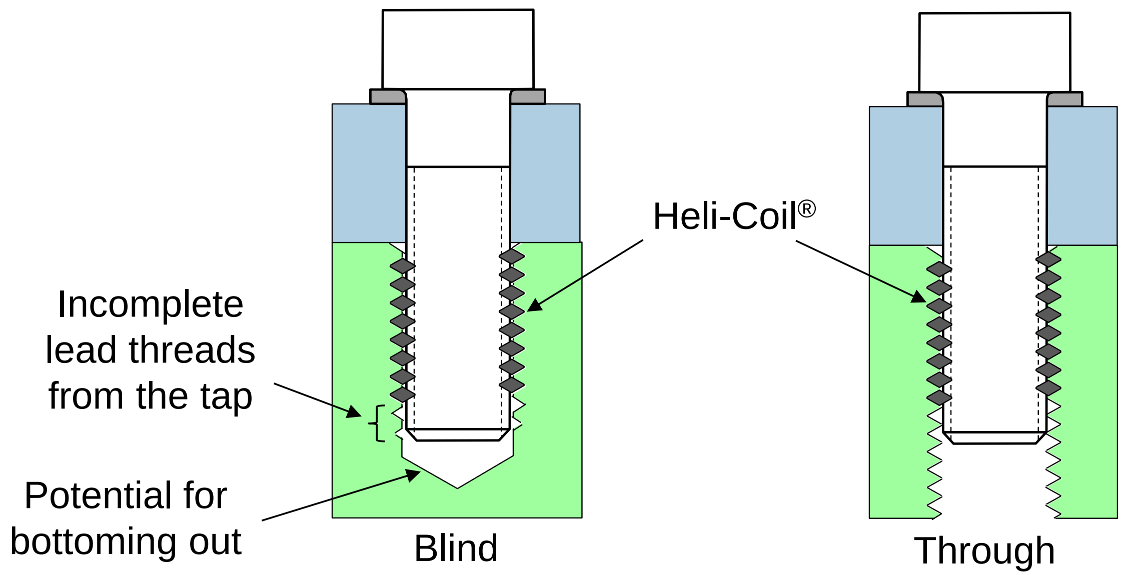
L_B is minimum distance from under-head surface to the end of the full-diameter body
L_G is maximum distance from under-head surface to the first compete thread

Per ASME B18.3, Table 1C, which provides dimensions L_G and L_B , Note 1:

$L_G - L_B$ = 5 times the pitch of the coarse thread (UNRC) for the applicable bolt diameter

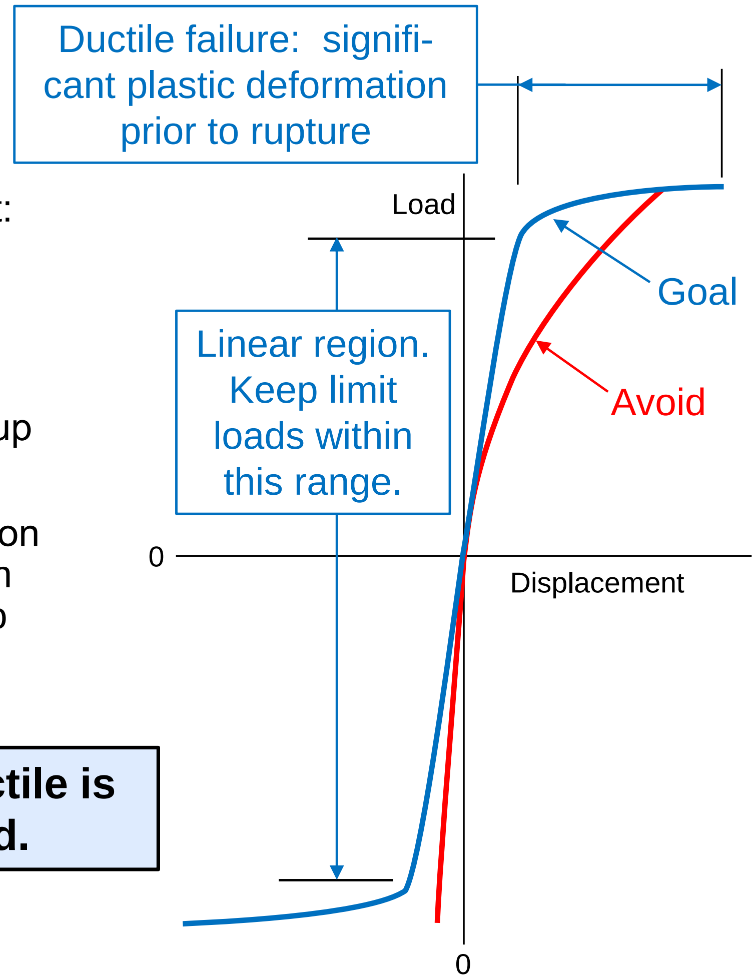
Bolted Joints:
Fixed Fasteners (with Tapped Holes or Threaded Inserts)

If a blind hole is not necessary, use a through hole to avoid potential interference with the tap's incomplete lead threads or the bottom of the hole.



Design Joints To Have High, Linear Stiffness but Ductile Failure

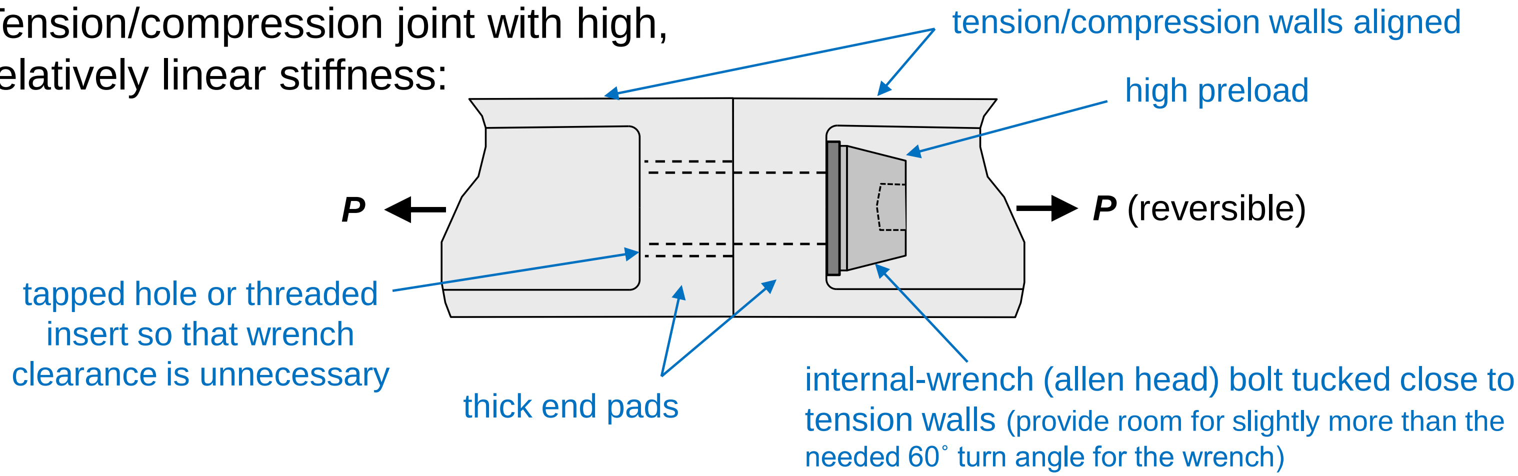
- High stiffness: keeps the structure's natural frequencies high, which usually helps avoid high dynamic loads
- Linear relationship between load and displacement: makes the structure more predictable with linear-elastic analysis (the vast majority of structural analyses, especially loads analysis)
- But we don't want the joint to be linear all the way up to rupture.
  - If there's an unanticipated high load or distribution of load (or an energy-limited load), ductility often allows loads to redistribute (or allows the joint to absorb energy) before anything ruptures.



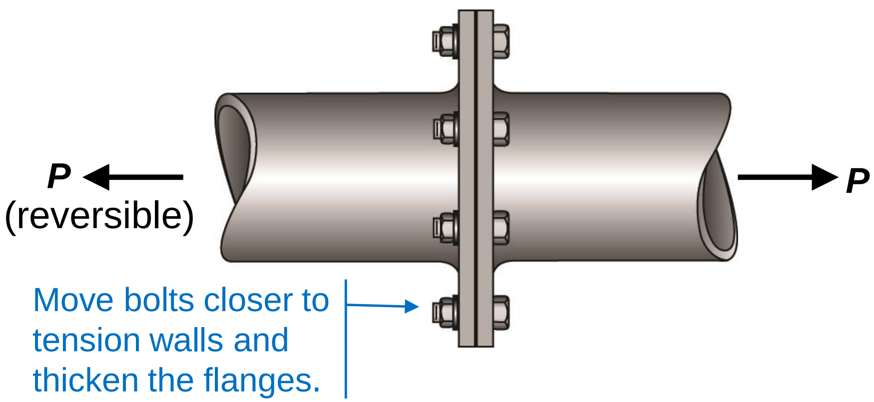
Designing to ensure that failure is ductile is very important but often neglected.

Good and Bad Examples of Bolted Joint Designs

Tension/compression joint with high, relatively linear stiffness:



Joint that is stiff in compression and flexible in tension (avoid):



- The flanges, with bolts far from the tension walls, will bend and pry on the bolts.
- Nonlinear stiffness makes it difficult to predict dynamic response or load distribution within the structural assembly.

Design Joints to Have Ductile Failure

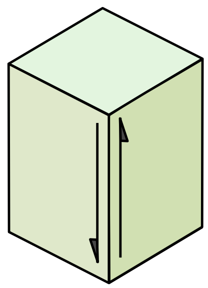
So that loads can redistribute before rupture occurs

We can't do this with bonded joints, but we can for bolted and welded joints.

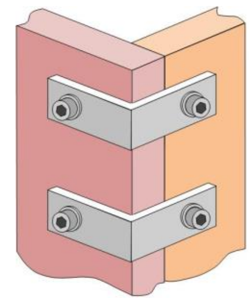
- Bolted tension joints:
  - Use a bolt that is designed for tensile use rather than one that has a thin head. You want failure to occur at the threaded cross section before the head fails.
  - If the bolt has a full-diameter body, as opposed to being fully threaded, use a bolt made of a material whose yield strength is no more than about 75% of its ultimate strength so that the full-diameter body will yield before rupture occurs at the threads, where the cross-sectional area is smaller.
    - Good choice: A-286 treated to 160 ksi ultimate strength
    - Avoid: titanium treated to 160 ksi ultimate strength for tension joints
  - Use the bolt with a nut, tapped hole, or threaded insert that is at least as strong as the bolt. (See the hardware specification for minimum strength.)
- Bolted and riveted shear joints: Design them to be bearing critical.
  - See Sec. 4
- Welded aluminum joints: Use full-penetration welds.
  - If the weld is of high quality, the material near the weld (in the heat-affected zone) will yield before the weld ruptures.

Methods of Attaching Panels

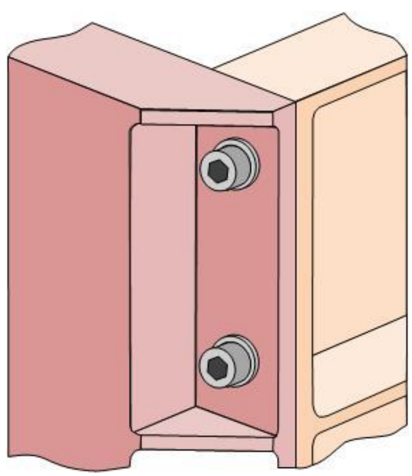
Recognize that panel edges transfer shear loads



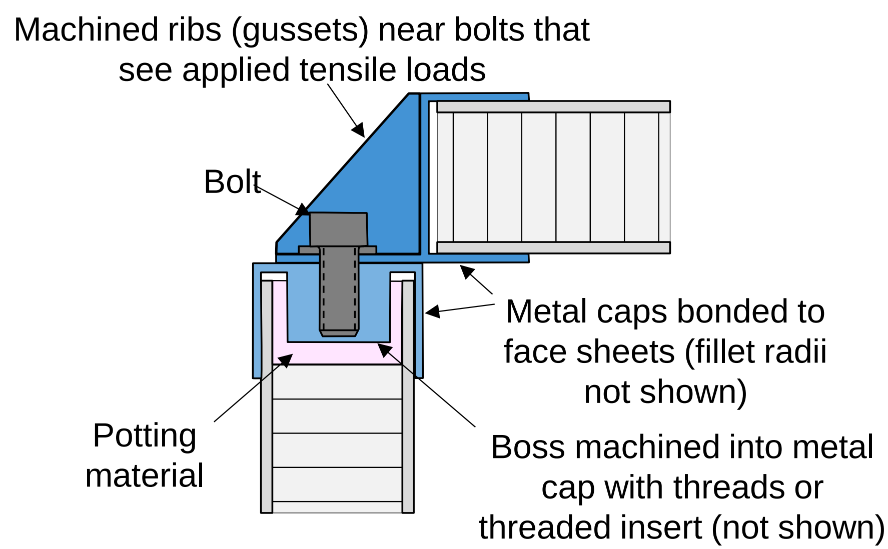
So don't design attachments that look like this:



Make them more like this instead:



Machined metal panels



Sandwich panels

Design Structures to Minimize Joints!

When disassembly is not required,
the best way to avoid problems with joints is to not have joints.

How?

- Direct load paths; fewer structural members
- One-piece construction
  – Integral machining
  – Additive manufacturing (3D printing)
  – Investment casting
- Forms of construction that can mount equipment without structural adapters
  – Machined isogrid (shown above right)
  – Orthogrid (machined waffle pattern)

Rideshare Adapter isogrid base plate

A structure with fewer joints, hence fewer parts, is usually ...

- **less costly**
- **more predictable**
- **more trouble-free**

From my course "Design and Analysis of Bolted Joints" (DABJ)

Often not recognized:
To Reduce Cost and Schedule Time, Reduce the Number of Parts!

Fewer parts (hence fewer joints) leads to less…

- **documentation and configuration control**
- **tooling**
- **assembly time**
- **analysis time**
- **risk**

In 1967, the McDonnell Douglas structural design team for the Skylab space station decided the Skylab decks should be made of machined isogrid panels.

- Provided a versatile pegboard for attachment; the team had no way of knowing what all would eventually be mounted to the decks.
- Open isogrid allowed cables to pass through without having to modify the deck structure.

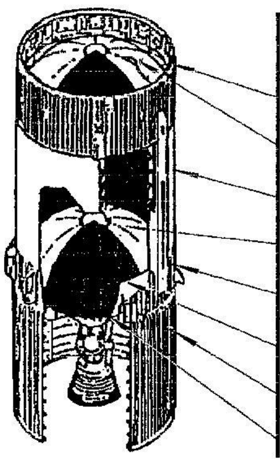
Image credit: NASA

Unfortunately, the common wisdom then was that manually machining such panels and scrapping the metal chips was far more expensive than making structures out of assemblies of many cheap parts.

A key Skylab-team design engineer, Ollie Harwood, suspected this mindset was wrong …

Justifying the First Use of Isogrid for Spaceflight Structures

To justify the use of machined isogrid panels for Skylab decks, the design team conducted a cost study in 1967 for the Saturn S-IVB boost stage:



SUBASSEMBLY	TYPE OF CONSTRUCTION	COST RATIO (MFG HRS/ LB)
FORWARD SKIRT	SKIN, STRINGER & FRAMES	3.8
FORWARD DOME	WELDED, FORMED & CHEM-MILLED GORES	1.8
TANK CYLINDER	WELDED MACHINED WAFFLE PANELS	1.0
COMMON BULKHD	HONEYCOMB SANDWICH - WELDED GORE FACES	3.3
AFT SKIRT	SKIN, STRINGER & FRAMES	4.3
LOX TANK DOME	WELDED, FORMED & CHEM-MILLED GORES	1.8
INTERSTAGE	SKIN, STRINGER & FRAMES	1.4
THRUST STRUCTURE	SKIN, STRINGER & FRAMES	4.9

Cost ratio is based on manufacturing dollars per pound, normalized to the lowest-cost assembly.

From O.P. (Ollie) Harwood, "Right for Flight: The Structural and Architectural Design of Machines that Fly"

The assemblies that cost the least per pound were those that had the fewest parts.

"In areas where leakage of hydrogen was not a problem—the skirts, for instance—it was decided, for economic reasons, to fall back on the familiar 'inexpensive' sheet metal skin-stringer-frame construction. That's why the reversed results were surprising. How could anyone in the design profession so completely miss the target? The answer, of course, is that with enough ignorance anything is possible."

Ollie Harwood, from "Right for Flight"

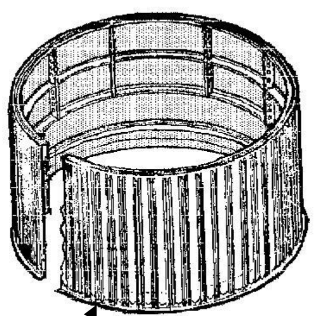
This is old data, but the cost difference would be even greater today.

Since 1967 the cost of machining has gone down while the cost of labor for assembly has greatly increased.

The Cost of Parts

The "after-the-fact" cost study on the Saturn S-IVB forward skirt was expanded to compare expected costs of alternate designs.

CONVENTIONAL SKIN, STRINGER, & FRAME



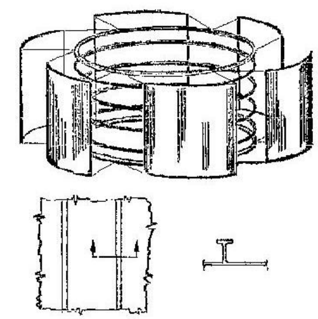
- Extruded Hat Section Stiffeners
- 3 Channel Frames with Stabilizing Intercostals
- One Extruded Hat Frame
- Extruded End Attach Angles

527 PARTS

COST RATIO 2.9

As built

STRINGERS INTEGRAL WITH SKIN

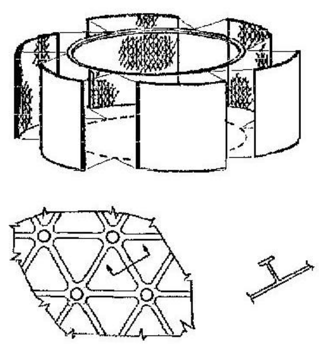


- Stringers Machined Integral with 6 Premium Sized Panels
- Intermediate Frames Rolled Standard Hat Sections
- Extruded End Attach Angles

103 PARTS

COST RATIO 1.2

INTEGRAL ISOGRID WAFFLE



- 6 Bolted Plates Stiffened with Triangular Pattern Waffle
- 2-piece Frame on Free End
- Integral End Attachment

10 PARTS

COST RATIO 1.0

From O.P.Harwood, "Right for Flight: The Structural and Architectural Design of Machines that Fly"

A composite skirt was also considered in the study but was slightly heavier than the isogrid design at a cost ratio of 9.3.

"Not a very good investment for no performance gain"

Ollie Harwood, from "Right for Flight"

More on the Saturn S-IVB Cost Study for Alternate Designs

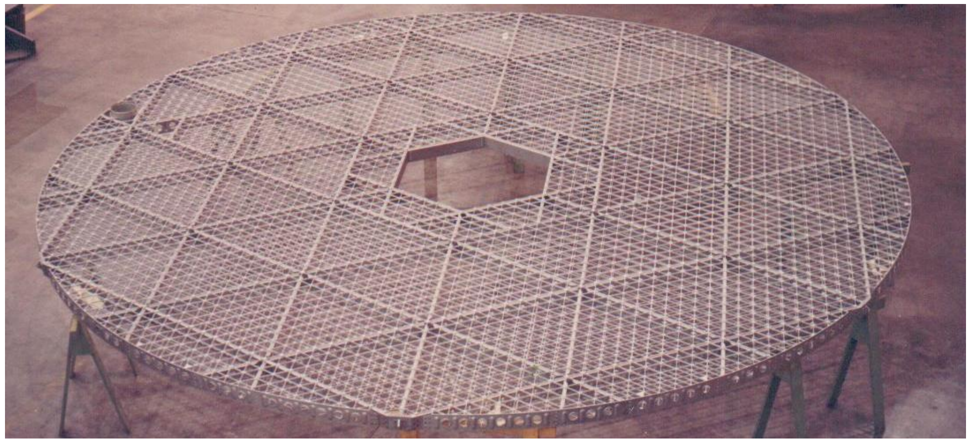
Regarding the study summarized on the previous slide,

"This study is somewhat suspect because, in the first place, it was just a study, involving no real hardware. Also, the hardware description was simplified to make the cost analysis more manageable.

"All the operations to fabricate parts were identified and analyzed in paralyzing detail. However, assembly time, much more difficult to define, was estimated as a percentage of fabrication time, though sensible thinking suggests that the efficiency of assembly for 527 parts (or 103) is noticeably less than for 10. Likewise, the interaction with other subsystems and associated changes were ignored for convenience, though this cost, as the original investigation suggests, would adversely influence a real program. In any case, the study, for all its simplifications and shortcomings, was one of the better efforts along this line and managed to confirm the adverse effect of higher part count."

Ollie Harwood, from "Right for Flight"

First Use of Machined Isogrid for for Spaceflight Structures: Skylab Decks



"Though it may not have been recognized at the time, this pattern was physically integrating all the subsystems that found their way into the Orbital Workshop. Not only that, but the manufacturing method, machining, was roundly criticized as an expensive way of doing business.

"Justifying the choice of 'expensive' machining led to some illuminating discoveries about the real costs of making aerospace structure, including the revelation that an assembly of individually simple and inexpensive parts can still cost much more than a large, fully machined unit which incorporates them all.

"The investigation helped to confirm, as suspected, that the cost of structure, or any other manufactured product, is greatly influenced by the number of parts in it. In low production quantities, those typical of aerospace contracts, this is particularly the case. The phenomenon seems quite independent of the manufacturing process. One of the main effects of reducing part-count is similar reduction of coordination effort, confusion (with its adverse effect on learning), and, most importantly, documentation."

Ollie Harwood, from "Right for Flight"

Isogrid Skylab Deck

Photo taken at the Smithsonian's Air and Space Museum, 2019

For design and analysis of isogrid, see Ref. 34 (Isogrid Design Handbook).

(written by Ollie and other McDonnell Douglas engineers, based mainly on experience from the Skylab program)

Overcoming the Inertia of Misconception

There is (or at least used to be) widespread belief that sheet-metal construction, with many parts that are individually cheap to fabricate, is economical.

As a young engineer in the 1980s, I remember the adage spread by senior designers at Martin Marietta in response to anyone suggesting integral machined construction, for which most of the material would be "wasted":

~~**"Paper is cheap; metal is expensive"**~~

The message of this saying was not true then, with manual machining, and it's much less true today, with CNC machining. The cost of labor—documenting, analyzing, assembling, inspecting—and tooling for assembly usually overwhelm the cost of raw material.

Instead, minimize parts!

"One's 'intuition' about low-production costs is not to be trusted. Nothing could be more misleading than the assumption that what an individual produces in a backyard or home 'garage' project would be economical for aerospace. The missing ingredient is the cost of labor, which the home builder donates to the project while he scrounges to minimize the cost of material.

"An aerospace company has the exact opposite point of view; labor must be paid for even if workers put in no more than their attendance, while material is relatively cheap."

Ollie Harwood, "Right for Flight"

A Common Dilemma

Typical process for a flight vehicle: Design and release drawings for the primary structure first.

– The structure has to be built and tested early because everything builds off of the structure.

– Form of construction selected based on a trade study, considering cost and weight

– Designed to accommodate the known items on the master equipment list (MEL)

– But MELs tend to grow: we end up adding stuff that no one thought about!

Result: a proliferation of additional parts and structural modifications

– Adapters for mounting components

– Inserts in sandwich panels

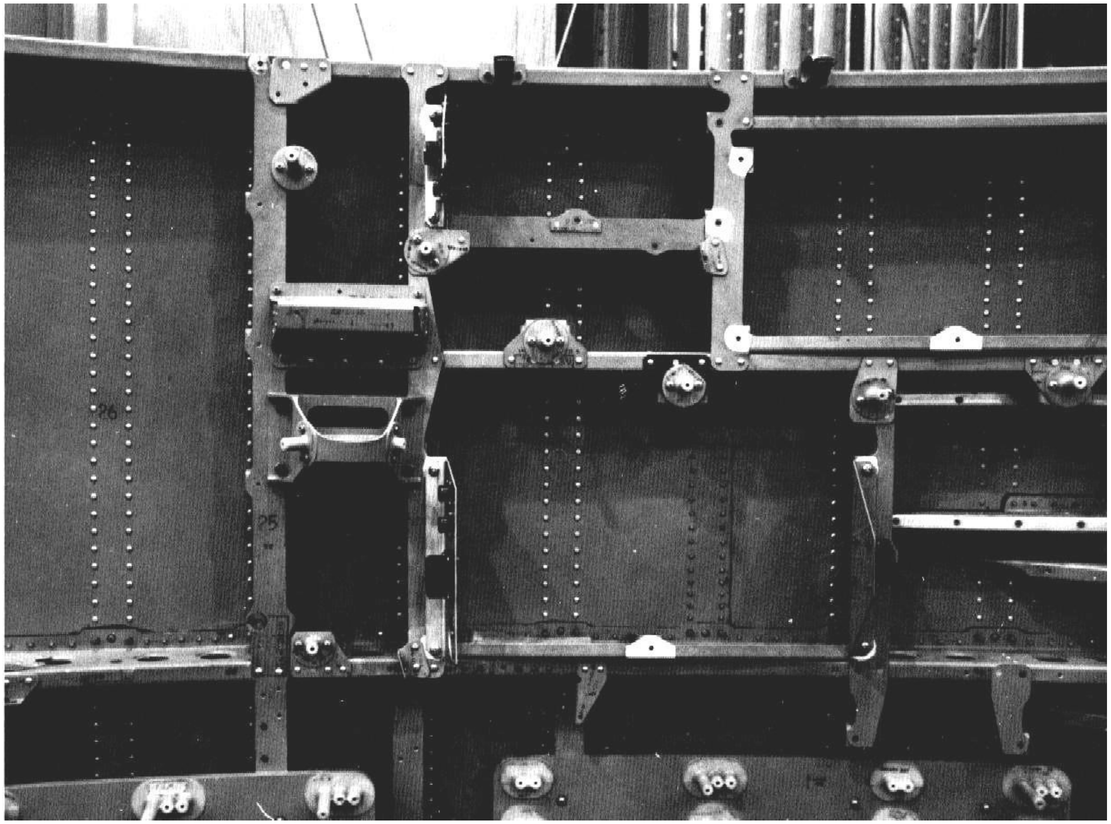
– Cutouts and local reinforcement

– All these things add cost and weight

"The structure envisioned when original choices were made bears little resemblance to the final result, negating the validity of 'trade studies'."

Ollie Harwood, from "Right for Flight"

Example: S-IVB Aft Skirt



Example of the rework necessary to mount equipment in a skin-stringer structure. The rows of rivets attach external stringers.

Design an Adaptable Structure

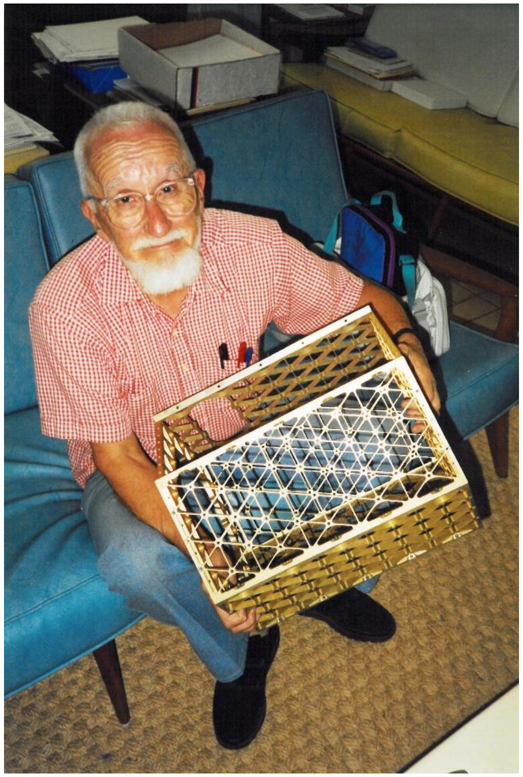
Referring to the common practice of modifying a structure after design release or after the structure is built in order to accommodate subsystems and equipment:

"The adjustment just described passes for 'design integration', an activity always involving the structure. 'Integration', however, is the wrong term. It is actually the patchwork created by forgetting or deferring ***real integration*** *at the beginning. Such integration must* ***start*** *with the structure."*

*"**Integration** and **adaptability** are of primary importance in the effective initial design of structure. In fact, it can be stated that **adaptation** is that which must be done when adaptability is missing."*

"Anticipate potential change with standard features and patterns."

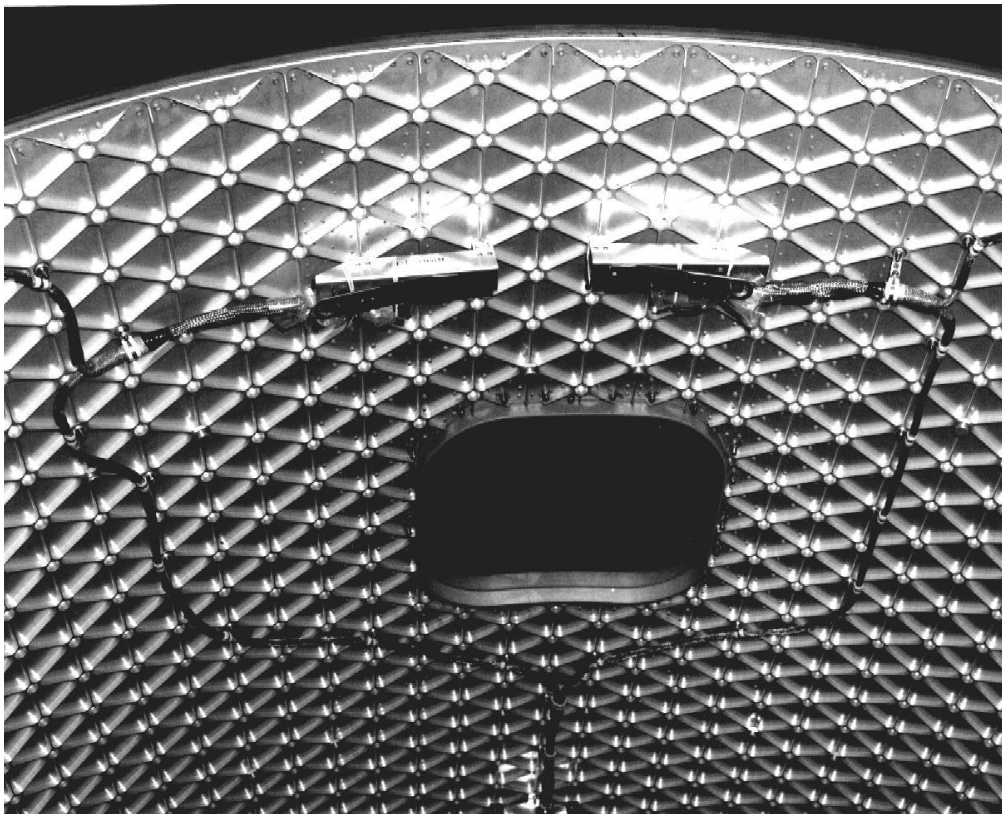
Ollie Harwood, from "Right for Flight"



Ollie with isogrid SmallSat bus structure, 1993

Example of an Adaptable Structure

Payload fairing for the Delta launch vehicle



An integrally machined isogrid structure provides an attachment point at each node—a pegboard—allowing the structure to adapt to late-added components.

I Listened to Ollie: Another Adaptable Structure

A structure used to support small space payloads during launch.

It sits between the primary payload and the launch vehicle's standard payload interface.

Rideshare Adapter (I steered the design for Design Net Engineering)

- About 22" tall
- 38.81"-dia bolt circle
- 6061-T6 aluminum alloy; conversion coating
- Weight: ~80 lb
- 8 parts
- 3 parts of different design
- Fabrication cost: ~ $35K in 2007
- 144 identical cap screws and Helicoils®
- Assembled in about 3 hours

Summary:
Structural Design Guidelines from Ollie Harwood

- Minimize parts. (Use judgment here; clearly there are limitations to this guidance.)
  - Make large parts rather than small parts.
  - Incorporate in the large parts any needed fittings and local reinforcements.
- Avoid materials that cannot produce fittings.
  - Ollie was not a fan of composites in applications when aluminum would do the job. He liked to remind people that a composite laminate is itself an assembly of individual parts and is thus labor intensive.
- Design an adaptable structure. Anticipate potential change with standard features and patterns.
- Minimize tooling with accurate, self-indexing parts.
  - Example: The machined pieces of the Skylab workshop floor (deck) fit together accurately and could be assembled in place without tooling.

continued

Summary:
Structural Design Guidelines from Ollie Harwood (continued)

- Anticipate the need for access with open structure.
  - *"If the openings aren't there at first, eventually they will be. As unplanned afterthoughts, they destroy the efficiency (assumed in trade studies)."*
- Modularize—minimize parts of different design.
- Triangulate.
  - The triangle is the only stable polygon, not needing skin.
  - Open triangles are penetrable and less at risk of being modified.

Summary:
A Closing Quote from Ollie

"Aerospace vehicles in general could be vastly improved if structural designers stopped resenting the intrusion of other subsystems, and worked at anticipating their needs. It would benefit both the structure and the subsystems living with it, minimizing argument time, and making the vehicle lighter and more economical.

"Provincialism within design 'disciplines' was demonstrated at a recent kick-off meeting for new program … (summer of 1988). A manager of structural design rose to proclaim, 'We will not compromise our structure an inch to accommodate avionics!' The industry teems with such defenders of lost causes. In each case, the structure loses!"

Summary

Key Messages of This Course

- Before designing a structure, make sure you identify and understand its requirements.
- Recognize the difference between requirements and criteria.
  - True requirements relate to functions and constraints.
  - As the team designing the structure, adopt criteria that help ensure structural integrity. Don't wait for your customer to specify the criteria as requirements.
  - But make sure the criteria are acceptable to your customer and other stakeholders.
- Take time to draw free-body diagrams and understand how the structure carries loads.
- Mentally trace load paths to envision possible failure modes.
- Learn to appreciate the benefits of ductility, and design it into the structure.

continued

Key Messages of This Course, continued

- For ultimate-strength analysis of structures made of ductile materials, recognize the limitations of FEA, especially linear-elastic FEA. Seek and use empirical or semi-empirical methods instead.
- In buckling analysis, account for inelastic effects, local buckling, and dimensional tolerances. Don't rely solely on FEA.
- Watch for situations that may lead to fatigue failure, and take the steps necessary to prevent it.
  – Positive margins of safety for strength do not preclude fatigue failure.
  – Ensuring the materials won't fail in fatigue is part of the job of ensuring structural integrity, regardless of whether your customer has levied a requirement for fatigue or fracture mechanics analysis.
- Develop simple fatigue screening criteria for use in the design process; there won't be time then for detailed fatigue analysis.
  – Based on simple, conservative loading spectra.
  – Fine tune the loading spectra to remove excess conservatism as needed throughout the program.

continued

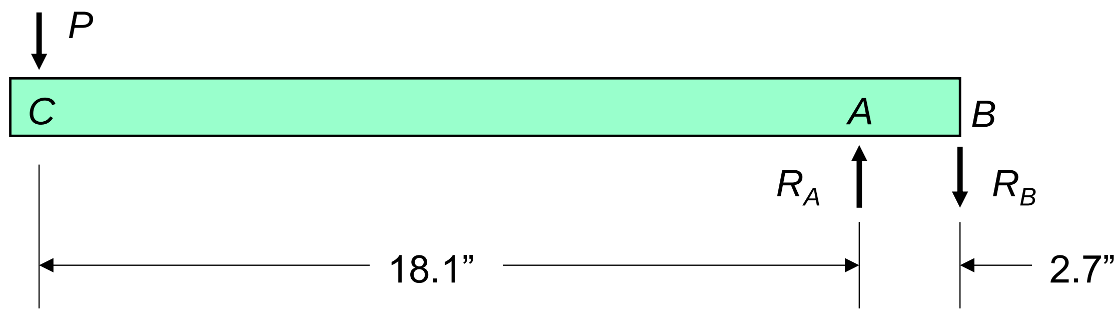
Key Messages of This Course, continued

- Use structural analysis as part of the design process.
- Stop analyzing bad designs! Work as a team to improve your designs!
- Design bolted joints to have …
  – high, (nearly) linear stiffness up to the maximum expected (limit) load
  – ductile ultimate failure
  – high preload
- Design to minimize parts and the need for joints, within reason.
- Design an adaptable, open structure.

Never stop learning!

Solutions to Class Problems

Solution to Class Problem 2-1



At point C: $\sum M_C = 0 = 18.1R_A - (18.1 + 2.7)R_B$ so $R_B = \dfrac{18.1}{18.1 + 2.7} R_A = 0.8702R_A$

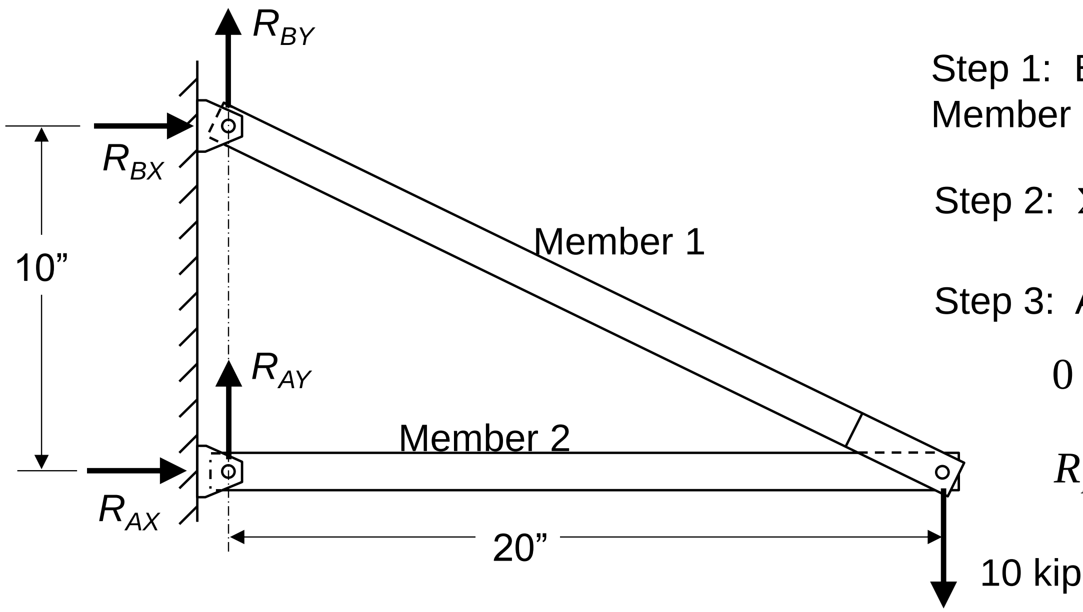
$$\sum F = 0 = -P - R_B + R_A$$

$$= -P - 0.8702R_A + R_A = -P + 0.1298R_A$$

so $R_A = \dfrac{P}{0.1298} = 7.70P$ $R_B = 0.8702R_A = 6.70P$ same as we got before

For a statically determinate problem, there is only one solution.
“Pick a convenient point about which to sum moments” simply means
“use the point that allows the simplest calculations.”

Solution to Class Problem 2-2



Step 1: By inspection, R_{AY} = 0 because Member 2 cannot carry any Y load.

Step 2: ΣF_Y = 0, so R_{BY} = 10 kip

Step 3: At point A, ΣM_Z = 0, so

$$0 = -20(10) - 10R_{BX}$$

$$R_{BX} = -\frac{20(10)}{10} = -20\text{ kip}$$

Step 4: ΣF_X = 0, so R_{AX} = 20 kip

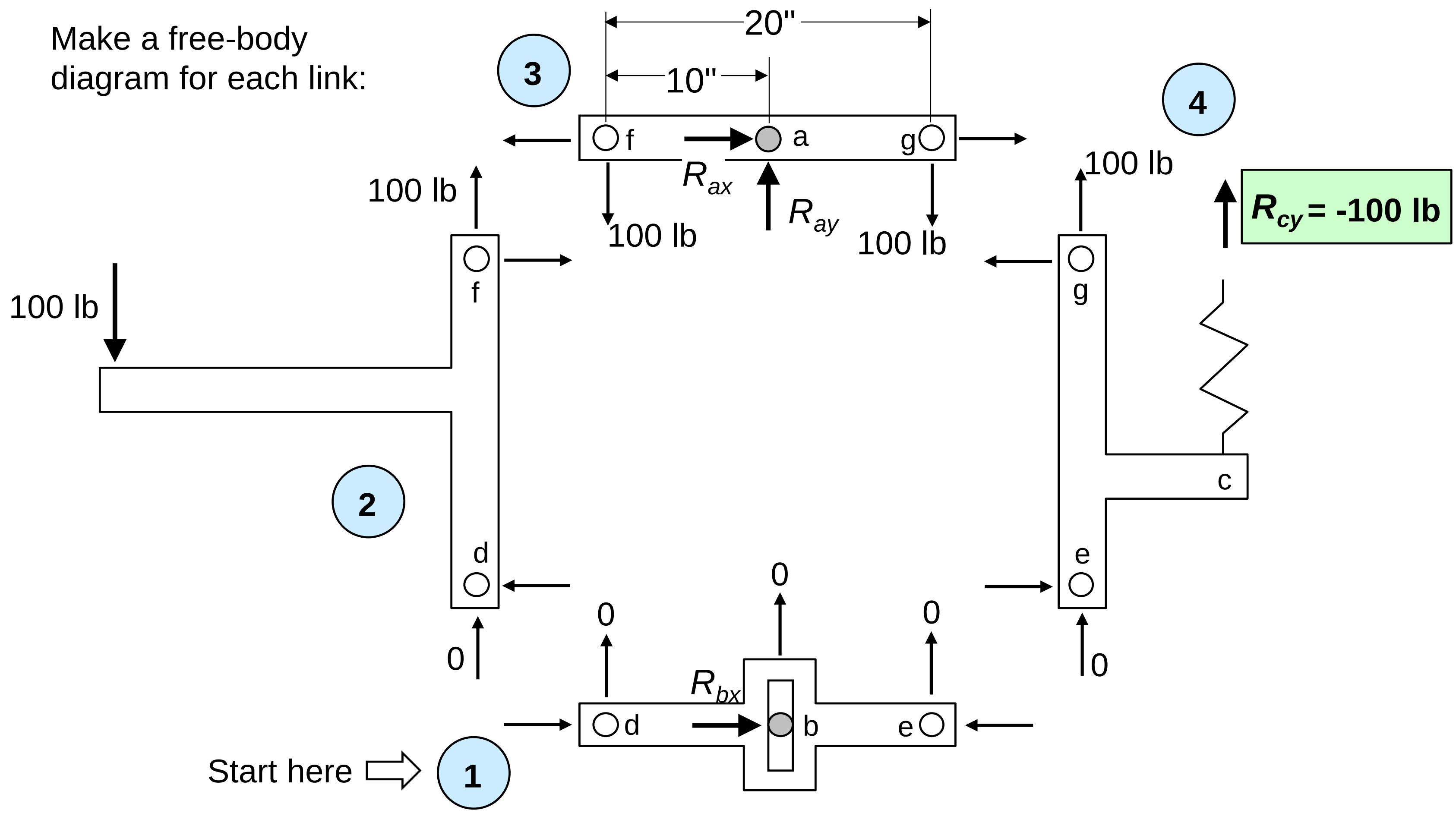
Step 5: At point A, ΣF_X = 0, so the force in Member 2 is F_2 = -20 kip (compression)

Step 6: The force in member 1 must balance reactions at point B, so

$$F_1 = \sqrt{R_{BX}^2 + R_{BY}^2} = \sqrt{(-20)^2 + (10)^2} = 22.4\text{ kip (tension)}$$

Solution to Class Problem 2-3

Make a free-body diagram for each link:



Solution to Class Problem 2-4

a. Fundamental frequency = 79 Hz and damping = 2% of critical

The fundamental frequency is within the range of input frequency. We will assume the sweep rate is slow enough for the instrument to achieve resonance.

$$Q \cong \frac{1}{2\zeta} = \frac{1}{2(0.02)} = 25$$

Peak response acceleration, $a_{max} = 25(2.5) = 62.5g$

b. Fundamental frequency = 112 Hz and damping = 1% of critical

The fundamental frequency is above the range of input frequency. Response will be highest at the maximum input frequency of 100 Hz. From SSAM Eq. 5.30,

$$\frac{f}{f_n} = \frac{100}{112} = 0.893 \qquad TR = \sqrt{\frac{1+[2(0.01)(0.893)]^2}{\left[1-(0.893)^2\right]^2+[2(0.01)(0.893)]^2}} = 4.9$$

Peak response acceleration, $a_{max} = 4.9(2.5) = 12.2g$

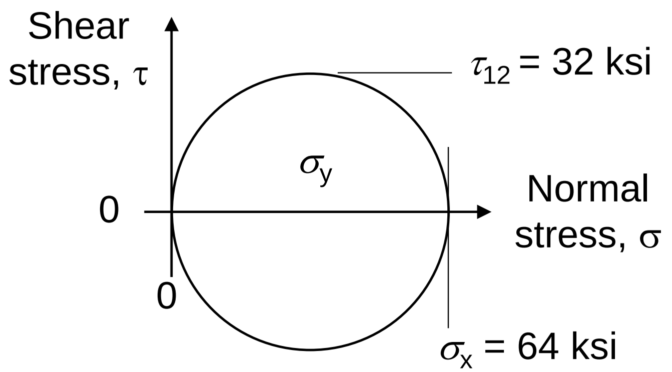
Solution to Class Problem 3-1

a. Calculate the principal stresses at maximum load.

Normal stress, $\sigma_x = \dfrac{P}{A} = \dfrac{48}{0.75} = 64\,\text{ksi}$ $\qquad \sigma_y = 0$ and $\tau_{xy} = 0$

By inspection, the principal normal stresses are 64 ksi and 0 ksi

Principal shear stress, $\tau_{12} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \dfrac{\sigma_x}{2} = 32\,\text{ksi}$



b. At what angle from the applied tensile force did the shear stress peak?

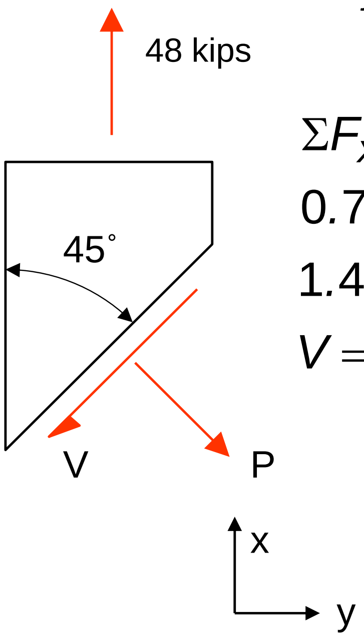
The principal shear stress is always ±45˚ from the direction of principal normal stress.

continued

Solution to Class Problem 3-1 (continued)

c. Draw a free-body diagram that shows the applied tensile force at one end and, at the other end, the forces acting on the surface on which the principal shear stress acts. Calculate the shear and normal forces on that surface needed for static equilibrium.

$$\Sigma F_y = 0 = -V \sin 45^\circ + P \cos 45^\circ$$



Therefore, $V = P$

$$\Sigma F_x = 0 = 48 - V \cos 45^\circ - P \sin 45^\circ$$

$$0.707V + 0.707P = 48\,\text{kip}$$

$$1.414V = 48\,\text{kip}$$

$$V = 33.9\,\text{kip}$$

d. Calculate the shear stress and see if it agrees with the principal shear stress calculated in part a.

The tensile area is given as 0.75 in^2. The force V acts over a surface area equal to

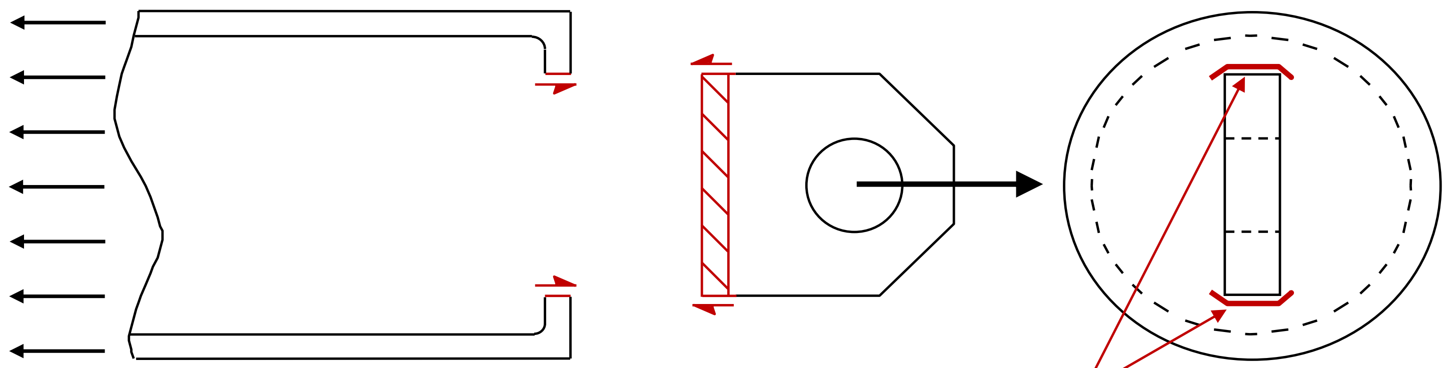
$$A = 0.75 / \sin 45^\circ = 1.061\,\text{in}^2$$

Making the shear stress $\tau = V / A = 33.9 / 1.061 = 32\,\text{ksi}$

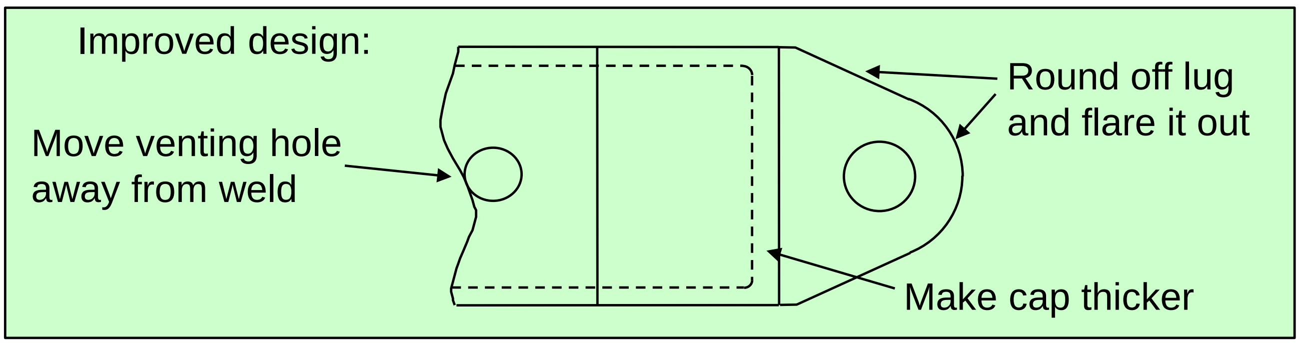
same as we got in part a

Class Problem 4-1: Recognizing Potential Failure Modes

The end cap of the machined fitting failed in shear.

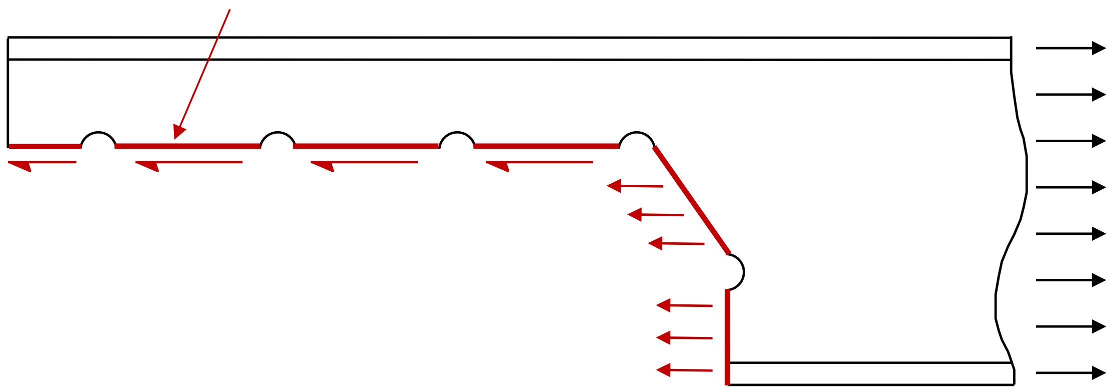


Failure started where stress concentrated at the stiffest load path and then propagated around the lug.

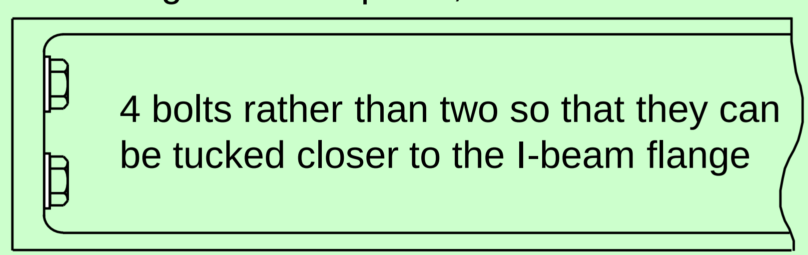


Problem 4-2

The web of the I-beam failed in shear through the upper line of fasteners. Failure then propagated down.



Improved design: All one piece, machined out of bar stock



Problem 4-3: Collapse of Hyatt Regency Walkways

Kansas City, Missouri, 1981

Image source: Dr. Lee Lowery, Jr., P.E. *Wikimedia Commons*

Image source: "Hyatt Regency Skywalk Collapse Remembered," by Randall P. Berhardt, August, 2016

114 people killed

180 injured

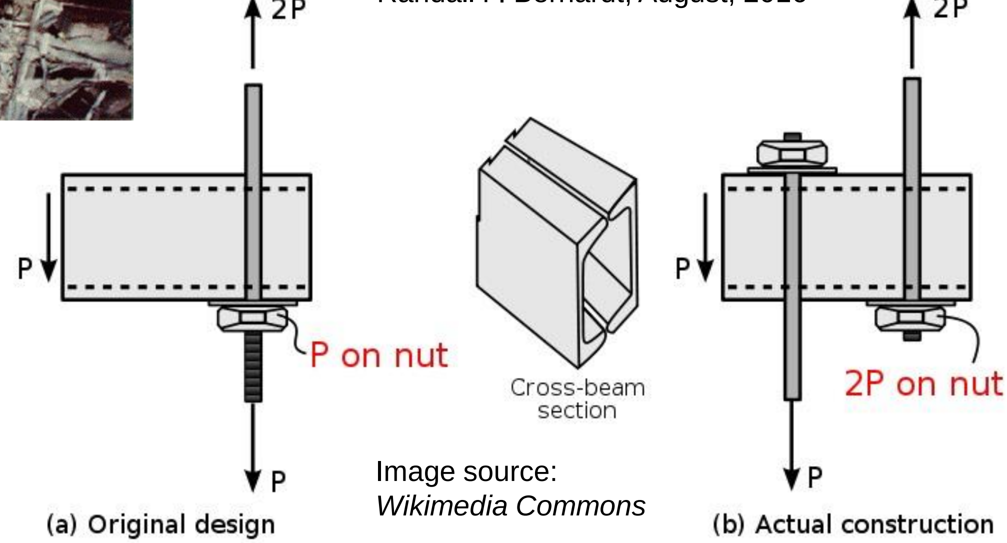


Image source: *Wikimedia Commons*

Solution to Class Problem 4-4, Eccentric Column

Given:

Limit compressive load, P_c = 37 kip
Ultimate factor of safety, FS_u = 1.4

From the inelastic-column Example Problem 4-e,

Section and material properties:

A = 2.69 in^2
I = 0.575 in^4
E = 10,100 ksi

Inelastic column critical stress and load:

F_{cr} = 29.7 ksi
P_{cr} = 79.9 kip

Allowable ultimate compressive stress, F_{cu} = 28 ksi

continued

Solution to Class Problem 4-4, Eccentric Column, continued

Given an allowable compressive stress of 28 ksi, here are three ways to work this problem:

1. Compute the stress, f_c, caused by the limit load P_c = 37 kip. Multiply that stress by the 1.4 factor of safety and compute the margin of safety by

$$MS_u = \frac{28}{1.4(f_c)} - 1$$

2. Compute the stress, f_{cu}, caused by the design ultimate load P_{cu} = 1.4(37) = 51.8 kip. Compute the margin of safety by

$$MS_u = \frac{28}{f_{cu}} - 1$$

3. Compute the allowable ultimate load, $P_{cu\text{-allow}}$, which causes a peak stress of 28 ksi. Compute the margin of safety by

$$MS_u = \frac{P_{cu-\mathrm{allow}}}{1.4(37)} - 1$$

Because stress is not proportional to the applied load here, these methods will give different results.

Which method provides the most meaningful margin of safety?

continued

Solution to Class Problem 4-4, Eccentric Column, continued

Equations used in each method (with P_c in kip units, f_c in ksi, and δ in inches):

Eq. 4.11 $\Rightarrow$ $$f_c = \frac{P_c}{A} + \frac{P_c \delta c}{I} = \frac{P_c}{2.69} + \frac{P_c \delta (1.85/2)}{0.575}$$

$$\boxed{f_c = P_c\left(0.3717 + 1.609\delta\right)}$$

Eq. 4.12 $\Rightarrow$ $$\delta = e \cdot \sec\left(\frac{L}{2}\sqrt{\frac{P_c}{EI}}\right) = 0.05\left[\sec\left(\frac{25}{2}\right)\sqrt{\frac{P_c}{10100(0.575)}}\right]$$

$$\boxed{\delta = 0.05\sec\left[0.1640\sqrt{P_c}\right]}$$

continued

Solution to Class Problem 4-4, Eccentric Column, continued

Method 1: Compute the stress, f_c, caused by the limit applied load of 37 kip.

For P_c = 37 kip, $\delta = 0.05 \sec\left[0.1640\sqrt{37}\right] = 0.0922$ in

$$f_c = 37\left[0.3717 + 1.609\left(0.0922\right)\right] = 19.3 \text{ ksi}$$

$$MS = \frac{28}{1.4\left(19.3\right)} - 1 = +0.04$$ therefore, okay

As we'll see, this is the wrong conclusion.

continued

Solution to Class Problem 4-4, Eccentric Column, continued

Method 2: Compute the stress, f_{cu}, caused by the ultimate applied load of 1.4(37) = 51.8 kip.

$$\delta = 0.05 \cdot \sec\left(\frac{25}{2}\sqrt{\frac{51.8}{10100(0.575)}}\right) = 0.1314 \text{ in}$$

$$f_c = 51.8\left[0.3717 + 1.609(0.1314)\right] = 30.2 \text{ ksi}$$

$$MS = \frac{28}{30.2} - 1 = -0.07 \quad \text{therefore, no good}$$

This is the right conclusion but the wrong margin of safety.

The applied load does not have to be reduced by 7% to satisfy our criteria.

The margin of safety should tell us how much the applied load can increase (positive margin) or must be reduced (negative margin) to just satisfy the design criteria.

continued

Solution to Class Problem 4-4, Eccentric Column, conclusion

Method 3: Compute the load, $P_{cu\text{-allow}}$, that causes a peak stress of 28 ksi.

Because 28 ksi is the allowable stress, $P_{cu\text{-allow}}$ is the allowable load.

Iterate to find $P_{cu\text{-allow}}$ in kip units:

P_{cu}	δ	f_c
60.0	0.1691	38.6
45.0	0.1103	24.7
50.0	0.1252	28.7
49.2	0.1225	28.0

→ Thus, $P_{cu\text{-allow}} = 49.2$ kip

A 39% reduction in capability (from 79.9 kip in Example Problem 4-1) caused by only a 0.050” misalignment!

The applied load would have to be reduced by 5% to satisfy our criteria.

Calculate the margin of safety based on load rather than stress:

$$MS_u = \frac{P_{cu\text{-allow}}}{1.4P} - 1 = \frac{49.2}{1.4(37)} - 1 = \underline{-0.05}$$

Redesign the strut

The margin of safety should be based on allowable applied load rather than allowable stress

Solution to Class Problem 5-1

Damage calculated from S-N curve for σ_m = 20 ksi

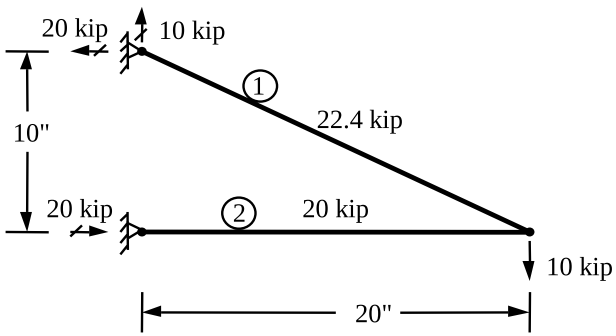
Max stress, ksi	Expected cycles	Design cycles (4X)	*N*	*D*
45.0	500	2000	7500	0.267
35.0	5000	20000	50000	0.400
30.0	50000	200000	500000	0.400
Total damage				**1.067**

Damage calculated using equiv. alternating stress and S-N curve for R = -1 (σ_m = 0)

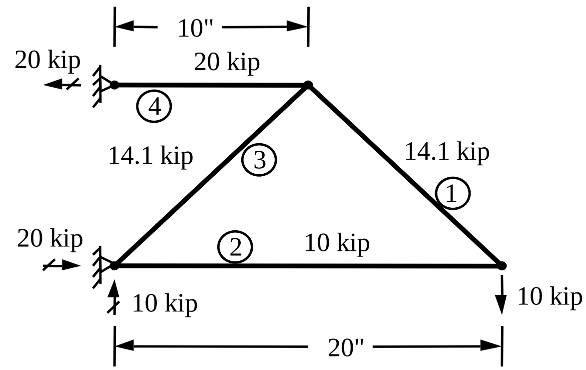
Max stress, ksi	Expected cycles	Design cycles (4X)	σ_a	σ_m	$\sigma_{a\text{-equiv}}$	*N*	*D*
45.0	500	2000	25.0	20	32.35	8500	0.235
35.0	5000	20000	15.0	20	19.41	120000	0.167
30.0	50000	200000	10.0	20	12.94	10000000	0.020
Total damage							**0.422**

Conclusion: The use of equivalent alternate stress per the Goodman method is unsafe in this case.

Solution to Class Problem 6-1: Arranging Truss Members



B



F

Mem-ber, i	Load, P_i	Length, L_i	$P_i\,L_i$	Req'd Area, A_i	$\frac{P_i^2 L_i}{2A_iE_i}$
1	22.4	22.4	500	0.448	1.25
2	20.0	20.0	400	0.400	1.00
Σ			900		2.25

Mem-ber, i	Load, P_i	Length, L_i	$P_i\,L_i$	Req'd Area, A_i	$\frac{P_i^2 L_i}{2A_iE_i}$
1	14.1	14.1	200	0.283	0.500
2	10.0	20.0	200	0.200	0.500
3	14.1	14.1	200	0.283	0.500
4	20.0	10.0	200	0.400	0.500
Σ			800		2.00

Proportional to weight

Inversely proportional to stiffness
(for a given applied load, *P*)

Truss F would be both lighter and stiffer than truss B.
45° angles are most efficient.

SDA Index

November 2024

Instar's Courses, Taught by Tom Sarafin

We often customize courses to meet client needs

We now offer these courses as live, virtual webinars in addition to in-person courses at customer facilities.

These courses are jointly offered by Instar Engineering and Consulting, Inc., and Applied Technology Institute.

Website: instarengineering.com
Contact: tom.sarafin@instarengineering.com

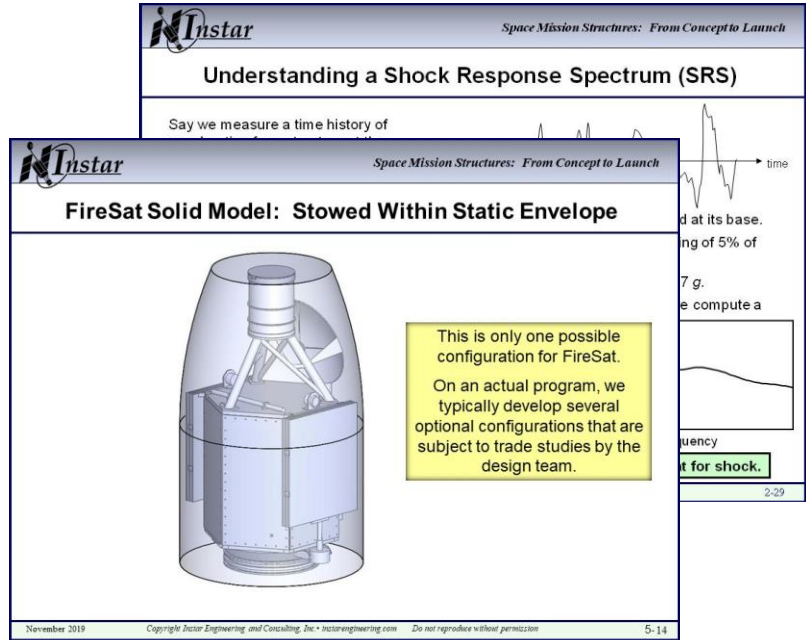
Website: aticourses.com
Contact: ati@aticourses.com

SMS—Space Mission Structures
From Concept to Launch

- Presents the structure for a space vehicle as a system
- Objectives: improve your understanding of ...
  - Structural functions, requirements, and environments
  - How structures behave and how they fail
  - How to develop structures that are cost-effective and dependable for space missions
  - Structural verification
- Target audience:
  - Systems engineers
  - Structural and mechanical design engineers
  - Stress analysts
  - Dynamics and loads engineers
  - Anyone else with an engineering degree interested in the subject
- Course length: 3 full days

SMS Topics

1. Overview of space mission structures
2. Launch environments and how structures respond
3. Assessing structural integrity: stress analysis
4. Overview of finite element analysis
5. Configuration development and preliminary structural design
6. Improving the loads-cycle process
7. Verification and quality assurance
8. Final verification and risk assessment



"Excellent presentation—a reminder of how much fun engineering can be."

"This is a great class presented with real-world applications."

"Many really good examples." "Good stuff, and a very clear presentation."

"I have a much better understanding of structure design issues, especially modeling and its limits. The instructor clearly understands and is effective at communicating this material."

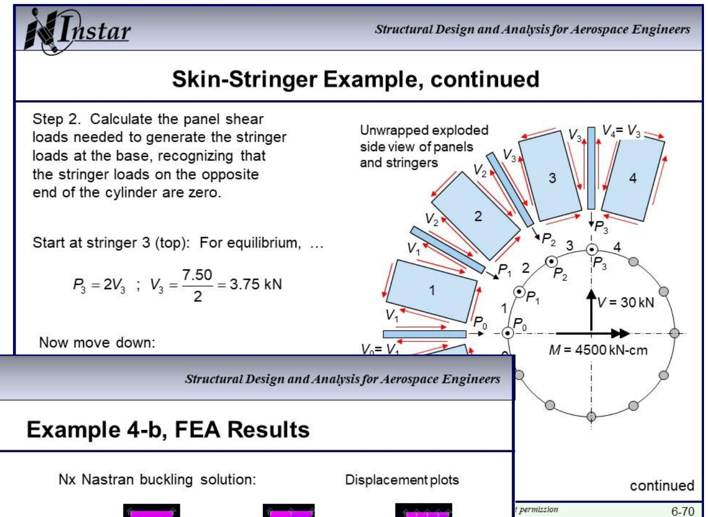
"Great course!"—former Chief Engineer for USBI

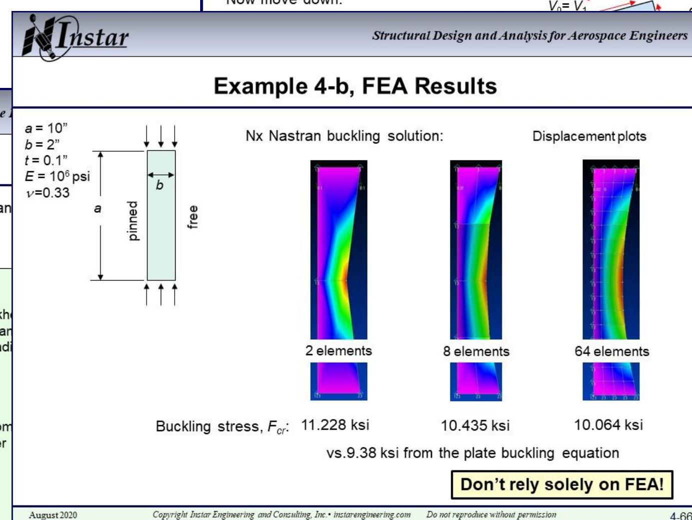
SDA—Structural Design and Analysis
for Aerospace Engineers

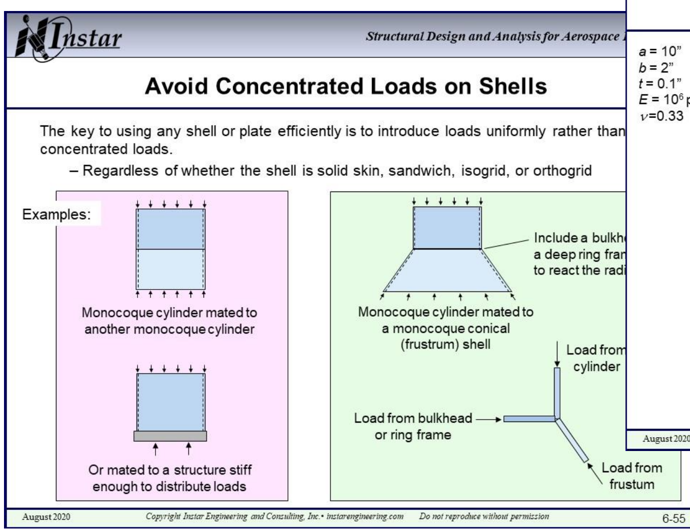
- This course is a companion or follow-on to Instar's flagship course, SMS—Space Mission Structures, from Concept to Launch.
  - SMS gives the big picture of spaceflight structures development.
  - SDA goes into much more detail on design and analysis of aerospace structures and is not specific to spacecraft. Much of the course applies to aircraft as well.
  - Although we recommend people working in the space industry start with SMS and follow with SDA, SMS is not a prerequisite.
- Target audience:
  - Structural and mechanical design engineers
  - Stress analysts
  - Anyone else with an engineering degree interested in the subject
- Course length: 3 full days

SDA Topics

1. Structural requirements and design criteria
2. Review of statics and dynamics
3. Mechanics of materials
4. Strength analysis
5. Fatigue of metals
6. Structural design

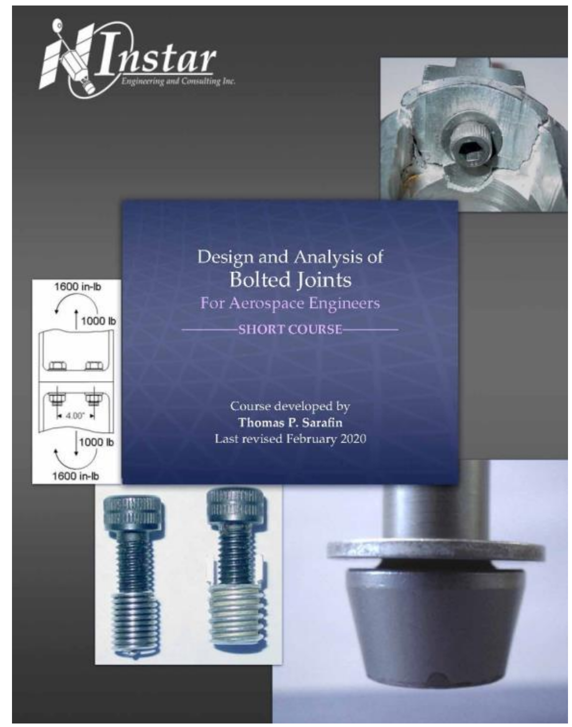






DABJ—Design and Analysis of Bolted Joints

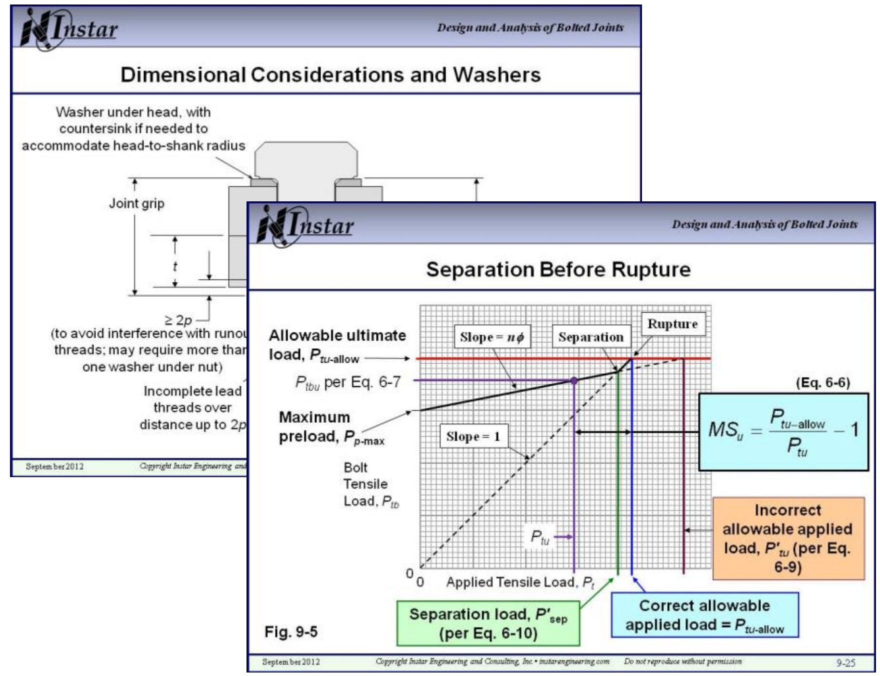
Includes a close look at NASA-STD-5020 (including Revs. A and B), “Requirements for Threaded Fastening Systems in Spaceflight Hardware”



- Objectives:
  - Build an understanding of how bolted joints behave and how they fail
  - Impart effective processes, methods, and standards for design and analysis, drawing on a mix of theory, empirical data, and practical experience
  - Share guidelines, rules of thumb, case histories, and valuable references
- Target audience:
  - Mechanical design engineers
  - Structural analysts
  - Others interested in the subject
- Course length: 3 full days

DABJ Course Topics

1. Overview
2. Screw threads: evolution and important characteristics
3. Developing a concept for the joint
4. Calculating bolt loads when ignoring preload
5. Failure modes and assessment methods
6. Thread stripping and pull-out strength
7. Selecting hardware and detailing the design
8. Mechanics of a preloaded joint under applied tension
9. Fastening system analysis per NASA-STD-5020B
10. Special topics: (a) finite element modeling of bolted joints and (b) deriving design tables for bolt sizing



"Well-researched, well-designed course."

"Interaction between instructor and experienced designers (in class) was <u>priceless</u>."

"Best course I have taken."

"Really good course. More people need to take this."

"Great course! Lots of lessons learned. The examples made it that much better."

"Strong emphasis on understanding physical principles vs. blindly applying textbook formulas."

STDI—Structural Test Design and Interpretation

- This 3-day course provides a rigorous look at structural testing and its roles in product development and verification for aerospace programs.
- The course starts with a broad view of structural verification throughout product development and the roles of testing, and then covers planning, designing, performing, interpreting, and documenting a test structural requirements and flight environments .
- Objectives are to improve your understanding of how to …
  - identify and clearly state test objectives
  - design (or recognize) a test that satisfies the identified objectives while minimizing risk
  - establish pass/fail criteria
  - design the instrumentation
  - interpret test data
  - write a good test plan and a good test report
- Target audience: All engineers and managers involved in ensuring that launch vehicles and their payloads are structurally safe and ready to fly
- Course length: 3 full days

STDI Course Topics

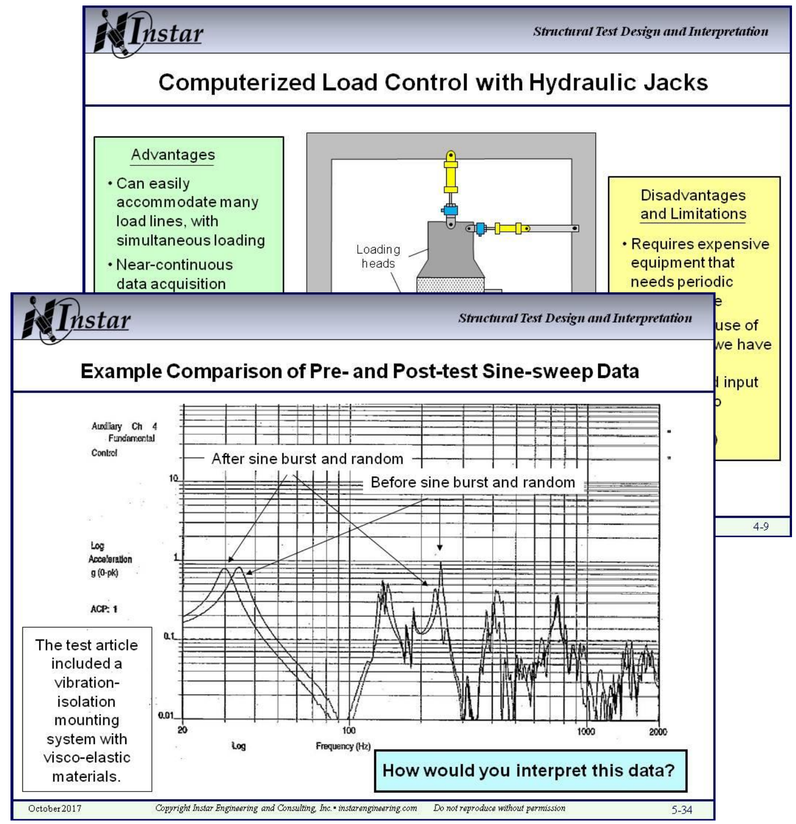
1. Overview of structural testing
2. Designing and documenting a test
3. Loads testing of small specimens
4. Static loads testing of large assemblies
5. Testing on an electrodynamic shaker
6. Notching and force limiting
7. Overview of other types of structural tests
8. Case history: vibration testing of a large spacecraft telescope

"Good job, Tom. These courses are a big help and give us a lot of great fundamental information."

"Tom Sarafin's courses never disappoint. This class offers a well-balanced blend of fundamentals, examples, and lessons learned that any aerospace engineer involved in structural test design and interpretation would benefit from."

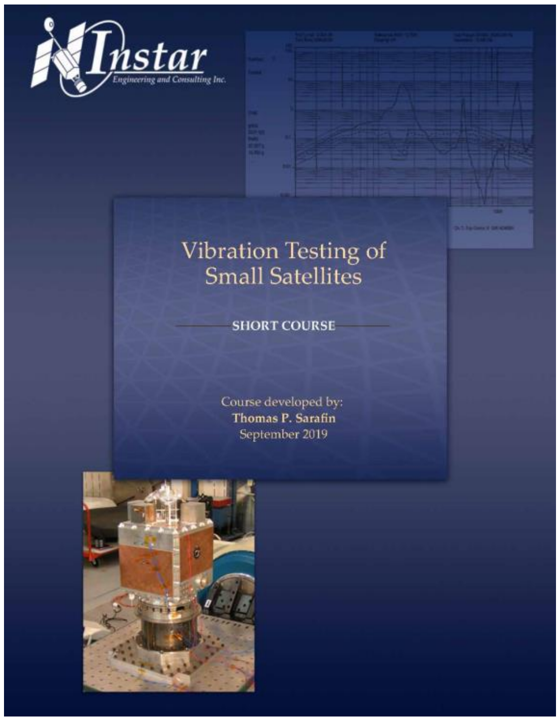
"This is a great course."

Can be combined with a one-day computer workshop on notching and force limiting (NFLW)



VTSS—Vibration Testing of Small Satellites

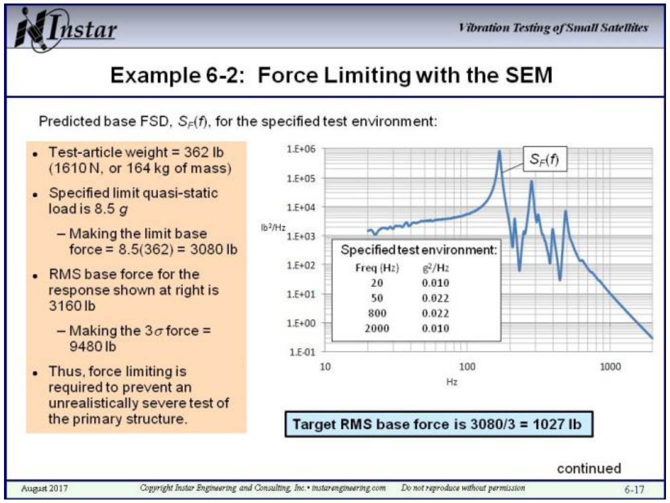
and NFLW—Notching and Force Limiting Workshop



Can be combined with a one-day computer workshop on notching and force limiting (NFLW)

- This 2-day course provides a tutorial, practical guidance, examples, and recommendations for testing a small satellite on an electrodynamic shaker. Addressed are sine-burst testing, random vibration testing, and low-level diagnostic sine sweeps. Notching, response limiting, and force limiting are addressed in detail, with examples.
- The course is primarily aimed at satellites in the 50 – 500 lb (23 – 230 kg) range, but most of it also applies to CubeSats. Most of the guidance applies to larger satellites as well if they will be tested on a shaker.
- The objectives are to improve your understanding of how to …
  - establish an effective vibration test program
  - identify and clearly state test objectives
  - design (or recognize) a test that satisfies the objectives while minimizing risk of an over test
  - establish pass/fail criteria and interpret test data
  - write effective test plans and test reports
- Teacher: Tom Sarafin
- Target audience: All engineers and managers involved in ensuring small spacecraft can withstand launch environments

VTSS Course Topics



1. Overview
2. Test configuration, fixtures, and instrumentation
3. Low-level sine sweep testing
4. Sine burst testing
5. Random vibration testing
6. Notching and force limiting
7. Test documentation and reviews
8. Designing a SmallSat to Withstand the Vibration Test

“Anyone involved with the mechanical structure of a spacecraft or test engineering of ensuring a spacecraft survives launch should take this course – very well taught!”

“This course was an outstanding look at the way fundamentals of vibration illuminate the execution details of a good vibration test. Tom’s teaching style and many years of experience make for a rich and entertaining course in what is a very complicated topic.”

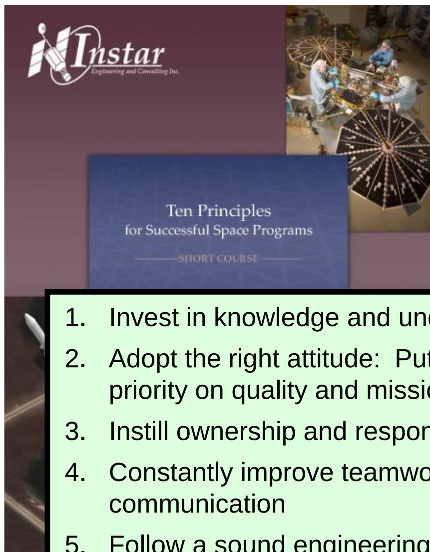
“Whether you’re a novice engineer with little to no experience in vibration testing of a more experienced engineer with vibration testing history, you’ll walk away from this class feeling you’ve learned a lot.”

NFLW—Notching and Force Limiting Workshop

NFLW is a one-day computer workshop that is available only as an optional follow-on to Instar's STDI (Structural Test Design and Interpretation) or VTSS (Vibration Testing of Small Satellites) course, unless the instructor gives permission otherwise.

- The instructor provides in advance several MicroSoft Excel spreadsheets to be used in class. He then guides the class to complete the first spreadsheet by filling in the applicable equations, as covered in STDI and VTSS. The class then uses that spreadsheet to work an example problem three ways, designing force limits, manual notches, and response limits. The class then similarly works two other examples.
- The objectives of this workshop are for you to "learn by doing". The goal is for you to be able to design technically justifiable force limits, manual notches, and response limits for future tests.
- Course length: One 6-hour day

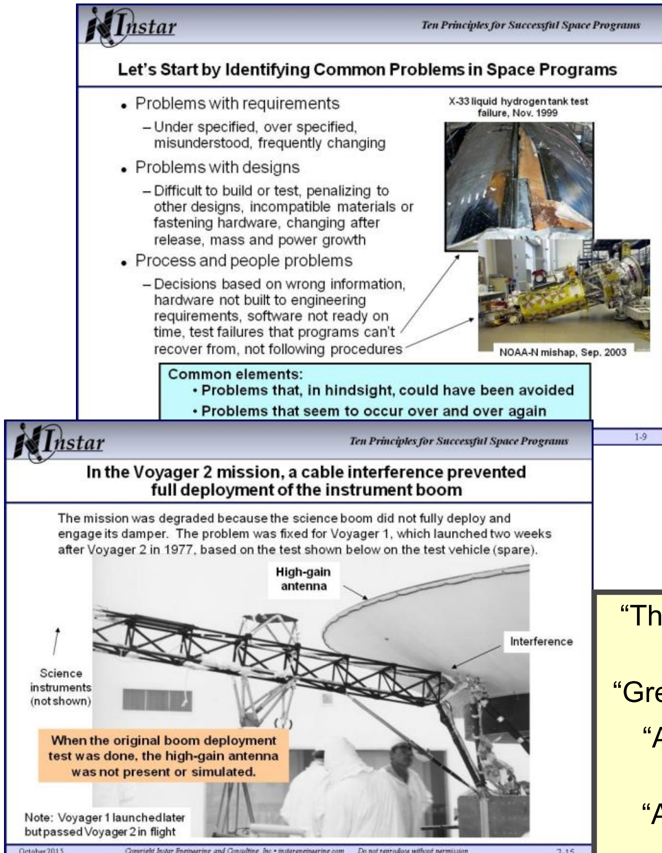
TenP—Ten Principles for Successful Space Programs
for Leaders, Managers, and Consultants



1. Invest in knowledge and understanding
2. Adopt the right attitude: Put consistent priority on quality and mission success
3. Instill ownership and responsibility
4. Constantly improve teamwork and communication
5. Follow a sound engineering approach
6. Reduce cost and risk simultaneously with good engineering
7. Keep everything as simple as possible
8. Establish an effective quality system that involves everyone
9. Be willing to accept risks, but only those you and other stakeholders truly understand
10. Make sure you—and everyone else—have enough time, resources, and freedom to follow the above principles

- This course addresses the most challenging problem in the space industry: How do we reduce cost and schedule time while also trying to ensure a successful mission?
- After examining the driving issues in space-system development, the instructor introduces ten principles for successful space programs.
- The course then explores key aspects of leadership, management, and engineering within the framework of those principles.
- Objectives:
  - Provide a fresh focus on quality and mission success
  - Build understanding
  - Spur thought
  - Help your program improve efficiency of its organization and processes—from the top level of management on down to how every engineer or technician approaches his or her job
- Target audience: all leaders, managers, supervisors, systems engineers, and consultants involved in procuring, specifying, designing, producing, or testing space vehicles
- Course length: 2 or 3 full days

TenP Topics



1. Why Are Space Missions So Challenging?
2. Finding Solutions: Ten Principles
3. Building a Mission Success Culture and an Effective Team
4. Instilling Ownership and Responsibility in Contractors
5. System Development and Requirements Development (3-day course version includes requirements-writing workshop)
6. Reducing Cost and Risk By Design
7. Verification Planning
8. Managing Risk with a Quality System
9. Responsibly Accepting Risk

"This course does a good job of relaying that no process is going to work unless the people implementing it take pride/ownership in making it work."

"Great presentation of how the culture of a space program should be developed."

"All engineers from every field should understand the importance of a quality product as taught by this course."

"Anyone in the space industry or even thinking of being in the space industry needs this class."

"My sincere hope is that the fundamental principles here can take hold to move us forward."